Landau Level Mixing and Skyrmions

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The effects of Landau level mixing and finite thickness of the two-dimensional electron gas on the relative stability of skyrmion and single spin-flip excitations at Landau level filling factor $\nu = 1$ have been investigated. Landau level mixing is studied by fixed-phase diffusion Monte Carlo and finite thickness is included by modifying the effective Coulomb interaction. Both Landau level mixing and finite thickness lower skyrmion excitation energies and favor skyrmions with fewer spin flips. However, the two effects do not work 'coherently'. When finite thickness is included the effect of Landau level mixing is strongly suppressed.

1 Introduction

In a seminal paper Sondhi et al.[1] predicted that under appropriate conditions (small enough Zeeman splitting) the quasiparticle excitations of the $\nu = 1$ integer quantum Hall state would be charged 'skyrmions' with topologically nontrivial spin textures. This prediction has since been confirmed by a number of experiments.[2]

We report here on numerical results obtained using trial wave functions recently proposed by MacDonald, Fertig, and Brey[3] which describe skyrmions with well defined spin quantum numbers. In our calculations both the finite thickness (FT) of the two-dimensional electron gas (2DEG) and Landau level mixing (LLM) have been included and shown to lower skyrmion excitation energies and favor skyrmions with fewer spin flips. However, we find that the two effects do not work efficiently together — the FT effect strongly suppresses the LLM effect.

2 Trial Wave Functions for the Spherical Geometry

In the spherical geometry N electrons are confined to move on the surface of a sphere of radius R in the presence of a radial magnetic field B. The total number of flux quanta piercing the surface of the sphere is $N_\phi = 2\,(R/l_0)^2$ where $l_0 = \sqrt{\hbar c/eB}$ is the magnetic length. $N_\phi = N - 1$ for Landau level filling factor $\nu = 1$ and if electron positions are given in terms of the complex coordinate $z = \tan(\theta/2)\exp(-i\phi)$ where θ and ϕ are usual spherical angles the ground state wave function is

$$\psi_{gs} = \prod_k \left(\frac{1}{1+|z_k|^2}\right)^{N_\phi/2} \prod_{i<j}(z_i - z_j) \tag{1}$$

and the hard-core quasihole skyrmion (antiskyrmion) wave function with K reversed spins is[3]

$$\begin{aligned}\psi_{sk}(K) = \quad & \prod_k \quad \left(\frac{1}{1+|z_k|^2}\right)^{(N_\phi+1)/2} \prod_{i<j}(z_i - z_j) \\ & \sum_{j_1 \cdots j_{N-K}} \quad (z_{j_1} \ldots z_{j_{N-K}})(\downarrow_{i_1} \ldots \downarrow_{i_K} \uparrow_{j_1} \ldots \uparrow_{j_{N-K}}).\end{aligned} \tag{2}$$

Here $\uparrow$ and $\downarrow$ are spinors describing up and down spin electron states.

3 Finite Thickness and Landau Level Mixing

The FT of the lowest subband wave function of a 2DEG leads to a significant softening of the short-range part of the electron-electron interaction. In our calculations this effect has been included through the use of the modified interaction $V(r) = e^2/\epsilon\sqrt{r^2+\beta^2}$ where β is a parameter characterizing the thickness.[4]

The importance of LLM for a given system is determined by the electron gas parameter $r_s = (\nu/2)^{1/2}(e^2/\epsilon l_0)/\hbar\omega_c$ where $\hbar\omega_c = \hbar eB/mc$ is the cyclotron energy. In the limit $r_s \to 0$ the cyclotron energy is much larger than the Coulomb energy and there is no LLM. However, in typical experiments r_s is of order 1 or higher and so LLM cannot be ignored. In the present calculation LLM has been included by using the trial wave functions ψ_{gs} and $\psi_{sk}(K)$ as guiding functions for a fixed-phase diffusion Monte Carlo simulation.[5,6]

The experimentally measured transport gap of the $\nu = 1$ state is equal to the energy for creating a skyrmion-antiskyrmion pair with infinite separation. In the absence of LLM particle-hole symmetry ensures that the energy required to create a skyrmion is equal to that for creating an antiskyrmion. However, in our calculations LLM is included and this equality can no longer be assumed. Thus in order to calculate a physically meaningful quantity we have computed the energy gap $\Delta_{S=K+1}$ for creating a spin K antiskyrmion and a quasielectron with a single spin-flip.[7]

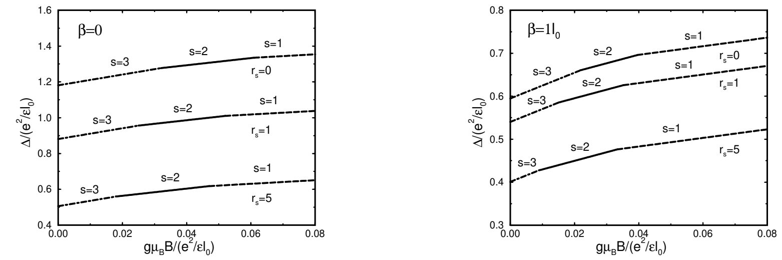

Figure 1: Energy gaps for creating the skyrmion excitations described in the text for $\beta = 0$ (left) and $\beta = 1l_0$ (right).

4 Results

Figure 1 shows results for the dependence on the parameter $\tilde{g} = g\mu_B B/(e^2/\epsilon l_0)$ of the energy gap for creating the skyrmion excitation described above for different values of total spin $S = K+1$ and for $r_s = 0, 1$ and 5. In these figures the lines

$$\Delta_S(\tilde{g}) = \Delta_S(\tilde{g} = 0) + S\tilde{g}\frac{e^2}{\epsilon l_0} \tag{3}$$

are plotted for $S = 1, 2$ and 3 only over the range of $\tilde{g}$ values for which they correspond to the lowest energy excitation. All results are for 30 electrons.

For $\beta = 0$ and $r_s = 0$ our results are consistent with previous calculations.[8] As r_s is increased we find the skyrmion energy gaps decrease by an amount which grows as the number of spin flips decrease. Thus the effect of LLM is to stabilize skyrmions with fewer spin flips.

For $\beta = 1l_0$ and $r_s = 0$ FT is seen to have the same effect as LLM for $\beta = 0$ — the energy gap is lowered and skyrmions with fewer spin flips become more stable. However once FT is included the additional lowering of the energy gaps due to LLM is strongly suppressed. Thus we conclude that while both LLM and FT have qualitatively similar effects on skyrmions, they do not work 'coherently'.

Despite the fact that the error bars on the energy gaps shown in Fig. 1 are quite small there is significant uncertainty in the value of the critical $\tilde{g}_c$ below which the single spin-flip excitation becomes unstable to skyrmion formation $(\Delta_{S=1}(\tilde{g}_c) = \Delta_{S=2}(\tilde{g}_c))$. These values together with their statistical errors are given in Table 1. Again, one can clearly see that both FT and LLM tend to lower $\tilde{g}_c$. Note also that the effect of LLM on $\tilde{g}_c$ for $\beta = 1l_0$ is quite weak, consistent with the observations given above.

Table 1: Critical $\tilde{g}$ below which the single spin flip excitation is unstable to skyrmion formation. Results are for 30 electrons.

	$r_s = 0$	$r_s = 1$	$r_s = 5$
$\tilde{g}_c$ for $\beta = 0$	0.061(5)	0.052(5)	0.047(5)
$\tilde{g}_c$ for $\beta = 1 l_0$	0.039(4)	0.035(4)	0.033(4)

5 Conclusion

To summarize, using skyrmion trial states recently introduced by MacDonald, Fertig, and Brey[3] we have performed variational and diffusion Monte Carlo calculations of the effect of FT and LLM on skyrmion excitations at $\nu = 1$. We find that both these effects lower the skyrmion excitation energies and stabilize skyrmions with fewer spin flips. However, we also find that the two effects do not work coherently together — when FT is included the effect of LLM is strongly suppressed.

Acknowledgements

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