Conductivity of Paired Composite Fermions

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We develop a phenomenological description of the $\nu = 5/2$ quantum Hall state in which the Halperin-Lee-Read theory of the half-filled Landau level is combined with a p-wave pairing interaction between composite fermions (CFs). The electromagnetic response functions for the resulting mean-field superconducting state of the CFs are calculated and used in an RPA calculation of the q and ω dependent longitudinal conductivity of the physical electrons, a quantity which can be measured experimentally.

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The $\nu = 5/2$ fractional quantum Hall state remains one of the most interesting phenomena in two dimensional electron physics [1]. Since its experimental discovery over a decade ago [2], the nature of this state has been a topic of debate. Evidence from exact diagonalization of small systems [3] points towards the 5/2 state being properly described as a spin-polarized Moore-Read state [4], a state which can be viewed as a chiral p-wave superconductor [4,5] of composite fermions (CFs) [6]. Among other ramifications, the Moore-Read state should theoretically exhibit excitations with exotic non-Abelian statistics [4]—something never before observed in nature.

Although there is a reasonably strong theoretical case that the $\nu = 5/2$ fractional quantum Hall effect (FQHE) state is, in fact, a Moore-Read state, the question remains, how can one test this hypothesis experimentally? While several experiments seem to be at least *consistent* with the 5/2 state being a Moore-Read state [3,7], we are still in need of a smoking gun. The analogy with superconductivity makes one think of how the classic experimental hallmarks of BCS-superconductivity [8] theory might be translated into the fractional quantum Hall regime. For example, in traditional superconductors, many measurable response functions display "coherence peaks" below the critical temperature which are extremely good evidence of BCS superconductivity. We ask whether such a phenomenon should exist for the Moore-Read state. To address this question, we have developed a phenomenological description of the FQHE state in which the Halperin-Lee-Read (HLR) [9] theory of the half-filled Landau level is combined with a p-wave pairing interaction between CFs. Within this theory we are able to predict various response functions of the Moore-Read state which may be measured experimentally. Recalling that surface acoustic wave (SAW) experiments [6] were particularly powerful in experimentally demonstrating the existence of CFs, we are particularly interested in the SAW signatures of the Moore-Read state.

In the HLR theory [9], each electron is modeled as a fermion bound to two quanta of "Chern-Simons" flux, the fermion plus flux being called a CF. For the 5/2 state, 4/5 of the electrons are required to fill the lowest two (essentially inert) Landau bands and the remaining (1/5) valence electrons are transformed to CFs. At the mean-field level, the external field precisely cancels the bound flux, and we model the valence electrons as free fermions in zero effective magnetic field. There is some indication that under certain conditions the residual interaction between the CFs can create a pairing instability [5,10]. To represent this physics, we add a pairing interaction between the CFs by hand. We thus use a model Hamiltonian for the CFs of the standard BCS form, $H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + H_{\rm BCS}$, where

$$H_{\rm BCS} = \frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\mathbf{k}\mathbf{k}'} c^\dagger_{\mathbf{k}+\mathbf{q}/2} c^\dagger_{-\mathbf{k}+\mathbf{q}/2} c_{-\mathbf{k}'+\mathbf{q}/2} c_{\mathbf{k}'+\mathbf{q}/2}, \quad (1)$$

$c^\dagger$ is the CF creation operator, $\xi_k = k^2/(2m) - \mu$ and m is CF effective mass which may be much larger than the underlying electron mass ($\hbar = c = 1$). The *ad hoc* mass renormalization will cause problems at the cyclotron energy scale but is expected to be reasonable at lower energies [9,11]. In the spirit of Ref. [9] we calculate the CF response of H, then transform this result [see Eq. (15) below] to determine the physical electron response.

In Eq. (1) the pairing interaction is taken to be of chiral p-wave form $V_{\mathbf{k}\mathbf{k}'} = -V e^{-i\theta_{\mathbf{k}}} e^{i\theta_{\mathbf{k}'}}$, where $\theta_{\mathbf{k}}$ is the angle of $\mathbf{k}$ on the Fermi surface. Note that this interaction is not time-reversal symmetric—it is only attractive in the $l = +1$ channel, not the $l = -1$ channel. Such an asymmetry is expected because, although the CFs see zero average magnetic field at the mean-field level, their residual interactions are not time-reversal symmetric.

If we define the gap function to be

$$\Delta_{\mathbf{q}} \equiv V\sum_{\mathbf{k}'}\langle c_{-\mathbf{k}'+\mathbf{q}/2}c_{\mathbf{k}'+\mathbf{q}/2}\rangle e^{i\theta_{\mathbf{k}'}}, \tag{2}$$

the BCS mean-field (MF) Hamiltonian can be written in pseudospin notation as

$$H_{\rm MF} = \frac{1}{2}\sum_{\mathbf{k}}\Psi_{\mathbf{k}}^\dagger[\xi_{\mathbf{k}}\tau_z - \Delta(\cos\theta_{\mathbf{k}}\tau_x + \sin\theta_{\mathbf{k}}\tau_y)]\Psi_{\mathbf{k}},$$

where τ_x, τ_y, τ_z are the usual Pauli spin matrices, $\Psi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}}^\dagger, c_{-\mathbf{k}})$ and $\Delta = |\Delta_{\mathbf{q}=\mathbf{0}}|$ is the temperature dependent energy gap found by solving the usual BCS gap equation. The restriction of $\Delta_{\mathbf{q}}$ to its zero wave vector component explicitly breaks gauge invariance. We fix this problem below.

We now add a perturbation Hamiltonian to the above $H_{\rm MF}$ given by

$$H' = \sum_{\mathbf{q}}[a_{0_{\mathbf{q}}}j_{0_{-\mathbf{q}}} + a_{1_{\mathbf{q}}}j_{1_{-\mathbf{q}}} + a_{b_{\mathbf{q}}}j_{b_{-\mathbf{q}}}], \tag{3}$$

where

$$(j_{0_{\mathbf{q}}}, j_{1_{\mathbf{q}}}) = \frac{1}{2}e\sum_{\mathbf{k}}\Psi_{\mathbf{k}+\mathbf{q}/2}^\dagger\left(\tau_z, \frac{k_\perp}{m}\tau_0\right)\Psi_{\mathbf{k}-\mathbf{q}/2}, \tag{4}$$

$$j_{b_{\mathbf{q}}} = \frac{1}{2}\sum_{k}\Psi_{\mathbf{k}+\mathbf{q}/2}^\dagger(-\cos\theta_{\mathbf{k}}\tau_y + \sin\theta_{\mathbf{k}}\tau_x)\Psi_{\mathbf{k}-\mathbf{q}/2}. \tag{5}$$

The first two terms in H' are the coupling of the scalar potential $a_{0_{\mathbf{q}}}$ to the density $j_{0_{\mathbf{q}}}$ and the transverse vector potential $a_{1_{\mathbf{q}}}$ to the transverse paramagnetic current $j_{1_{\mathbf{q}}}$ (for simplicity we work in Coulomb gauge here so the longitudinal vector potential is zero). The third term in H' is the coupling of CFs to the phase fluctuations of the order parameter, described by $a_{b_{\mathbf{q}}} = (\Delta_{\mathbf{q}} - \Delta_{-\mathbf{q}}^*)/(2i)$ which will be self-consistently calculated. Such self-consistent treatment of phase fluctuations enables one to calculate gauge invariant responses to external perturbations despite the fact that $H_{\rm MF}$ is not gauge invariant [12]. Magnitude fluctuations are neglected since they can be shown to decouple due to approximate particle-hole symmetry at the Fermi surface [13].

We define the response functions Q_{ij} for the mean-field Hamiltonian $H_{\rm MF}$ by $j_i(\mathbf{q}, \omega) = Q_{ij}(\mathbf{q}, \omega)a_j(\mathbf{q}, \omega)$, where the indices i and j can be 0, 1, or b. Here $j_i(\mathbf{q}, \omega)$ is the Fourier transform of the time-dependent expectation value of $j_{i_{\mathbf{q}}} + \delta_{i1}(n/m)a_{1_{\mathbf{q}}}$ and thus includes the diamagnetic contribution to the transverse current.

When the constraint, following from (2), that $a_{b_{\mathbf{q}}} = V\langle j_{b_{\mathbf{q}}}\rangle$ is included, the standard RPA analysis [12] can be used to obtain the gauge invariant CF electromagnetic response functions $K_{\mu\nu}$ defined by $j_\mu(\mathbf{q}, \omega) = K_{\mu\nu}(\mathbf{q}, \omega)a_\nu(\mathbf{q}, \omega)$, where the indices μ and ν can now be 0 or 1. We obtain $K_{\mu\nu} = Q_{\mu\nu} - Q_{\mu b}Q_{b\nu}/(Q_{bb} - 1/V)$, where the second term on the right-hand side corresponds to the usual vertex corrections required for a conserving approximation.

Following Mattis and Bardeen [14] (see also [15]), in the extreme anomalous limit $v_F q \gg \max[\omega, \Delta]$ the expressions for these response functions can be simplified substantially. We obtain

$$Q_{00}(\mathbf{q}, \omega) = -\frac{m^2}{4\pi^2}F_0(q)\Omega(1, -\cos\theta_q, \omega) - \frac{m}{2\pi}, \tag{6}$$

$$Q_{10}(\mathbf{q}, \omega) = i\sin\theta_{\mathbf{q}}\frac{m}{4\pi^2}F_1(q)\Omega(0, 1, \omega), \tag{7}$$

$$Q_{11}(\mathbf{q}, \omega) = \frac{1}{4\pi^2}F_2(q)\Omega(1, \cos\theta_q, \omega) - \frac{q^2}{24\pi m}, \tag{8}$$

$$Q_{b0}(\mathbf{q}, \omega) = i\frac{m^2}{4\pi^2}F_0(q)\frac{\omega}{\Delta}\cos\frac{\theta_{\mathbf{q}}}{2}\Omega(0, 1, \omega), \tag{9}$$

$$Q_{b1}(\mathbf{q}, \omega) = \frac{m}{4\pi^2}F_1(q)\frac{\omega}{\Delta}\sin\frac{\theta_{\mathbf{q}}}{2}\Omega(0, 1, \omega), \tag{10}$$

$$Q_{bb}(\mathbf{q}, \omega) = \frac{m^2}{4\pi^2}F_0(q)\Omega(1, -1, \omega) + \Lambda(\mathbf{q}) + \frac{1}{V}, \tag{11}$$

where θ_q is the angle between the vectors $\mathbf{k} + \mathbf{q}/2$ and $\mathbf{k} - \mathbf{q}/2$ when constrained to the Fermi surface, and $Q_{0b}(\mathbf{q}, \omega) = -Q_{b0}(\mathbf{q}, \omega)$, $Q_{b1}(\mathbf{q}, \omega) = Q_{1b}(\mathbf{q}, \omega)$, $Q_{10}(\mathbf{q}, \omega) = Q_{01}(\mathbf{q}, \omega)$. In these equations, $F_\alpha(q) = (2k_f^{\alpha-1}/q)[1 - q^2/(2k_f)^2]^{(\alpha-1)/2}$, and $\Omega(\omega) = \Omega_1(\omega) + i\Omega_2(\omega)$ with

$$\Omega_1(r, s, \omega) = \pi\int_{\max[\Delta-\omega, -\Delta]}^{\Delta}[1 - 2f(E + \omega)]\frac{sE(E + \omega) + r\Delta^2}{[\Delta^2 - E^2]^{1/2}[(E + \omega)^2 - \Delta^2]^{1/2}}dE, \tag{12}$$

$$\Omega_2(r, s, \omega) = -2\pi\int_{\Delta}^{\infty}[f(E) - f(E + \omega)]\frac{sE(E + \omega) + r\Delta^2}{[E^2 - \Delta^2]^{1/2}[(E + \omega)^2 - \Delta^2]^{1/2}}dE - \pi\int_{\Delta-\omega}^{-\Delta}[1 - 2f(E + \omega)]\frac{sE(E + \omega) + r\Delta^2}{[E^2 - \Delta^2]^{1/2}[(E + \omega)^2 - \Delta^2]^{1/2}}dE, \tag{13}$$

$$\Lambda(\mathbf{q}) = \int\frac{d^2k}{(2\pi)^2}\left(\frac{1 - f(\xi_{\mathbf{k}-\mathbf{q}/2}) - f(\xi_{\mathbf{k}+\mathbf{q}/2})}{\xi_{\mathbf{k}-\mathbf{q}/2} + \xi_{\mathbf{k}+\mathbf{q}/2}} - \frac{1 - 2f(E_{\mathbf{k}})}{2E_{\mathbf{k}}}\right), \tag{14}$$

where $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta|^2}$ and f is the Fermi function. The one dimensional integrals for $\Omega(\omega)$ are easily evaluated numerically. In the extreme anomalous limit $\Lambda(\mathbf{q}) \simeq -[m/(2\pi)]\ln(v_F q/[2\Delta(0)])$ is large and the vertex corrections to the Coulomb gauge are small.

Note that this mean-field treatment gives a finite temperature phase transition. This is an artifact of our calculation. Vortices in a Chern-Simons "superfluid" cost a finite amount of energy to create and interact only via short-range interactions. As a result there is no finite temperature Kosterlitz-Thouless transition and fluctuations will push T_c to zero. We assume here that including these fluctuations primarily has the effect of smoothing the finite temperature transition into a crossover, but the qualitative features of our results remain.

In all of what follows we take $\Delta(0)/E_F = 0.01$. This is consistent with $k_f = (4\pi n/5)^{1/2} \sim 10^8\ \mathrm{m}^{-1}$, $\Delta(0) \sim 0.1$ K, and a CF effective mass $m \sim 10 m_b$, where m_b is the electronic band mass.

Coherence effects are most clearly seen in the q and ω dependent conductivity. Figure 1 shows the transverse conductivity $\sigma_{yy}^{\mathrm{CF}} = e^2 K_{11}/i\omega$ for CFs as a function of temperature for $q = 0.1 k_F$. For low frequencies, $\omega \lesssim 0.2\Delta(0)$, $\mathrm{Re}\sigma_{yy}^{\mathrm{CF}}$ shows a Hebel-Slichter coherence peak just below T_c. This peak appears because for small q the p-wave nature of the pairing is irrelevant and the coherence factors which determine electromagnetic absorption are type II, the same coherence factors which govern the temperature dependence of the NMR relaxation time $1/T_1$ in conventional superconductors. For the same low frequencies $\mathrm{Im}\sigma_{yy}^{\mathrm{CF}}$ increases dramatically below T_c, reflecting the large increase in $\mathrm{Re}K_{11}(q, \omega)$ due to the enhanced CF diamagnetic response in the paired state. The kink clearly visible in $\mathrm{Re}\sigma_{yy}^{\mathrm{CF}}$ for $\omega = \Delta(0)$ occurs when the threshold condition $\omega = 2\Delta(T)$ is satisfied.

It is natural to ask if a similar coherence peak is observable in the SAW response of the 5/2 state. To address this we calculate the *electronic* longitudinal (EL) conductivity, $\sigma_{xx}^{\mathrm{EL}}$, following HLR using the Chern-Simons RPA. The only modification to the HLR result is due to the off-diagonal part of the mean-field CF response function. The resulting expression for the conductivity is

$$\sigma_{xx}^{\mathrm{EL}} = \frac{ie^2\omega K_{00}/q^2}{1 + \frac{4\pi i\tilde{\phi}}{q}K_{10} + \frac{(2\pi\tilde{\phi})^2}{q^2}(K_{00}K_{11} - K_{10}^2)}, \qquad (15)$$

where $\tilde{\phi} = 2$ is the number of flux quanta attached to each CF. Just as in the HLR case, in the limit of small q this expression is dominated by K_{11} and to a good approximation $\sigma_{xx}^{\mathrm{EL}} \simeq [e^2/(2\pi\tilde{\phi})^2] i\omega K_{11}^{-1} = [e^2/(2\pi\tilde{\phi})]^2/\sigma_{yy}^{\mathrm{CF}}$.

Figure 2 shows $\sigma_{xx}^{\mathrm{EL}}$ vs T/T_c for the same parameters as in Fig. 1. Note that there is no sign of the Hebel-Slichter peak at low frequencies. This is due to the rapid increase in $\mathrm{Im}\sigma_{yy}^{\mathrm{CF}}$ below T_c which suppresses $\sigma_{xx}^{\mathrm{EL}}$, masking the relatively small coherence peak. We conclude that to observe this peak in any experiment which directly measures $\sigma_{xx}^{\mathrm{EL}}$ (e.g., SAW) it would be necessary to measure the real and imaginary parts of $\sigma_{xx}^{\mathrm{EL}}$ with sufficient accuracy to carry out the inversion to obtain $\sigma_{yy}^{\mathrm{CF}}$ [17]. While

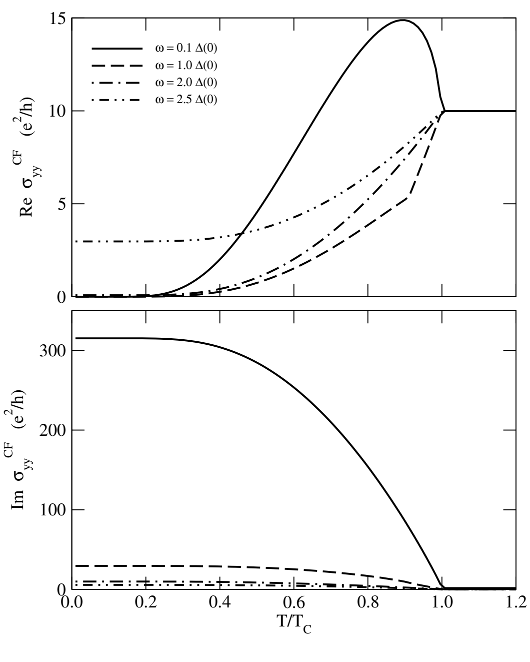

FIG. 1. Real and imaginary parts of the transverse conductivity of composite fermions, $\sigma_{yy}^{\mathrm{CF}}$, in a p-wave "superconducting" state as a function of temperature for $\omega/\Delta(0) = 0.1$, 1.0, 2.0, and 2.5. For low frequencies, $\omega \lesssim 0.2\Delta$, $\mathrm{Re}\sigma_{yy}^{\mathrm{CF}}$ shows a Hebel-Slichter coherence peak and $\mathrm{Im}\sigma_{yy}^{\mathrm{CF}}$ shows a strongly enhanced diamagnetic response. Results are for $\Delta(0) = 0.01 E_F$ and $q = 0.1 k_F$.

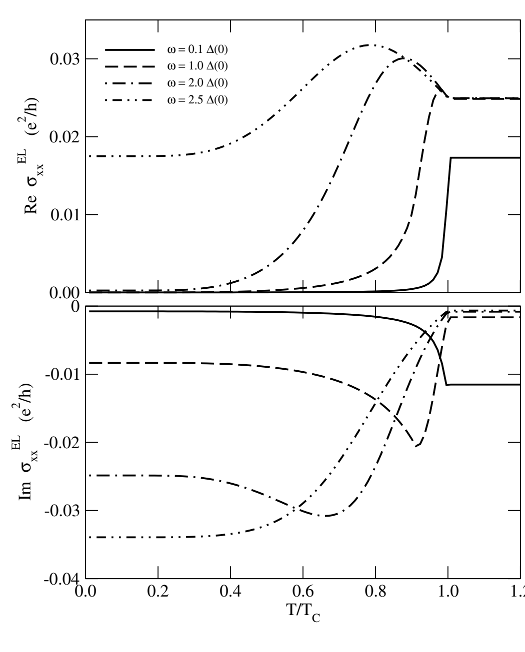

FIG. 2. Real and imaginary parts of the longitudinal conductivity of electrons, $\sigma_{xx}^{\mathrm{EL}}$, for the same parameters as in Fig. 1. In this regime, $\sigma_{xx}^{\mathrm{EL}} \propto 1/\sigma_{yy}^{\mathrm{CF}}$, and because of the strongly enhanced $\mathrm{Im}\sigma_{yy}^{\mathrm{CF}}$ for $\omega \lesssim 0.2\Delta$ (see Fig. 1) there is no sign of the Hebel-Slichter peak in $\sigma_{xx}^{\mathrm{EL}}$ [16].

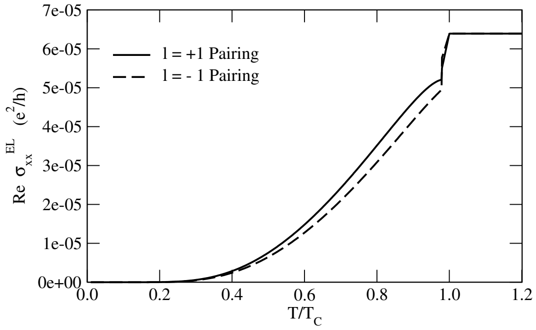

FIG. 3. Longitudinal conductivity of electrons for $\omega = 0.5\Delta(0)$ and $q = 0.75k_F$. Results are shown for pairing angular momentum parallel and antiparallel to the applied field. The small difference indicates the p-wave nature of the pairing is difficult to observe in $\sigma_{xx}^{\mathrm{EL}}$, even at large wave vectors. Results are for $\Delta(0) = 0.01E_F$.

such accuracy would be difficult to achieve, it is worth noting that similar difficulties which arise in obtaining the microwave conductivity from surface impedance measurements of ordinary superconductors were eventually overcome, leading to the observation of BCS coherence peaks [17]. Alternatively, if it were possible to design an experiment which directly measures $1/\rho_{yy}^{\mathrm{EL}} \equiv \sigma_{yy}^{\mathrm{CF}}$, we predict a coherence peak would be observed.

All the results shown to this point are for $q \ll k_F$. This is the regime for which the HLR theory is expected to be qualitatively correct. It must be emphasized that in this limit the p-wave nature of the pairing is irrelevant and the results would be the same for the s wave (up to factors of 2 from the fact that we need two spin states), or any l wave, CF superconductors. The p-wave nature of the pairing becomes relevant only when q is large enough to span parts of the Fermi surface where the phase of the order parameter is significantly different.

A measure of the relevance of the p-wave pairing can be seen by comparing results for which the applied magnetic field is parallel and antiparallel to the pair angular momentum. This corresponds to changing the sign of $\tilde{\phi}$ in (15). For $q \ll k_F$, including all results presented above, there is no measurable difference for these two cases. For $q \sim k_F$, a difference in $\sigma_{xx}^{\mathrm{EL}}$ appears, but it is small. A typical result is shown in Fig. 3.

To summarize, we have developed a phenomenological model of the 5/2 state by adding a chiral p-wave pairing interaction between CFs by hand. The electromagnetic CF response functions for this model were then calculated, including self-consistent fluctuations of the order parameter to ensure gauge invariance, and used in a Chern-Simons RPA calculation of the experimentally measurable electronic longitudinal conductivity. For small q the CF transverse conductivity exhibits a Hebel-Slichter peak, but this peak is not directly observable in the electronic longitudinal conductivity. The smoking gun is therefore hidden but, in principle, can be observed by accurate measurement of the real and imaginary parts of the electronic longitudinal conductivity, or direct measurement of the inverse of the electronic transverse resistivity. We note that the method developed here can be used to make predictions for other electromagnetic response experiments, such as microwave conductivity and resonant Raman scattering [18], and can be applied to analyze a variety of other paired CF states [19,20].

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