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Composite fermions and Landau-level mixing in the fractional quantum Hall effect

V. Melik-Alaverdian and N. E. Bonesteel

National High Magnetic Field Laboratory and Department of Physics, Florida State University, Tallahassee, Florida 32306-4005

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The reduction of the energy gap due to Landau-level mixing, characterized by the dimensionless parameter $\lambda=(e^2/\epsilon l_0)/\hbar\omega_c$, has been calculated by variational Monte Carlo for the fractional quantum Hall effect at filling fractions $\nu=1/3$ and 1/5 using a modified version of Jain's composite-fermion wave functions. These wave functions exploit the Landau-level mixing already present in composite-fermion wave functions by introducing a partial Landau-level projection operator. Results for the energy gaps are consistent with experimental observations in n-type GaAs, but we conclude that Landau-level mixing alone cannot account for the significantly smaller energy gaps observed in p-type systems.

The quantum Hall effect (QHE) occurs when a two-dimensional electron gas placed in a strong transverse magnetic field develops an energy gap. In the fractional QHE this energy gap is entirely due to many-body correlations between the electrons.[1] It is generally believed that the essential physics of the fractional QHE can be understood in the limit where the cyclotron energy $\hbar\omega_c=\hbar eB/m^*c$ is much greater than the Coulomb energy scale $e^2/\epsilon l_0$.[1] Here B is the applied magnetic field, m^* is the effective mass of the electron or hole, ϵ is the dielectric constant, and $l_0=\sqrt{\hbar c/eB}$ is the magnetic length. In this limit the kinetic energy is completely quenched, the electrons (or holes) are entirely in the lowest Landau level provided the filling fraction is less than 1, and the Coulomb energy is the only energy scale in the problem.

Jain has constructed a class of trial wave functions based on the idea that the fractional QHE can be viewed as an effective integer QHE for composite fermions—electrons bound to an even number of statistical flux quanta.[2] Although Jain's wave functions naturally explain many features of the observed fractional QHE hierarchy, they suffer from a problem from the point of view of computation. With the exception of the "parent" ground states, those with Landau-level filling fractions $\nu=1/q$ where q is an odd integer, Jain's wave functions are not entirely in the lowest Landau level and it is therefore necessary to explicitly project them onto the lowest Landau level, a task which is numerically difficult, especially for systems with a large number of electrons. Even for the parent filling fractions the unprojected excited-state wave functions constructed using composite fermions, in particular, the quasielectron wave function, also suffer from intrinsic Landau-level mixing.[3,4] Of course in real experiments there will always be some Landau-level mixing characterized by the dimensionless parameter $\lambda=(e^2/\epsilon l_0)/\hbar\omega_c$. The intrinsic Landau-level mixing of Jain's wave functions is therefore not entirely unphysical. The problem is that the amount of Landau-level mixing in a given composite-fermion wave function is fixed, whereas in real experiments it depends on the parameter λ.

Recently Manoharan *et al.*[5] have measured the fractional QHE energy gaps in high-quality two-dimensional p-type systems realized in GaAs/Al_xGa_{1-x}As quantum wells. At filling fraction $\nu=1/3$ the energy gap they observe is $\Delta_h\simeq 0.023e^2/\epsilon l_0$, roughly a factor of 2 smaller than the corresponding energy gap $\Delta_e\simeq 0.05e^2/\epsilon l_0$ observed in high-quality n-type systems.[6,7] For typical two-dimensional carrier densities in GaAs heterostructures $\lambda\simeq 1$ for n-type systems while $\lambda\simeq 5$ for p-type systems. This led Manoharan *et al.* to suggest that the reduced energy gap they observed in p-type systems might be due to Landau-level mixing.

Motivated by these experiments we have calculated the λ dependence of the energy gap for $\nu=1/3$ and 1/5 using a new class of variational wave functions. These wave functions are constructed by applying a partial Landau-level projection operator to Jain's composite-fermion wave functions. In this way we have exploited the intrinsic Landau-level mixing already present in these wave functions by introducing a variational parameter that controls the amount of this mixing. Previously Yoshioka[8] studied the effect of Landau-level mixing on the excitation spectra of the fractional QHE using exact diagonalization on small systems, including both the lowest and first excited Landau levels, for $\nu=1/3$. Where it is possible to compare, our variational results are in good agreement with Yoshioka's results; however, since our calculations are based on variational Monte Carlo we can study significantly larger systems, as well as filling fraction $\nu=1/5$. The results we have obtained for the energy gap are consistent with experiments in n-type GaAs, but we conclude that Landau-level mixing alone cannot account for the smaller energy gaps observed in p-type systems.

We work in the spherical geometry introduced by Haldane.[9] In this geometry the electrons, which are taken to be fully spin polarized, are confined to the surface of a sphere of radius R and move in the magnetic field of a magnetic monopole placed at the center of the sphere. The magnetic-field strength at the surface of the sphere is $B=S(l_0/R)^2$ and the field is described by the vector potential $\mathbf{A}=\mathbf{e}_\phi(\hbar S/eR)\cot\theta$, where $2S=q(N-1)$ for $\nu=1/q$. In this gauge the appropriate generalization of Laughlin's ground-state wave function is[9]

$$\psi\propto\prod_{i<j}(u_iv_j-v_iu_j)^q=\prod_i u_i^{q(N-1)}\prod_{i<j}(z_i-z_j)^q. \quad (1)$$

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Here $u_i=\cos\theta_i/2\exp -i\phi_i/2$ and $v_i=\sin\theta_i/2\exp i\phi_i/2$ are the spinor coordinates of the ith electron, where θ_i and ϕ_i are the spherical coordinates of the electron, $z_i=v_i/u_i$ is the complex stereographic coordinate, and N is the total number of electrons. We have introduced the stereographic coordinates z in order to write ψ in a way that is formally similar to Laughlin's droplet wave function. Throughout this paper all wave functions are understood to be normalized and the proportionality sign will be used whenever necessary. Jain showed that ψ can also be written[2]

$$\psi\propto\prod_i u_i^{q(N-1)}\prod_{i<j}(z_i-z_j)^{q-1}\times\begin{vmatrix}1 & z_1 & \cdots & z_1^{N-2} & z_1^{N-1}\\ \vdots & \vdots & & \vdots & \vdots\\ 1 & z_N & \cdots & z_N^{N-2} & z_N^{N-1}\end{vmatrix}. \tag{2}$$

In this form the determinant corresponds to one filled *pseudo*-Landau-level of composite fermions, where the flux quanta bound to the electrons are represented, roughly, by the $\Pi_{i<j}(z_i-z_j)^{q-1}$ Jastrow factor.[2]

To determine the energy gap we must calculate the energy difference between ψ and the appropriate excited-state wave function. The excited state we have used is constructed by promoting a composite fermion from the lowest pseudo-Landau-level to the first excited pseudo-Landau-level. It is possible to construct an entire low-energy band of excited states in this way.[10] Here we are interested in the state constructed by removing a composite fermion from the lowest pseudo-Landau-level at the bottom of the sphere and reintroducing it into the first excited pseudo-Landau-level at the top of the sphere. The resulting wave function is

$$\psi'\propto\prod_i u_i^{q(N-1)}\prod_{i<j}(z_i-z_j)^{q-1}\times\begin{vmatrix}1 & z_1 & \cdots & z_1^{N-2} & \dfrac{\bar z_1}{1+|z_1|^2}\\ \vdots & \vdots & & \vdots & \vdots\\ 1 & z_N & \cdots & z_N^{N-2} & \dfrac{\bar z_N}{1+|z_N|^2}\end{vmatrix}. \tag{3}$$

This wave function describes a state with a charge $+e/q$ quasihole at the bottom of the sphere, and a charge $-e/q$ quasielectron at the top of the sphere. The total angular momentum quantum number of ψ' is $l=N$ and so it is orthogonal to the $l=0$ ground state. In the $N\to\infty$ limit ψ' describes a state with a quasielectron-quasihole pair with infinite separation. The energy gap for creating such a pair is precisely the energy gap that appears in the activated temperature dependence of the longitudinal resistance.

The energy difference between ψ' and ψ has been calculated previously for $\nu=1/3$ and was found to be $\Delta\simeq 0.05e^2/\epsilon l_0+0.16\hbar\omega_c$.[4] The contribution to Δ that is proportional to $\hbar\omega_c$ comes from the nonzero overlap of Jain's quasielectron wave function with the first excited Landau level. We have exploited this intrinsic Landau-level mixing in ψ' by introducing a partial Landau-level projection operator and considering the wave function

$$\psi'_\alpha\propto[1+(\alpha-1)P_{\rm LLL}]\psi'. \tag{4}$$

Here $P_{\rm LLL}$ is an operator that projects fully into the lowest Landau level. When $\alpha=1$ ψ'_α is simply Jain's unprojected wave function while in the limit $\alpha\to\infty$ ψ'_α becomes the fully projected version of Jain's wave function. The parameter α can therefore be used as a variational parameter to control the intrinsic Landau-level mixing present in ψ'. For the simple case considered here (one quasielectron) it is possible to perform the partial projection analytically with the following result:

$$\psi'_\alpha\propto\prod_i u_i^{q(N-1)}\prod_{i<j}(z_i-z_j)^{q-1}\begin{vmatrix}1 & z_1 & \cdots & z_1^{N-2} & \bar z_1/(1+|z_1|^2)+(1-\alpha)(q-1)/(2S+2)\sum_{i\neq 1}1/(z_i-z_1)\\ \vdots & \vdots & & \vdots & \vdots\\ 1 & z_N & \cdots & z_N^{N-2} & \bar z_N/(1+|z_N|^2)+(1-\alpha)(q-1)/(2S+2)\sum_{i\neq N}1/(z_i-z_N)\end{vmatrix}. \tag{5}$$

Both ψ and ψ'_α have the conventional form of a Jastrow factor multiplying a Slater determinant. We have applied standard variational Monte Carlo techniques for such wave functions to calculate their properties in what follows.[11]

Before proceeding it is necessary to discuss the finite thickness of the two-dimensional electron or hole gas. The wave functions of electrons or holes in the lowest subband of a two-dimensional system have a finite extent perpendicular to the plane of the system. This "thickness" has the effect of softening the short-range part of the Coulomb interaction. Since it is precisely the short-range interactions that determine the energy gap in the fractional QHE it is crucial to include this effect in any realistic calculation before comparing with experiment. Following Yoshioka[8] and Zhang and Das Sarma[12] the thickness corrections have been included using the Fang-Howard wave function for the lowest subband in an inversion layer,

$$\xi(w)=\left(\frac{b^3}{2}\right)^{1/2}we^{bw/2}, \tag{6}$$

where w is the coordinate perpendicular to the two-dimensional gas. The finite thickness then has the effect of modifying the Coulomb interaction so that the effective electron-electron interaction is

$$U(r)=\frac{e^2}{\epsilon l_0}\int_0^\infty dw' \int_0^\infty dw |\xi(w)|^2 \frac{1}{\sqrt{r^2+(w-w')^2}} |\xi(w')|^2. \tag{7}$$

The thickness is characterized by the dimensionless parameter $\beta=1/(bl_0)$. For the systems we are considering here $\beta\simeq 1$.

To calculate the dependence of Δ on λ for a given value of β we must minimize the total energy of ψ'_α as a function of α. This calculation is greatly simplified by the following observation. Because ψ' contains only a single composite fermion in the first excited pseudo-Landau-level it can be decomposed as follows:

$$\psi' = \gamma\psi'_0 + (1-\gamma^2)^{1/2}\psi'_1. \tag{8}$$

Here ψ'_0 is the projected state with all N electrons in the lowest Landau level and ψ'_1 is orthogonal to ψ'_0 with $N-1$ electrons in the lowest Landau level and one electron in the first excited Landau level.

It follows from (4) and (8) that the partially projected state can be written

$$\psi'_\alpha \propto \alpha\gamma\psi'_0 + (1-\gamma^2)^{1/2}\psi'_1. \tag{9}$$

If we define the expectation value of the Coulomb interaction in units of $e^2/\epsilon l_0$ to be $V[\alpha]=\langle\psi'_\alpha|V|\psi'_\alpha\rangle/(e^2/\epsilon l_0)$, where $V=\Sigma_{i<j}U(r_{ij})$ with $U(r)$ as defined in (7), then from (9) it follows that

$$V[\alpha]=\frac{\gamma^2(\alpha^2-\alpha)V[\infty]+\alpha V[1]+(1-\alpha)(1-\gamma^2)V[0]}{1+\gamma^2(\alpha^2-1)}. \tag{10}$$

When the expectation value of the kinetic energy in ψ'_α is included we obtain the following expression for the energy gap as a function of λ and α:

$$\Delta[\alpha,\lambda]/(e^2/\epsilon l_0)=\{(1-\gamma^2)/\lambda+\gamma^2(\alpha^2-\alpha)V[\infty]+\alpha V[1]+(1-\alpha)(1-\gamma^2)V[0]\}/[1+\gamma^2(\alpha^2-1)]-E_0, \tag{11}$$

where $E_0\equiv\langle\psi|V|\psi\rangle$ is the ground-state energy in units of $e^2/\epsilon l_0$. It is therefore only necessary to determine five expectation values, E_0, $V[0]$, $V[1]$, $V[\infty]$, and γ, by variational Monte Carlo. Once these quantities have been calculated it is straightforward to minimize $\Delta[\alpha,\lambda]$ with respect to α and determine the energy gap as a function of λ. In addition, once the value of α, which minimizes the energy of ψ'_α for a given λ, is known, it is possible to calculate various expectation values for that value of λ.

Figure 1 shows the density profile of the excited state at $\nu=1/3$ as a function of θ for different values of λ. The results are shown for $N=30$ and $\beta=0$. Note that as λ increases only the quasielectron is affected by the partial Landau-level projection. This is because Jain's quasihole wave function is equivalent to Laughlin's and is therefore entirely in the lowest Landau level. For $\lambda=0$ the quasielectron is fully projected onto the lowest Landau level and its density profile is nearly indistinguishable from that obtained by Morf and Halperin for Laughlin's quasielectron trial state.[13] As λ increases the quasielectron charge becomes less localized. This delocalization occurs because as the first excited Landau level is mixed into the wave function the quasielectron has more degrees of freedom, allowing it to "spread out" and lower its Coulomb energy at the price of some kinetic energy. This delocalization of the quasielectron charge with increasing λ is the physical origin of the reduction of the energy gap by Landau-level mixing.

It is important to point out that the effect of Landau-level mixing on the bulk of the wave function is not included in our calculations and we do not expect the energy of either ψ or ψ'_α to be valid for finite values of λ. However, we do expect the *difference* in these energies to give a reasonable value for the energy gap, because in this difference the energy associated with the bulk of the wave function cancels, leaving only the energy associated with the local excitations. Note that in our calculation the reduction of the energy gap comes entirely from the quasielectron, because the quasihole is unaffected by the partial Landau-level projection operator. This is consistent with Yoshioka's exact diagonalization calculations, which showed that the excitation energy of the quasielectron was significantly more sensitive to Landau-level mixing than that of the quasihole.[8]

Figure 2 shows the energy gap as a function of λ for various values of the thickness parameter β. The results are for a system with 30 electrons. Both increasing β and increasing λ have the effect of reducing the energy gap, consistent with previous calculations.[8,12] The key result of our calculation is the observation that, as β increases, the effect of Landau-level mixing on the energy gap becomes weaker. This weakening of the Landau-level mixing effect can be understood as follows. The thickness correction softens the short-range part of the Coulomb interaction, which in turn reduces the interaction energy the quasielectron stands to gain by delocalizing. It follows that as β increases the quasielectron charge delocalizes less for a given value of λ, and the energy gain also decreases. Note that for the experimentally relevant value $\beta\simeq 1$ the energy gap is not much different for $\lambda=1$ and $\lambda=5$. For $\lambda=1$ the energy gap is roughly $0.06e^2/\epsilon l_0$, consistent with the experimentally measured value of $\Delta_e\simeq 0.05e^2/\epsilon l_0$, but for $\lambda=5$ the theoretical energy gap is much larger than the experimentally observed $\Delta_h\simeq 0.023e^2/\epsilon l_0$. We must therefore conclude that either (1) Landau-level mixing alone cannot account for the reduced energy gap observed in p-type GaAs quantum wells; or (2) the variational wave function ψ'_α, which limits the Landau-level mixing to that within the first excited Landau level, is

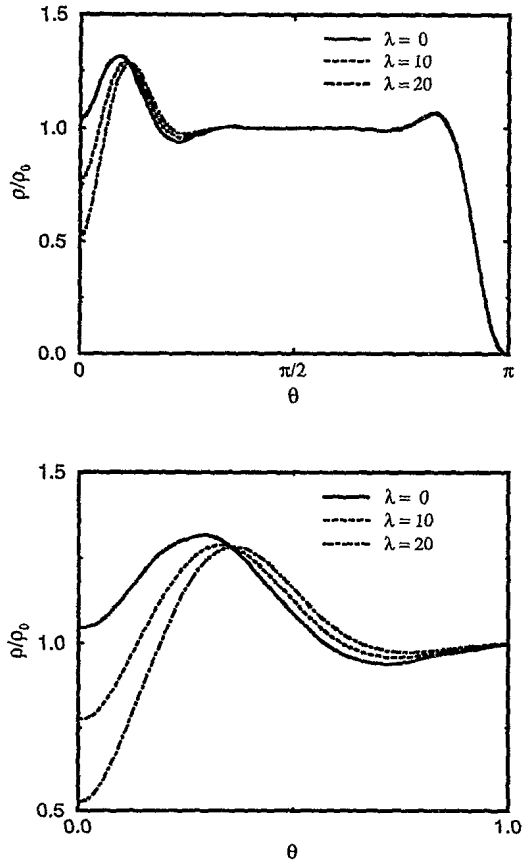

FIG. 1. (Top) Density profile of the excited-state wave function on a sphere with a quasielectron at the top of the sphere ($\theta=0$) and a quasihole on the bottom of the sphere ($\theta=\pi$) for different values of the Landau-level mixing parameter λ, and (bottom) blowup of the quasielectron density. As λ increases the quasielectron charge becomes less localized, leading to a reduction of the energy gap. The results shown are for 30 electrons and $\beta=0$.

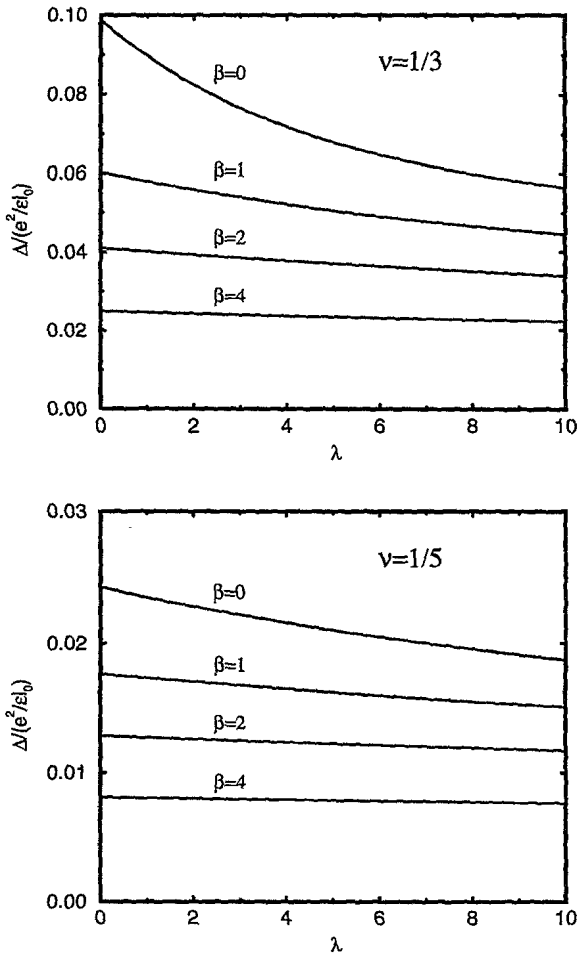

FIG. 2. Energy gap for creating a well-separated quasielectron-quasihole pair at $\nu=1/3$ and $\nu=1/5$ as a function of the Landau-level mixing parameter $\lambda=(e^2/\epsilon l_0)/\hbar\omega_c$ for various values of the thickness parameter β (defined in the text). The results shown are for 30 electrons.

not capable of accurately describing the true Landau-level mixing at $\lambda=5$. We believe that (1) is the case here because of the physical argument given above for the reduction of the dependence of the energy gap on λ with increasing β.

To conclude, we have modified Jain's composite-fermion wave functions by applying a partial Landau-level projection operator. These wave functions have been used to calculate the effect of Landau-level mixing on the energy gap in the fractional QHE. The main result is the observation that as the thickness parameter β is increased the effect of Landau-level mixing on the energy gap is suppressed. In particular, for the experimentally relevant value of $\beta=1$ Landau-level mixing has almost no effect on the energy gap. Thus, even though our results are in good agreement with the experimentally measured energy gap in n-type GaAs systems at $\nu=1/3$, we conclude that Landau-level mixing alone cannot account for the factor of 2 smaller energy gaps observed in p-type GaAs quantum wells.

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