Fluctuation Driven First-Order Pairing Transition of Chern-Simons Fermions

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Motivated by recent speculation that the $\nu = 5/2$ fractional quantum Hall state can be viewed as a 'superconductor' of composite fermions the effect of a fluctuating Chern-Simons gauge field on BCS pairing is investigated. For the simplest model one can construct, spin-1/2 Chern-Simons fermions interacting via an attractive s-wave pairing interaction, it is shown that fluctuations of the Chern-Simons gauge field stabilize the Fermi sea against pairing for weak enough attraction and drive the pairing transition which occurs as the attraction is increased first order.

1 Introduction

The suggestion of Halperin, Lee, and Read[1] that a two-dimensional electron gas at Landau level filling factor $\nu = 1/2$ can be viewed as a 'Fermi sea' of composite fermions[2] has generated a great deal of interest. The mathematical formulation of this description is based on representing physical electrons by Chern-Simons (CS) fermions coupled to a CS gauge field.[3]

Numerical calculations[4,5,6] suggest that the $\nu = 5/2$ fractional quantum Hall state[7] may be a p-wave 'superconductor' of composite fermions described by the Pfaffian wave function introduced by Moore and Read.[8] It is therefore natural to ask what effect the CS gauge field may have on BCS pairing.

In this short note I introduce the simplest possible model for addressing this issue. The model describes a two-dimensional gas of spin-1/2 fermions coupled to a CS gauge field and interacting via an attractive s-wave interaction. In the absence of the CS gauge field the model is, of course, unstable to a paired state for arbitrarily weak pairing interaction. When fluctuations of the CS gauge field are included the pairing transition is driven first order and the CS Fermi sea is *stable* for weak enough pairing interactions. The more physically relevant and somewhat subtler case of p-wave pairing of spin-polarized CS fermions interacting via the Coulomb repulsion is discussed by the author in Ref. 9.

2 BCS Pairing of Chern-Simons Fermions

Consider the reduced BCS model for a two-dimensional gas of spin-1/2 fermions interacting via an s-wave attractive interaction for which the Euclidean-time

Lagrangian density is

$$\mathcal{L}_0 = \psi^*(\partial_\tau - \mu)\psi - \frac{1}{2m}\psi^*\nabla^2\psi - V_0\psi_\uparrow^*\psi_\downarrow^*\psi_\uparrow\psi_\downarrow. \tag{1}$$

It is a straightforward procedure to decouple the pairing interaction via a Hubbard-Stratonovich transformation and integrate out the fermions to obtain the $T = 0$ condensation energy[10]

$$\begin{aligned}\frac{E_0(\Delta)}{N(0)} &= \frac{\Delta^2}{\lambda} - \int_{-\omega_0}^{\omega_0}\left(\sqrt{\epsilon^2+\Delta^2} - |\epsilon|\right)\, d\epsilon \\ &= \frac{\Delta^2}{\lambda} - \Delta^2 \ln\left(\frac{\omega_0}{\Delta} + \sqrt{\left(\frac{\omega_0}{\Delta}\right)^2 + 1}\right) - \omega_0\left(\sqrt{\Delta^2+\omega_0^2} - \omega_0\right)\end{aligned} \tag{2}$$

where Δ is the gap parameter, ω_0 is the energy cutoff, $N(0) = m/2\pi$ is the density of states and $\lambda = N(0)V_0$ is the BCS coupling constant. For small Δ the $\Delta^2\ln(1/\Delta)$ energy gain always wins over the Δ^2 energy cost and we obtain the usual result that the system is unstable to a paired state for arbitrarily weak V_0.

Now consider the pairing of CS fermions for which the Lagrangian density is

$$\begin{aligned}\mathcal{L}_{\rm CS} &= \psi_\sigma^*(\partial_\tau - ia_0 - \mu)\psi_\sigma - \frac{1}{2m}\psi_\sigma^*(\nabla - i\mathbf{a} + e\mathbf{A})^2\psi_\sigma \\ &\quad - \frac{ia_0}{2\pi\tilde{\phi}}\hat{\mathbf{z}}\cdot(\nabla\times\mathbf{a}) - V_0\psi_\uparrow^*\psi_\downarrow^*\psi_\uparrow\psi_\downarrow.\end{aligned} \tag{3}$$

Here $(a_0, \mathbf{a})$ is the CS gauge field and $\mathbf{A} = (yB, -xB, 0)/2$ is the vector potential describing the uniform magnetic field $\mathbf{B} = (2\pi\tilde{\phi}n/e)\hat{\mathbf{k}}$. Following the usual arguments[1] $\mathbf{B}$ is cancelled by the mean-field expectation value of the CS gauge fields and the CS fermions form a sharp Fermi surface.

Following the same procedure used to obtain (2) the pairing interaction in (3) can be factorized by a Hubbard-Stratonovich transformation and the fermions integrated out. The resulting effective action can then be expanded to second order in the CS gauge fields (random-phase approximation), and these fields themselves can be integrated out to obtain the condensation energy

$$E(\Delta) = E_0(\Delta) + E_{\rm CS}(\Delta) - E_{\rm CS}(0) \tag{4}$$

where

$$E_{\rm CS}(\Delta) = \int_0^\infty \frac{d\nu}{2\pi}\int\frac{d^2q}{(2\pi)^2}\tan^{-1}\frac{{\rm Im}\left[K_{00}(q,\nu;\Delta)K_{11}(q,\nu;\Delta) - \dfrac{q^2}{2\pi\tilde{\phi}}\right]}{{\rm Re}\left[K_{00}(q,\nu;\Delta)K_{11}(q,\nu;\Delta) - \dfrac{q^2}{2\pi\tilde{\phi}}\right]}. \tag{5}$$

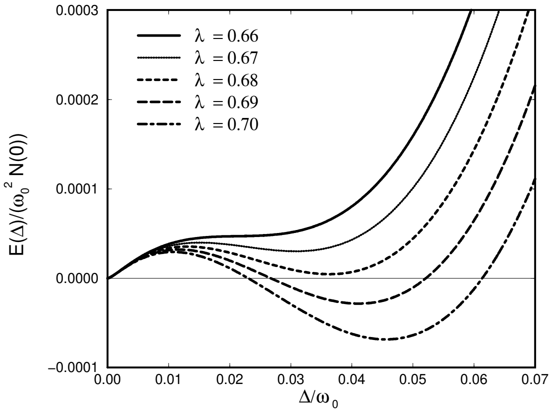

Figure 1: Condensation energy E plotted as a function of Δ for $\alpha = 1$ and different values of the BCS coupling constant $\lambda = V_0 N(0)$ illustrating the first-order pairing transition discussed in the text.

Here $K_{\mu\nu}(\Delta)$ is the electromagnetic response function defined by $j_\mu = K_{\mu\nu} A_\nu^{\rm ext}$ for an s-wave superconductor with gap Δ. The indices take the values 0 and 1 for time-like and transverse components, respectively.

3 First-Order Transition

In the paired state at $T = 0$ there is no damping for frequencies $\nu \leq 2\Delta$. This implies that $\mathrm{Im}K_{00}(\Delta) = \mathrm{Im}K_{11}(\Delta) = 0$ for $\nu \leq 2\Delta$ which in turn implies that there is no contribution to $E_{\rm CS}(\Delta)$ for these frequencies. Following Ubbens and Lee[11] $E_{\rm CS}(\Delta) - E_{\rm CS}(0)$ can then be estimated by calculating the $\nu \leq 2\Delta$ contribution of these frequencies to $-E_{\rm CS}(0)$,

$$\begin{aligned} E_{\rm CS}(\Delta) - E_{\rm CS}(0) &\simeq \int_0^{2\Delta} \frac{d\nu}{2\pi} \int \frac{d^2q}{(2\pi)^2} \tan^{-1} \frac{(6+\tilde{\phi}^2)k_F\nu}{\tilde{\phi}^2 m q^3} \\ &\simeq \alpha N(0)\omega_0^{1/3}\Delta^{5/3} \end{aligned} \tag{6}$$

where $\alpha \sim (\tilde{\phi}^2/(6+\tilde{\phi}^2))^{2/3}((k_F^2/2m)/\omega_0)^{1/3}$. For small Δ this term dominates the condensation energy and prevents any continuous zero temperature pairing transition. It follows that any transition will involve a discontinuous jump in Δ and so be first order. Such a first-order transition is illustrated in Fig. 1 where $E(\Delta)$ is plotted vs. Δ for $\alpha = 1$ and various values of λ. A similar effect, in which a continuous quantum Hall-insulator transition is driven first order by a fluctuating CS gauge field, has been studied by Pryadko and Zhang.[12]

4 Conclusions

To summarize, pairing of CS fermions has been shown to be qualitatively different than pairing of conventional fermions. In contrast to the well known result that a conventional Fermi sea is unstable for arbitrarily weak attractive interactions, the CS Fermi sea is stable against pairing for weak enough attractive interactions and the pairing instability which eventually occurs as the attraction is increased is first order. These results may be relevant to the experimentally observed compressible-incompressible transition of the $\nu = 5/2$ state in tilted fields.[13]

Acknowledgements

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