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BRAIDING AND ENTANGLEMENT IN NONABELIAN QUANTUM HALL STATES

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Certain fractional quantum Hall states, including the experimentally observed $\nu = 5/2$ state, and, possibly, the $\nu = 12/5$ state, may have a sufficiently rich form of topological order (i.e. they may be *nonabelian*) to be useful for quantum information processing. For example, in some cases they may be used for topological quantum computation, an intrinsically fault tolerant form of quantum computation which is carried out by braiding the world lines of quasiparticle excitations in 2+1 dimensional space time. Here we briefly review the relevant properties of nonabelian quantum Hall states and discuss some of the methods we have found for finding specific braiding patterns which can be used to carry out universal quantum computation using them. Recent work on one-dimensional chains of interacting quasiparticles in nonabelian states is also reviewed.

Keywords: Quantum computation; Quantum Hall effect; Nonabelian statistics.

1. Introduction

Fractional quantum Hall states are examples of states possessing "topological" order.[1] Such states are not characterized by local order parameters but rather by global properties, notably ground state degeneracies on topologically nontrivial surfaces and fractionalized quasiparticle excitations with exotic quantum statistics. A particularly rich form of topological order, which may be realized in certain experimentally observed fractional quantum Hall states, is so-called "nonabelian" order.[2] In states with nonabelian order, when N fractionalized quasiparticles are present, and their coordinates fixed in space, there is a degeneracy which grows exponentially with N. If these quasiparticles are well separated (on a scale set by the magnetic length) these states cannot be distinguished by local measurements,

making this Hilbert space an ideal place to store quantum information if one wants to protect it from decoherence due to coupling to the environment. If the quasi-particles are then adiabatically dragged around one another, unitary operations are carried out on this Hilbert space and, if the quasiparticles are kept sufficiently far apart, the resulting unitary transformation is expected to depend only on the topology of the space-time "braid" used to carry out the exchange. This built in fault tolerance, together with the topological protection from decoherence, makes nonabelian fractional quantum Hall matter a potentially useful "raw material" for one day building quantum computers.[3,4]

In this proceedings, we review some of our work on topological quantum computation, focusing on the problem of finding specific braiding patterns which can be used to carry out universal quantum computation using nonabelian quasiparticles. We also discuss recent work on models of *interacting* nonabelian quasiparticles. For more details the reader is referred to the original papers, Refs. 5–7, although we note that the work on bidirectional search discussed in Sec. 3 is presented here for the first time.

2. Read-Rezayi States and Topological Hilbert Space

The first proposed nonabelian fractional quantum Hall state was the Moore-Read state[2] — an incompressible state with Landau level filling fraction $\nu = 1/2$. There is compelling evidence[8] that the experimentally observed $\nu = 5/2 \ (= 2 + 1/2)$ fractional quantum Hall state is described by this state. The Moore-Read state is now understood to be the first in an infinite sequence of nonabelian quantum Hall states introduced by Read and Rezayi.[9] This sequence is labeled by an integer index k with the kth state being the exact ground states of a $k + 1$-body Hamiltonian with filling fraction $\nu = k/(k + 2)$. There is some numerical evidence suggesting that the $\nu = 12/5 = 3 - 3/5$ state is described by a $k = 3$ Read-Rezayi state.[9,10] There are also reasons to believe that the bosonic analogs of the Read-Rezayi states (with filling fractions $\nu = k/2$) may be realizable in rotating Bose gases.[11] All this is to suggest that nonabelian states are a very real (albeit rare) part of the natural world.

The quasiparticles in the Read-Rezayi state with index k are described (up to details which are irrelevant for our discussion here) by $SU(2)_k$ Chern-Simons theory,[12] and we will refer to them as $SU(2)_k$ particles. The mathematical description of $SU(2)_k$ particles is similar to that of particles with ordinary spin. This analogy is seen most clearly using the language of quantum groups.[12] In this language, each quasiparticle has a topological charge S which can take any half-integer value from between 0 and $k/2$. The so-called "fusion rules" for this topological charge then take the form

$$S_1 \otimes S_2 = |S_1 - S_2| \oplus \cdots \min(S_1 + S_2; k - (S_1 + S_2)), \tag{1}$$

which, in the limit $k \to \infty$ becomes the usual triangle rule for adding spin. As a particular example, provided $k \geq 2$, for two particles with topological charge $1/2$
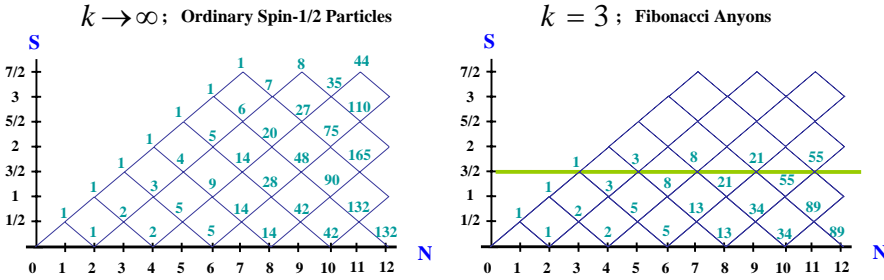
Fig. 1. Bratteli diagrams for $SU(2)_k$ particles with topological charge 1/2. The case $k \to \infty$ corresponds to ordinary spin-1/2 particles. The case $k = 3$ corresponds to quasiparticles in the $k = 3$ Read-Rezayi state.

this rule implies that

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1, \tag{2}$$

which indicates that these particles can be in states with total topological charge 0 or 1 or be in any quantum superposition of the two.

The structure of the Hilbert space of a collection of $SU(2)_k$ particles which each have topological charge 1/2 can be visualized using what are known as Bratteli diagrams. Figure 1 shows Bratteli diagrams for the $k \to \infty$ case (corresponding to ordinary spin-1/2 particles) and for the $k = 3$ case (corresponding to quasiparticles in the $k = 3$ Read-Rezayi state). In these diagrams N is the total number of particles and S their total topological charge. For the ordinary spin case, the branching structure of the diagram reflects the fact that every time one adds a spin-1/2 particle to a collection of particles the total spin of the particles either increases or decreases by 1/2, subject to the constraint that it cannot be less than 0. The numbers labeling the vertices give the number of paths in the diagram from the origin to that vertex and therefore indicate the dimensionality of the Hilbert space of a given number of particles with a given total spin. (Note that we ignore the $2S + 1$-fold degeneracy associated with the total S_z quantum number, a degeneracy which is not present in the nonabelian case.) For the $k = 3$ case, the only difference is that, in addition to the constraint that the total topological charge cannot be less than 0, there is an additional constraint that it cannot be larger than $k/2 = 3/2$. This truncation changes the path counting, as shown in the figure, with the consequence that the Hilbert space degeneracy grows as the Fibonacci sequence $(1,1,2,3,5,8,13,\cdots)$. For large N, this means that for $k = 3$ the Hilbert space dimensionality grows as $\sim \phi^N$ where $\phi = (\sqrt{5} + 1)/2$ is the golden mean. For general k, the Hilbert space dimensionality of N $SU(2)_k$ particles grows asymptotically as d^N where $d = 2\cos(\pi/(k+2))$ is the so-called *quantum dimension* of the topological charge 1/2 particles. (For excellent reviews of the mathematics of nonabelian particles, see Refs. 14, 15.)

The nonabelian statistics of these particles manifests itself when they are adiabatically moved around one another, causing their world lines to form braids in 2+1
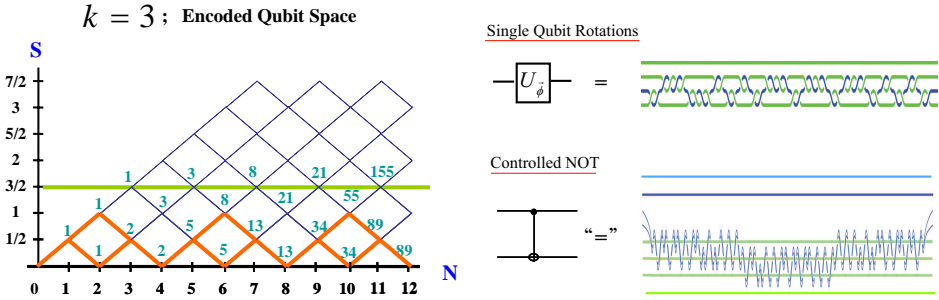
Fig. 2.   Encoded qubit space and braids corresponding to a single qubit operation (here the gate U_ϕ approximates $i\sigma_x$) and a two-qubit controlled-NOT gate.

dimensional space time. Each braid is associated with a precise unitary operation acting on the topological Hilbert space, with the important property that this unitary transformation is exactly the same for any two braids which are topologically equivalent (provided the particles are kept sufficiently far apart during the braiding). This implies a built-in fault tolerance which, as noted above, makes the idea of quantum computing by braiding nonabelian quasiparticles particularly appealing. It has been shown that the braid group representations of $SU(2)_k$ particles are sufficiently rich to carry out arbitrary quantum computation provided $k > 2$ and $k \neq 4$.[13] This includes the case $k = 3$ which, as described above, may be relevant to the experimentally observed $\nu = 12/5$ state.

3. Qubit Encoding and Bidirectional Search for Quantum Gates

The topological Hilbert space described in Sec. 2 does not have the familiar tensor product structure of a collection of independent qubits. To carry out "conventional" quantum computation in this Hilbert space it is therefore necessary to *encode* qubits in a subspace of this Hilbert space. Fig. 2 shows one such encoding scheme in which qubits are encoded using quadruplets of quasiparticles. The effective qubit Hilbert space corresponds to those states whose paths in the Bratteli diagram are indicated by extra thick lines. In this encoding one can see that, starting from the origin, every four particles has total topological charge 0, and has two internal states, which we take to be our qubit states.

In a topological quantum computer, quantum gates (the quantum analogs of classical Boolean logic gates) are carried out by "braiding" the worldlines of nonabelian quasiparticles.[3,4] While there are well defined and easy to implement rules for determining the unitary operation which corresponds to a given braid, it is a much harder problem to find braids which approximate a desired unitary operation. In practice, to find such braids it is necessary to carry out brute force searches over braids up to some given length, looking for those braids which yield unitary operations which are closest to the desired operation. In earlier work on this topic[5,6] it was found that straightforward brute force search could be used to
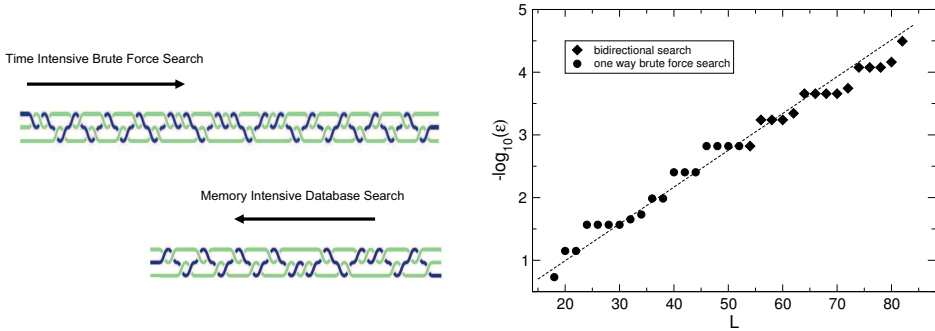
Fig. 3. An "injection" braid (see Ref. 5 for definition) obtained through bidirectional search with an accuracy of a few parts in 10^5 and a plot of $|\ln \epsilon|$ vs. L, where ϵ is the accuracy of the braid and L is the braid length (defined to be the number of elementary interchanges in the braid). In the plot, round dots indicate braids obtained by a straightforward "one-way" brute force search, and diamonds indicate results of a deeper load-balanced "bi-directional" search. The full injection braid is obtained by sewing together the right side of the top segment (obtained through one-way brute force search) with the left side of the bottom segment (obtained through a fast, but memory intensive, database search).

produce braids which approximate desired gates to roughly 1 part in 10^3. A set of quantum gates with roughly this accuracy, obtained by carrying out just such brute force searches,[5,6] are shown in Fig. 2. This set includes a sample single qubit operation, which involves braiding the quasiparticles within a given encoded qubit, and a two-qubit controlled-NOT gate, which involves braiding quasiparticles from two encoded qubits. (The set of all single qubit operations together with a controlled-NOT gate form a universal set of quantum gates which can be used to carry out any quantum algorithm).

While these braids can be improved using "standard" quantum computing ideas, e.g. the Solovay-Kitaev construction,[6] they are still less than ideal. As a reasonable benchmark, it is desirable to have braids which approximate a given target gate to a degree of accuracy below the so-called fault tolerant threshold[16] of 1 part in $\sim 10^5$.

Recently we have significantly improved our brute force search algorithm by implementing a so-called *bidirectional* search. What this means is that, when carrying out a brute force search over braids, we don't just search from the start of the braid working forward toward a desired target gate, but we also search *backward* from the desired target gate. By "load balancing" the algorithm, so that the forward search is time intensive (straightforward exhaustive brute force), while the backward search is memory intensive (database based) we have achieved significantly deeper searches. Figure 3 shows the results of a bidirectional search for a braid we have previously shown is extremely useful for quantum computation — an "injection" braid (a braid which permutes particle positions while carrying out the identity operation on the topological Hilbert space, see Refs. 5, 17 for more details). This braid is accurate to a few parts in 10^5, as can be seen from the plot,

also shown in Fig. 3, of the log of the error ϵ (the deviation of the unitary operation produced by the braid from the desired target gate, measured using operator norm) vs. braid length L. The linear behavior of this plot shows that the braid accuracy improves exponentially with L (reflecting the fact that the number of braids grows exponentially with L) and that it is, indeed, possible to "compile" braids to an accuracy close to the fault tolerant threshold by brute force search. We note that similar bidirectional searches have recently been carried out by Xu and Wan[18] for a related gate construction.

4. Chains of Interacting Nonabelian Quasiparticles

As described in Sec. 2, the key difference between abelian and nonabelian quantum Hall states is the appearance in the latter of an exponentially large Hilbert space of low energy states when localized quasiparticle excitations are present. In the limit that these quasiparticles are well separated these states become energetically degenerate and cannot be distinguished by local measurements. But if the quasiparticles come close enough together (within a few magnetic lengths) this degeneracy is lifted. This lifting of the degeneracy gives the topological Hilbert space *dynamics* which can be described by a Hamiltonian acting on the degrees of freedom associated with the different possible fusion channels of the quasiparticles.

In any realistic fractional quantum Hall experiment there will always be a finite density of localized quasiparticles present. Given this fact, it seems clear that understanding the dynamics of interacting nonabelian quasiparticles will likely be relevant for understanding future experiments on nonabelian states. One can also optimistically hope that learning more about the possible states of interacting nonabelian particles will lead to predictions for experimental signatures of nonabelian order.

As described in Sec. 2, for $k \geq 2$ the fusion rule (1) for $SU(2)_k$ particles implies $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$. This means that, just as for ordinary spin-1/2 particles, when combining two particles with topological charge 1/2 the resulting state can either have topological charge 0 or 1. For ordinary spin-1/2 particles the former would be referred to as a singlet and the latter as a triplet. The same terminology will be used here for $SU(2)_k$ particles, though it should be noted that here there is no S_z degeneracy, i.e. there is only one "triplet" state.

Now consider a one-dimensional chain of $SU(2)_k$ particles. A particularly useful basis for describing the total topological charge 0 sector of the Hilbert space of these particles is the set of non-crossing valence bond states. Examples of these states are the states $|\alpha\rangle$ and $|\beta\rangle$ shown in Fig. 4(a).

For ordinary spin-1/2 particles, the bonds in Fig. 4(a) would represent ordinary singlet bonds. For $SU(2)_k$ particles the interpretation is slightly different. In this case these bonds connect particles that, if brought together, would fuse to trivial topological charge 0. The fact that these states are non-crossing (meaning none of the singlet bonds cross each other) implies that the total topological charge of all
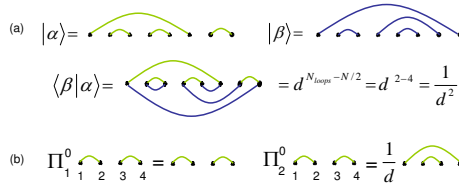
Fig. 4. Two non-crossing valence bond states and their overlap (a); and the action of singlet projection operators on representative valence-bond states (b).

the particles between any two particles connected by a bond is 0. This fact is crucial for generalizing valence-bond states to nonabelian particles — it guarantees that two particles connected by a bond can always be brought together without leading to any "braiding" induced transitions.

Figure 4(a) also illustrates the rule for calculating the overlap of two valence bond states $|\alpha\rangle$ and $|\beta\rangle$. This rule is a simple modification of the well known rule for calculating such overlaps for ordinary spin-1/2 particles. One simply overlays the two valence bond configuration and counts the number of closed loops N_{loops}. For N ordinary spin-1/2 particles the overlap would be $\langle\alpha|\beta\rangle = 2^{N_{\text{loops}}-N/2}$. For $SU(2)_k$ particles the 2 in this expression is replaced by the quantum dimension d (defined in Sec. 2), as shown in Fig. 4(a).

Now consider chains of $SU(2)_k$ quasiparticles in which neighboring pairs of particles are assumed to be close enough together to split the degeneracy of the singlet (topological charge 0) and triplet (topological charge 1) states. If we assume that that the singlet lies lowest, then the Hamiltonian for this system can be written,

$$H = -\sum_i J_i \Pi_i^0; \qquad J_i \geq 0. \tag{3}$$

Here J_i is the singlet-triplet splitting for particles i and $i+1$, and Π_i^0 is a projection operator which projects onto the singlet sector of these two particles. The action of the projection operator Π_i^0 on the valence-bond basis is easily worked out and shown, for two representative cases, in Fig. 4(b).

For the uniform $SU(2)_k$ chain, (3) with $J_i = J > 0$, a great deal is known. In Ref. 19 it was shown that the projection operators Π_i^0 satisfy a Temperley-Lieb algebra.[20] Using this fact it is possible to map these 1+1 dimensional quantum models onto the extreme anisotropic limit of the two-dimensional classical Andrews-Baxter-Forrester "restricted solid on solid" (RSOS) models.[21] The RSOS models are an infinite class of exactly solvable models whose critical properties are known to correspond to the so-called minimal sequence of two-dimensional conformal field theories.[22] Using this mapping Feiguin *et al.*[19] were able to deduce that the ground states of all the uniform $SU(2)_k$ chains were critical, and conformally invariant (i.e. with dynamical exponent $z = 1$) with central charge $c = 1 - 6/((k+2)(k+3))$. Numerical results for the case $k = 3$, the so-called "golden chain", confirmed that, indeed, the ground state was conformally invariant with central charge $c = 7/10$.[19]

In Ref. 7, random chains of the form (3), i.e. chains in which the J_i's are randomly distributed, were studied using the real-space renormalization group (RG) method pioneered by Ma and DasGupta[23] and applied with great success by Fisher to random Heisenberg spin chains[24] and the random transverse field Ising model.[25] In this approach the following steps are iterated: i) search for the bond with the largest J_i; ii) assume that the particles connected by J_i form a singlet, i.e. fuse to 0; iii) remove (or decimate) this bond, leaving behind only an effective interaction (determined using second order perturbation theory) between the two particles on either side of the bond. Once all the particles have been decimated the resulting state will be a particular non-crossing valence bond state known as a random singlet state.[24]

The main result of this analysis was the observation that all the $SU(2)_k$ random chains with antiferromagnetic bonds flow to random singlet states for which the bond strength distribution is the same as that for ordinary spin-1/2 random singlet phases (corresponding to the $k \to \infty$ limit).

5. Entanglement Scaling

In Ref. 7 the scaling of entanglement entropy in random $SU(2)_k$ chains was also studied, motivated by work of Refael and Moore[26] who studied entanglement in ordinary random spin chains and showed that it had universal properties similar to those of conformally invariant quantum critical points.

The entanglement entropy is defined by considering a subsystem (denoted A) of the entire chain consisting of a contiguous segment of L particles. If, starting with the ground state wave function for the entire chain, we trace out the degrees of freedom that are not in region A then we obtain a reduced density matrix ρ_A. The entanglement entropy of region A is then defined to be the Von Neumann entropy of this reduced density matrix ρ_A: $S(L) = -\text{Tr}\,\rho_A \log_2 \rho_A$. Roughly speaking, this quantity tells us how much information we would learn about region A if we measured the state of all the particles outside of region A.

In Ref. 26 it was shown that the entanglement entropy of certain random one-dimensional models at their critical points scales logarithmically, with a universal coefficient. Such logarithmic scaling is known to occur for critical points with conformal symmetry, in which case the universal coefficient is proportional to the central charge of the corresponding conformal field theory.[27,28] Due to this similarity, Refael and Moore proposed using entanglement scaling to generalize the notion of central charge to random 1+1 dimensional quantum critical points which *do not* have conformal symmetry (i.e. for which the dynamic exponent z is not 1, but rather ∞, as it is in the case of the random singlet phase). Furthermore, they made the interesting hypothesis that this *effective* central charge might satisfy a generalized c-theorem[29] meaning that it would always decrease along RG flows. As supporting evidence Refael and Moore showed that this is indeed the case for the RG flow from the uniform Heisenberg spin chain and critical transverse field Ising model to the
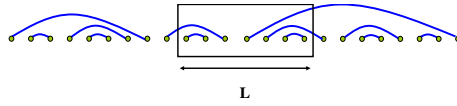
Fig. 5. A random singlet state. By counting the number of bonds leaving the box of length L and multiplying by the entanglement per bond one can determine the entanglement entropy $S(L)$ of the region enclosed by the box.

corresponding random chains. It should, however, be noted that shortly after their work, it was shown by Santachiara[30] that this "theorem" is violated for certain models (to be precise the Z_n parafermionic Potts model for $n \geq 42$). Nonetheless, it is still of interest to investigate cases where this generalized c-theorem might hold. One motivation is that, while the straightforward entanglement scaling may not provide a generalized "central charge" which satisfies a c-theorem, there may exist a closely related quantity which does.

As described in Sec. 4, the bond strength distributions of the infinite randomness fixed points of the random $SU(2)_k$ chains are all identical to that of the random singlet phase for ordinary spin-1/2 particles originally studied by Fisher.[24] It follows that the entanglement entropy of a region of size L in the random $SU(2)_k$ chains can be computed following the procedure of Refael and Moore[26] which we now outline.

Figure 5 shows a particular random singlet state with a box around a region of size L. The entanglement entropy of this region is simply equal to the number of bonds leaving the box multiplied by the entanglement associated with each bond. To calculate the entanglement entropy for a random chain one simply averages the resulting entropy over realizations of disorder. As shown by Refael and Moore[26] the average number of bonds leaving a region of size L in a random singlet phase scales as $\frac{1}{3} \ln L$. Since this scaling depends entirely on the bond strength distribution it should also hold for the random $SU(2)_k$ models. The only thing that is different for these models is the entanglement per bond. It was shown in Ref. 7 that for $SU(2)_k$ particles the entanglement per bond is $\log_2 d$ where d is the quantum dimension of the particles. This result can be understood as a simple consequence of the fact that the Hilbert space dimensionality of N $SU(2)_k$ particles scales as d^N for large N, as described in Sec. 2.

The end result for the entanglement scaling in the random $SU(2)_k$ chains is then[7]

$$S(L) \simeq \frac{\ln d}{3} \log_2 L. \tag{4}$$

From this one can extract an "effective central charge" for these models of $\tilde{c} = \ln d$. The cases $k = 2$ and $k \to \infty$ correspond to the transverse field Ising and random spin chain limits studied by Refael and Moore,[26] respectively, and in both cases (4) agrees with their results. All other k values then correspond to an infinite class of new infinite randomness fixed points. It is interesting to note that, just as for the models studied by Refael and Moore, these new models obey a generalized

c-theorem: for each k the effective central charge of the random model, $\ln d$, is less than the central charge of the uniform model $c = 1 - 6/(k+1)(k+2)$.

Acknowledgments

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