PAIRING INSTABILITY OF A DOUBLE LAYER COMPOSITE FERMION METAL

N. E. Bonesteel
NHMFL and Department of Physics, Florida State University
Tallahassee, FL 32308, USA
E-mail: nick@fangio.magnet.fsu.edu

ABSTRACT

A symmetrically doped double-layer electron system with total Landau level filling fraction $\nu = 1/m$ decouples into two even denominator ($\nu = 1/2m$) composite fermion 'metals' when the layer spacing is large. Out-of-phase fluctuations of the statistical gauge fields in this system mediate a singular attractive pairing interaction between composite fermions in different layers. A strong-coupling analysis shows that for any layer spacing d this pairing interaction results in an instability to a paired quantum Hall state with an energy gap proportional to $1/d^2$.

1. Introduction

Composite fermions were introduced by Jain in order to understand the observed hierarchy of fractional quantum Hall states.[1] A composite fermion is an electron confined to move in two dimensions and tied to an even number of statistical flux quanta. Jain showed that the fractional quantum Hall effect for electrons can be viewed as an effective *integer* quantum Hall effect for composite fermions. Halperin, Lee, and Read (HLR)[2] took Jain's suggestion further, arguing that at Landau level filling fraction $\nu = 1/2$ the statistical flux attached to composite fermions can, at the Hartree level, exactly cancel the physical flux of the applied magnetic field. The composite fermions then form a new type of metal, and a growing number of experiments appear to support this description.[3] HLR also showed that fluctuations of the statistical gauge field in this metal give rise to singular inelastic scattering of sufficient strength to lead to a breakdown of Fermi liquid theory. Though experimental proof of the non-Fermi liquid nature of the composite fermion metal remains ellusive, it has generated a great deal of excitement in the theoretical community.[4]

Double-layer electron systems have been realized in both double quantum wells[5] and wide single quantum wells.[6] In a symmetrically doped double-layer system with total Landau level filling fraction $\nu = 1$ each layer has filling fraction $\nu = 1/2$. It follows that in the limit where the layer spacing d is large it should be possible to view this system as two decoupled $\nu = 1/2$ composite fermion metals. This description will be referred to as the double-layer composite fermion metal (DLCFM) description in what follows. The main result of this note is that in the absence of interlayer tunneling the DLCFM is *always* unstable to a paired quantum Hall state for any d,

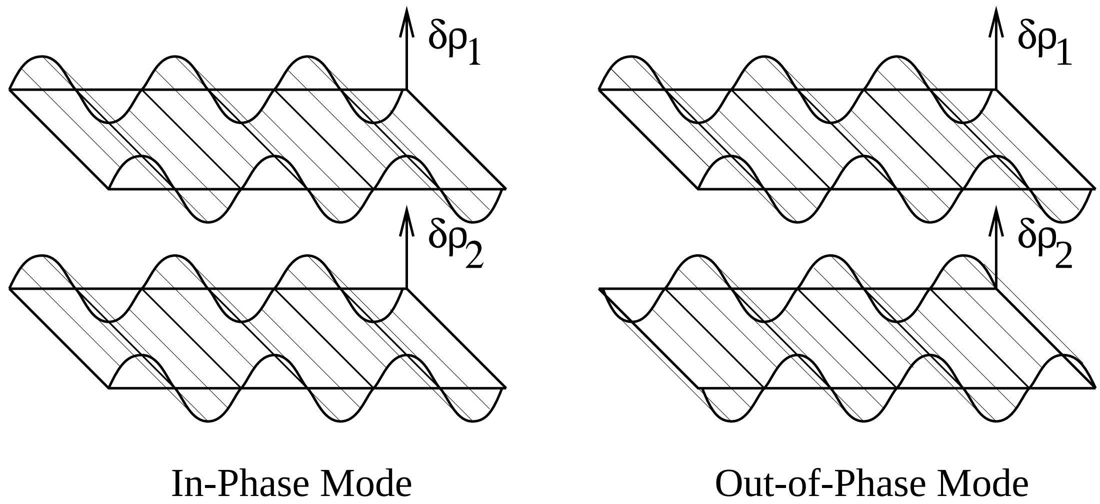

Fig. 1. In-phase and out-of-phase density fluctuations in a double-layer system. In a double-layer composite fermion metal these fluctuations are tied respectively to an in-phase and out-of-phase statistical gauge field. At long wavelengths and low frequencies the out-of-phase gauge field dominates the effective interaction between composite fermions and leads to the pairing instability discussed in the text.

however large.

2. Gauge Fluctuations and Pairing Interaction

In a DLCFM there are two statistical gauge fields, $a^{(1)}$ and $a^{(2)}$, one associated with each layer.[7] The transverse parts of these gauge fields are defined by the relation

$$\delta\rho_s = (\nabla \times a^{(s)})/\pi, \tag{1}$$

where $s = 1, 2$ is a layer index and $\delta\rho_s$ is the deviation of the density from the mean density in layer s. It is natural to describe the density fluctuations, and consequently the gauge field fluctuations, of this system in terms of in-phase and out-of-phase modes (Fig. 1). If the in-phase and out-of-phase gauge fields are defined as $a^{(\pm)} = a^{(1)} \pm a^{(2)}$, then within the random-phase approximation the relevant gauge field propagators at low frequency and long wavelengths are[7]

$$D^{(+)}(q, i\omega_n) \simeq \left(e^2 q/4 + |\omega_n| k_f/(4\pi q)\right)^{-1} \tag{2}$$

for the in-phase gauge fluctuations, and

$$D^{(-)}(q, i\omega_n) \simeq \begin{cases} \left(e^2 d q^2/4 + |\omega_n| k_f/(4\pi q)\right)^{-1} & \text{for } q \lesssim d^{-1} \\ \left(e^2 q/4 + |\omega_n| k_f/(4\pi q)\right)^{-1} & \text{for } q \gtrsim d^{-1} \end{cases} \tag{3}$$

for the out-of-phase fluctuations. The current-current interactions mediated by these gauge fields in the interlayer Cooper channel are then

$$V_{12}^{\pm}(k, k'; i\omega_n) = \pm \left(\frac{\mathbf{k} \times \hat{\mathbf{q}}}{m^*} \right)^2 D^{\pm}(q, i\omega_n) \tag{4}$$

where $\hat{\mathbf{q}} = (\mathbf{k} - \mathbf{k}')/|\mathbf{k} - \mathbf{k}'|$ and m^* is the effective mass of the composite fermions. Fluctuations in $a^{(-)}$ are more singular at low frequencies because the Coulomb interaction suppresses the in-phase density fluctuations but not the out-of-phase density fluctuations. As a consequence the effective interaction is dominated by the out-of-phase fluctuations and for the remainder of this note we will ignore the in-phase fluctuations, as well as the less singular density-density and density-current interactions in the system.[7]

The dominant out-of-phase mode mediates an *attractive* pairing interaction between composite fermions in opposite layers. This attractive pairing interaction appears because $a^{(-)}$ couples to composite fermions in different layers as if they were oppositely charged. The fluctuating $a^{(-)}$ field strongly inhibits the coherent propagation of a single composite fermion while a pair made up of composite fermions, one from each layer, is neutral with respect to $a^{(-)}$. Such a composite fermion pair can then propagate coherently through the fluctuating $a^{(-)}$ field, much like a meson propagating coherently through a strongly fluctuating gluon field.

3. Theory of Singular Pairing

Using the Nambu formalism the matrix Green's function is

$$G(k, i\omega_n) = [i\omega_n Z_n - \epsilon_k \tau_3 - \phi_n \tau_1]^{-1} \tag{5}$$

where $\omega_n = (2n+1)\pi T$ is a fermion Matsubara frequency, Z_n is the mass renormalization, and $\Delta_n = \phi_n / Z_n$ is the gap function. The Eliashberg equations for this system are then given by

$$\begin{aligned} \omega_n(1 - Z_n) &= -\pi T \sum_m \frac{Z_m \omega_m}{(|Z_m \omega_m|^2 + \phi_m^2)^{1/2}} \lambda_{m-n} \\ \phi_n &= \pi T \sum_m \frac{\phi_m}{(|Z_m \omega_m|^2 + \phi_m^2)^{1/2}} \lambda_{m-n} \end{aligned} \tag{6}$$

where the coupling constants λ are obtained by averaging the effective interactions (4) over the Fermi surface:

$$\lambda_{m-n} = \frac{k_f}{2\pi m^*} \int_0^{2k_F} D^{(-)}(q, |\omega_m - \omega_n|) \sqrt{1 - (q/2k_F)^2}\, dq. \tag{7}$$

For simplicity I consider the case of s-wave pairing, though the pairing interaction mediated by $a^{(-)}$ is attractive in all angular momentum channels. Presumably residual interactions not considered here will determine which pairing channel dominates in the actual system.

Performing the integral (7) one obtains

$$\lambda_{m-n} \sim \frac{E_f}{(e^2/l_0)}\left(\frac{l_0}{d}\right)^{2/3}\left(\frac{e^2/l_0}{|\omega_m-\omega_n|}\right)^{1/3} \tag{8}$$

where $E_f = k_f^2/2m^*$ and $l_0 = \sqrt{\hbar c/eB}$ is the magnetic length, (for $\nu = 1/2$, $k_f = l_0^{-1}$). As discussed by HLR,[2] the electron band mass must be renormalized so that $m^* \sim e^2 l_0$. For simplicity in what follows I will take $E_f = e^2/l_0$ so that

$$\lambda_{m-n} = \gamma\left(\frac{\omega_0}{|\omega_m-\omega_n|}\right)^{1/3} \tag{9}$$

where $\gamma = (l_0/d)^{2/3}$ is a dimensionless 'coupling constant', and $\omega_0 = e^2/l_0$.

The two Eliashberg equations can be combined to obtain a single equation for Δ_n:

$$\Delta_n = T\sum_m \lambda_{m-n}\frac{1}{(\omega_m^2+\Delta_m^2)^{1/2}}\frac{\omega_n\Delta_m-\omega_m\Delta_n}{\omega_n}. \tag{10}$$

Note that there is a cancellation when $\omega_n = \omega_m$ which removes the divergence in λ_{m-n} when $m = n$. This cancelation can be understood as a consequence of Anderson's theorem.[8] The quasistatic ($\omega < T$) gauge fluctuations which are responsible for the destruction of Fermi liquid behavior in the 'normal state', drop out of the gap equation because they can be viewed effectively as a random time-reversal invariant potential.

3.1. Calculation of T_c

Because of the scaling behavior of the coupling constants it is possible to factor out all the temperature dependence in the linearized version of (10) obtaining:

$$\Delta_n = \gamma\left(\frac{\omega_0}{2\pi T}\right)^{1/3}\sum_m |m-n|^{-1/3}(1-\delta_{m,n})\left(\frac{\Delta_m}{2m+1}-\frac{\Delta_n}{2n+1}\right)\mathrm{sgn}(2m+1). \tag{11}$$

Note that unlike conventional BCS theory there is no need for a frequency cutoff in the gap equation. Because the effective interaction falls off as $\omega^{-1/3}$ it is possible to take the Matsubara sum to infinity. The resulting expression for T_c in this limit is

$$T_c \simeq 4.3\omega_0\gamma^3 \propto 1/d^2 \tag{12}$$

where only the proportionality constant needs to be determined numerically.

3.2. Calculation of Δ at $T = 0$

The zero temperature Eliashberg equation on the imaginary frequency axis can be written

$$\Delta(i\omega) = \frac{1}{2}\int_{-\infty}^{\infty} d\omega'\frac{1}{(\omega'^2+\Delta^2)^{1/2}}\frac{\omega\Delta(i\omega')-\omega'\Delta(i\omega)}{\omega}\lambda(i\omega-i\omega'). \tag{13}$$

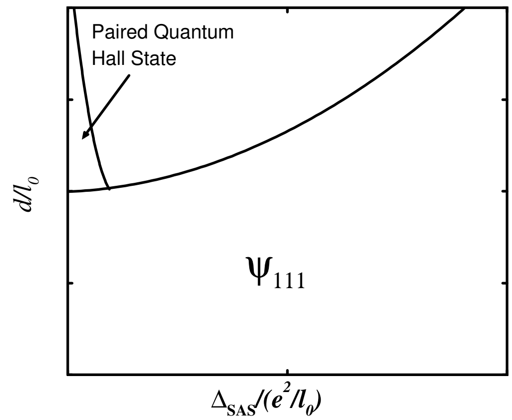

Fig. 2. Proposed phase diagram for a bilayer $\nu = 1$ state including the paired state discussed in this paper.

To solve this equation first consider the approximation $\Delta(i\omega) = \text{Const}$. The equation for $\Delta(0)$ is then obtained by taking the $\omega \to 0$ limit of the rhs of (13) with the result

$$\begin{aligned} 1 &= \frac{\gamma {\omega_0}^{1/3}}{3} \int_{-\infty}^{\infty} d\omega' \frac{1}{(\omega'^2 + \Delta^2)^{1/2}} \frac{1}{\omega'^{1/3}} \\ &= \gamma \frac{\Gamma(1/6)\Gamma(1/3)}{3\pi^{1/2}} \left(\frac{\omega_0}{\Delta}\right)^{1/3}. \end{aligned} \tag{14}$$

The zero temperature gap is then given by $\Delta(0) \simeq 22.1\gamma^3\omega_0$. A fully self consistent solution of the gap equation can be done numerically and yields

$$\Delta(0) \simeq 8.4\gamma^3\omega_0 \propto 1/d^2. \tag{15}$$

Thus the superconducting energy gap at zero temperature, like the mean-field transition temperature, falls off as $1/d^2$ as the layer spacing is increased. The important point here is that for any d, no matter how large, the system is unstable to a paired state. Such a paired state will be incompressible and thus exhibit the quantum Hall effect.[9] This result leads to the proposed phase diagram shown in Fig. 2. Ψ_{111} represents the 'conventional' $\nu = 1$ double-layer quantum Hall effect state.[10] The paired quantum Hall state discussed in this paper appears in the limit of small tunneling, as shown. It is an open theoretical question whether this paired state is adiabatically connected to Ψ_{111}. The experimental observation of this paired quantum Hall state would provide strong evidence for the existence of gauge fluctuations in composite fermion metals.

4. Conclusions

For large enough layer spacing a DLCFM with total filling fraction $\nu = 1/m$

decouples into two composite fermion metals, each with even denominator filling fraction $\nu = 1/2m$ (in this paper we have considered the case $m = 1$, but the results generalize trivially). Out-of-phase fluctuations of the statistical gauge fields in this system mediate a singular interlayer pairing interaction and a strong coupling analysis of this interaction shows that the DLCFM is, in the absence of disorder and interlayer tunneling, *always* unstable to a paired quantum Hall state. The key points in the analysis are (1) at finite temperature the low frequency fluctuations which lead to divergences in the normal and anomalous self energies cancel by Anderson's theorem leaving a sensible result; and (2) the gap equation can be solved essentially exactly by scaling and noting that the the integrals are all ultraviolet convergent (unlike conventional BCS theory). The experimental observation of this paired quantum Hall state would provide striking confirmation of the gauge theory of composite fermion metals.

Acknowledgements

I would like to acknowledge useful discussions with E. Miranda and J.R. Schrieffer. This work was supported by the National High Magnetic Field Laboratory at Florida State University.

References

1. J.K. Jain, *Phys. Rev. Lett.* **63** (1989) 199; *Phys. Rev. B* **40** (1989) 8079; *ibid.* **41** (1990) 7653.
2. B.I. Halperin, P.A. Lee, and N. Read, *Phys. Rev. B* **47** (1993) 7312.
3. R.R. Du *et al.*, *Phys. Rev. Lett.* **70** (1992) 2944; W. Kang et al., *Phys. Rev. Lett.* **71** (1993) 3850 (1993); R.L. Willett *et al.*, *Phys. Rev. Lett.* **71** (1993) 3846.
4. Y.B. Kim *et al.*, *Phys. Rev. B* **47** (1993) 7312; B.L. Altshuler, L.B. Ioffe, and A.J. Millis, *Preprint*; C. Nayak and F. Wilczek, *Nuc. Phys. B* **417** (1994) 359.
5. See, for example, S.Q. Murphy *et al.*, *Phys. Rev. Lett.* **51** (1995) 5138.
6. See, for example, Y.W. Suen *et al.*, *Phys. Rev. Lett.* **68** (1992) 1379.
7. N.E. Bonesteel, *Phys. Rev. B* **48** (1993) 11484.
8. P.W. Anderson, *J. Phys. Chem. Solids* **11** (1959) 26.
9. S.C. Zhang, T.H. Hansson, and S. Kivelson, *Phys. Rev. Lett.* **62** (1989) 82; D.H. Lee and S.C. Zhang, *ibid.* **66** (1991) 1220; S.C. Zhang, *Int. J. Mod. Phys. B* **6** (1992) 25.
10. K. Moon *et al.*, *Phys. Rev. B* **51** (1995) 5138.