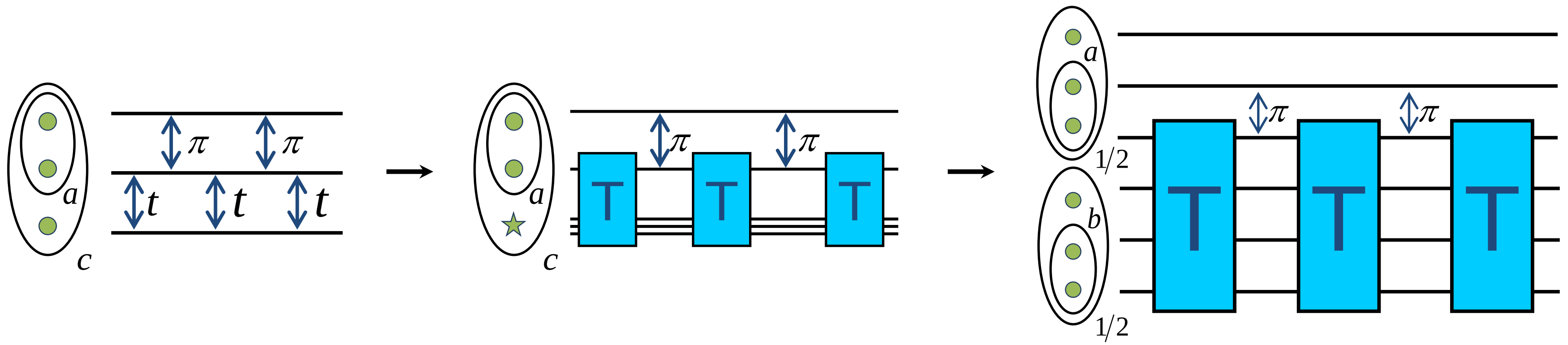
Designing Pulse Sequences for Exchange-Only Quantum Computation

Nick Bonesteel Florida State University



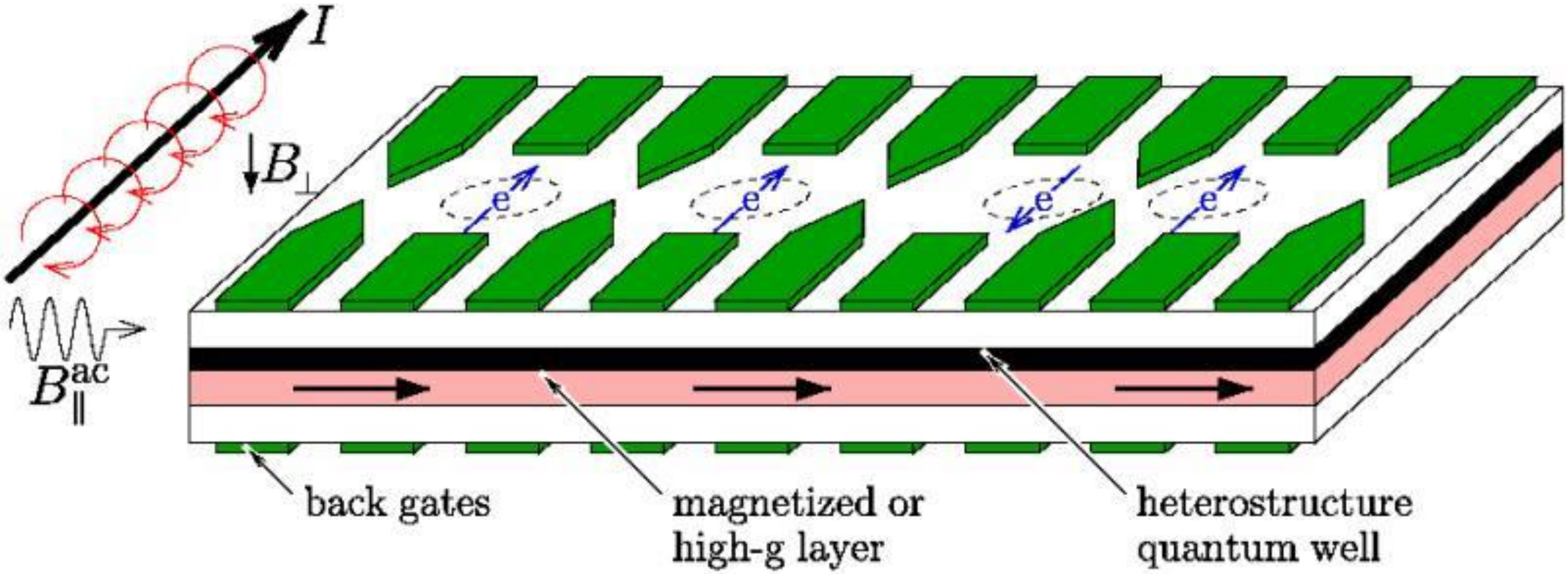
Work done in collaboration with:

Daniel Zeuch, Peter Gruenberg Institut, Research Center Juelich

Brookhaven National Laboratory, Nov. 20, 2019

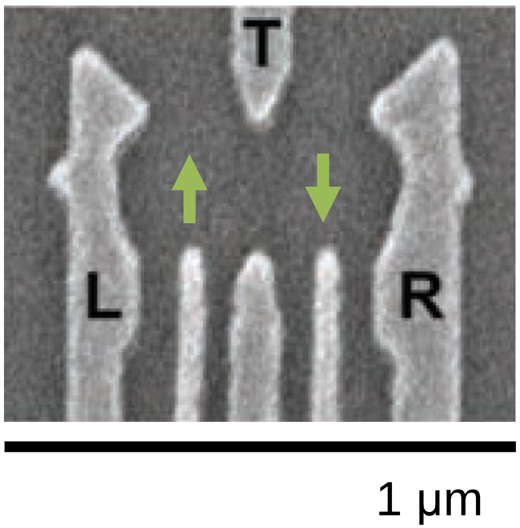
Early Vision of a Solid State Quantum Computer

Loss & DiVincenzo, Phys. Rev. B (1998)

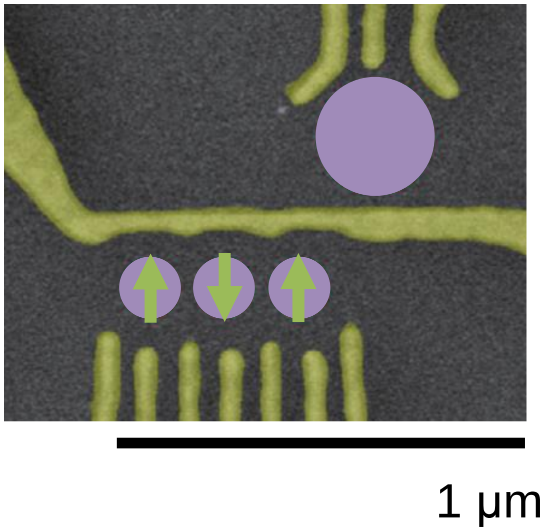


Decades of Slow Steady Progress

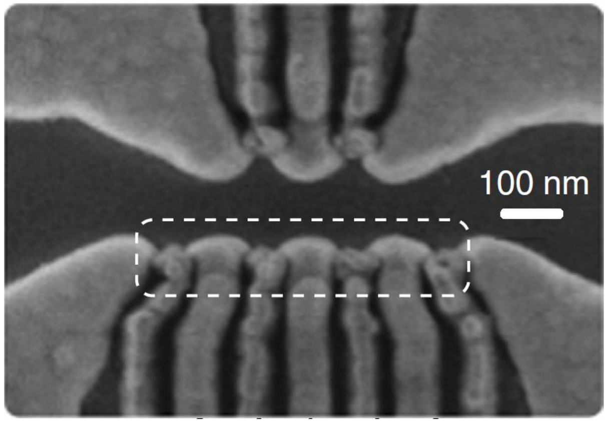
Petta *et al*., *Science* (2005)



Medford *et al*., *Nature Nanotechnology* (2013)

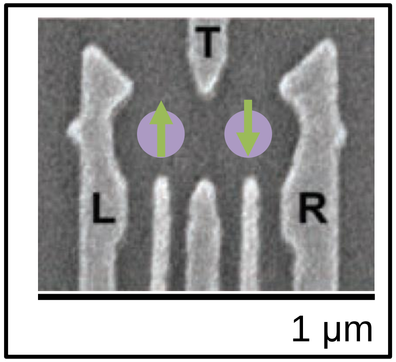


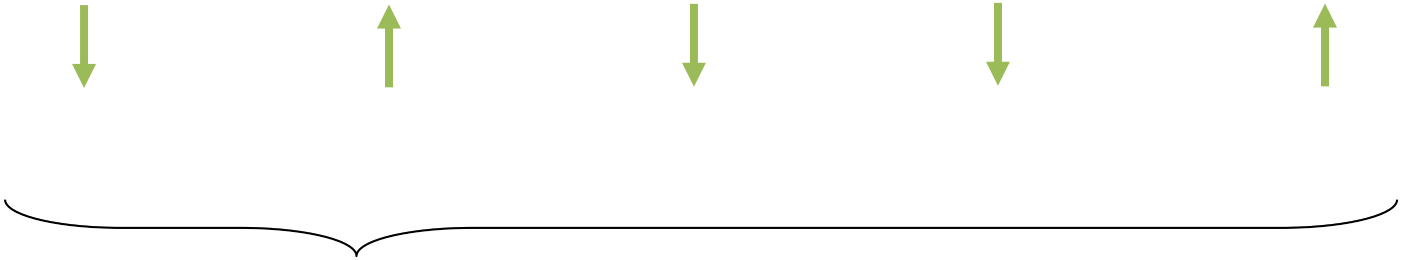
Andrews *et al*., *Nature Nanotechnology* (2019)



Basic Idea

- Use electron spins as qubits



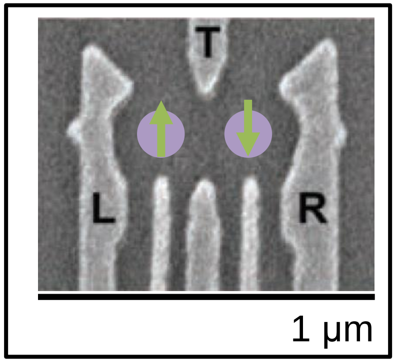


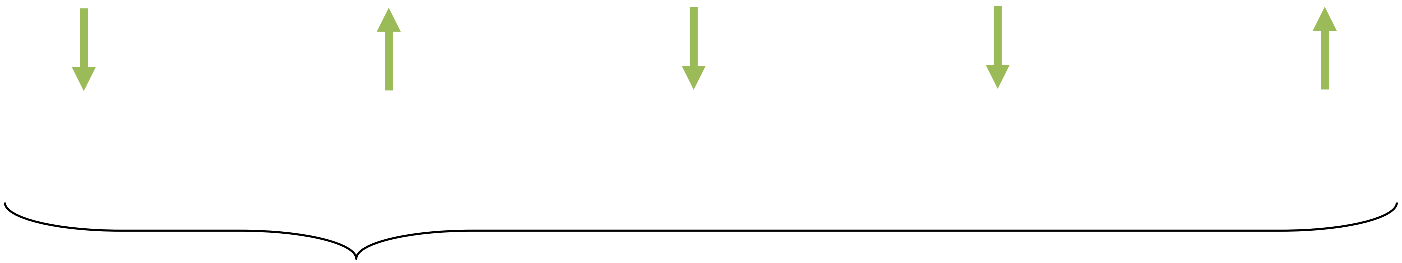
spin-1/2 chain: electrons
in quantum dots

Exchange-Based QC

- Quantum gates through spin exchange

$$H_i = J\ \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$





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Bacon, Kempe, Lidar & Whaley, *Phys. Rev. Lett.* (2000)

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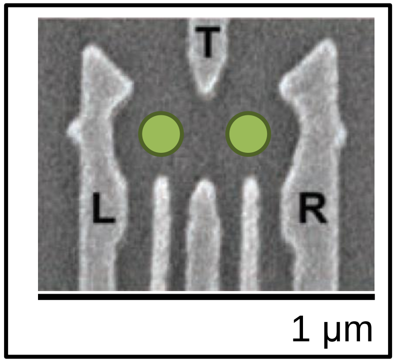
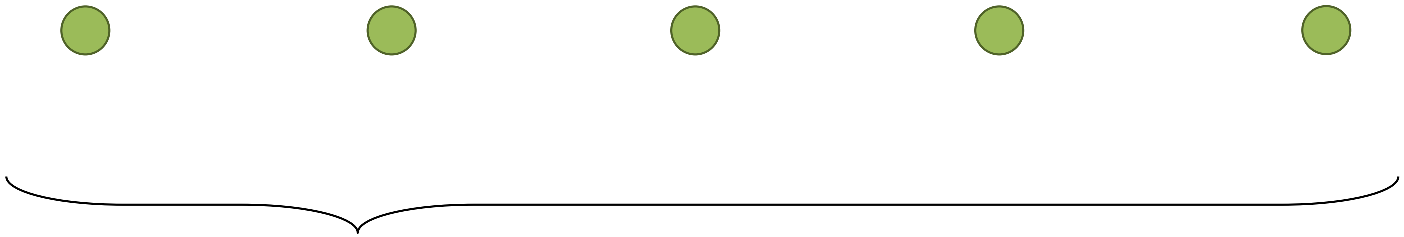


$$H_i = J\ \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$



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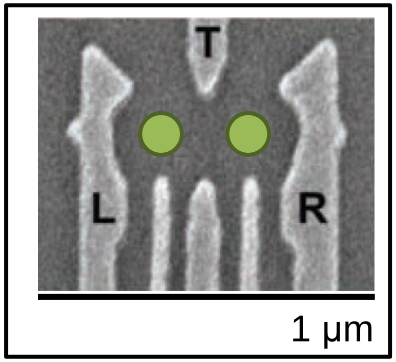

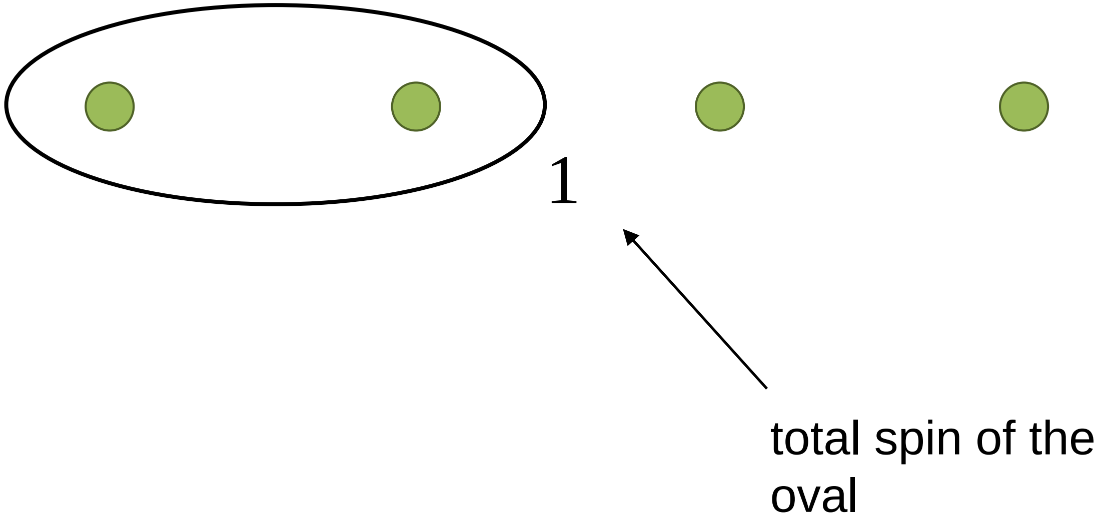

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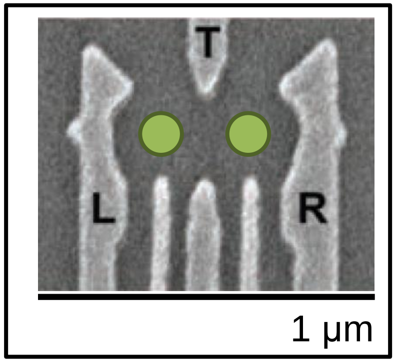

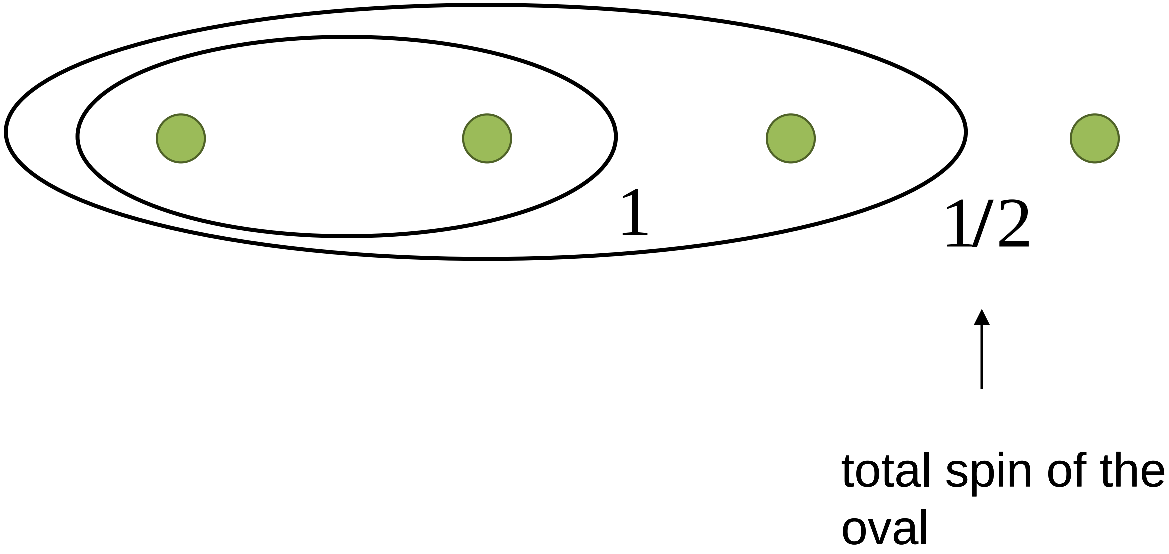

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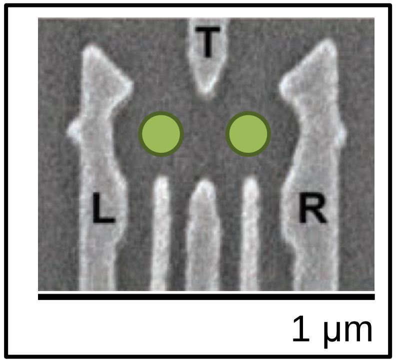

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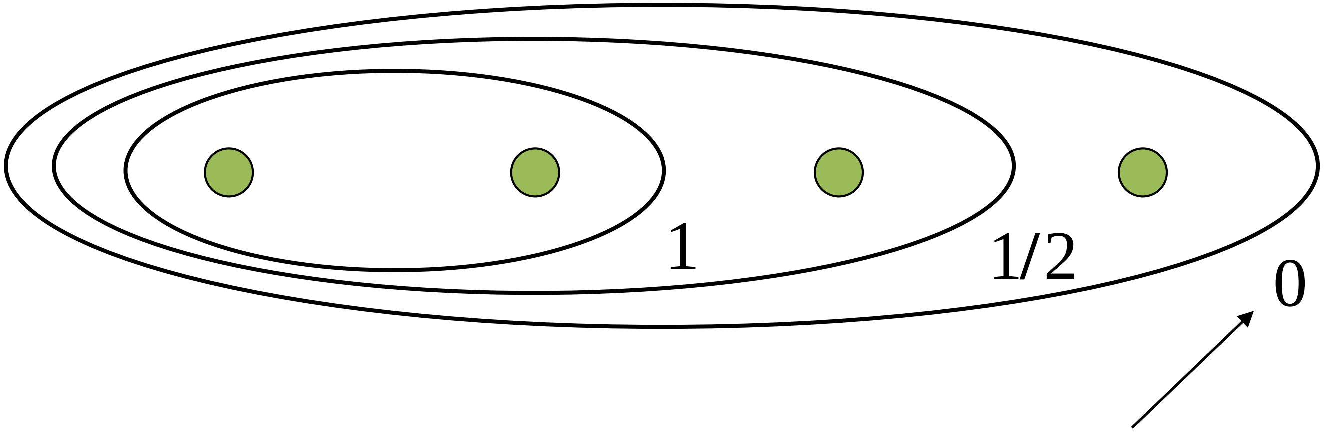
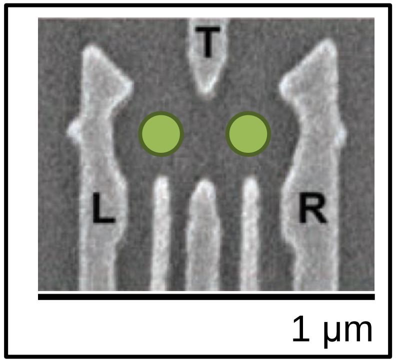


Exchange-Based QC



- Quantum gates through spin exchange

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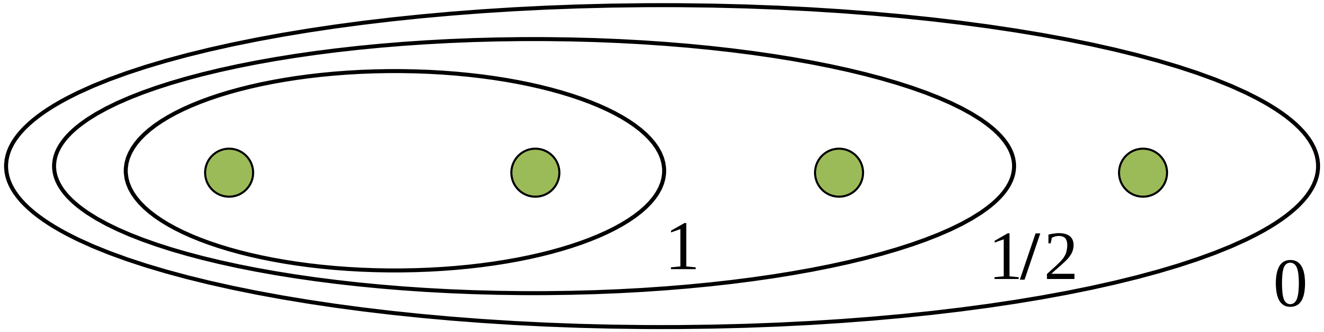
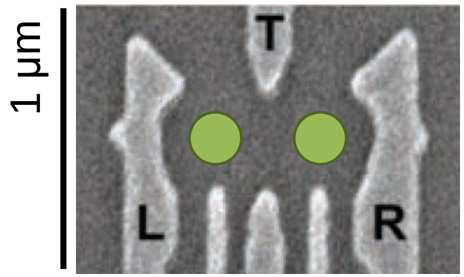


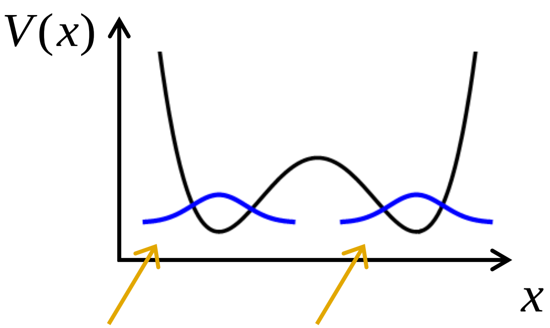
$$s_1 \otimes s_2 = |s_1 - s_2|,\ |s_1 - s_2 + 1|,\ \ldots,\ s_1 + s_2$$

Bacon, Kempe, Lidar & Whaley, *Phys. Rev. Lett.* (2000)

Controlling Exchange

Petta *et al.*, *Science* (2005)

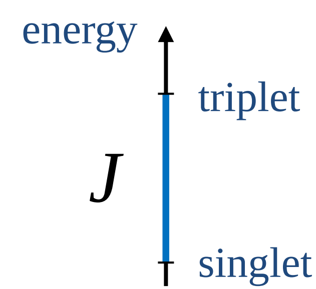




Electron wave functions in quantum dot potential $V(x)$

- Exchange Hamiltonian

$$H = J\, \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$



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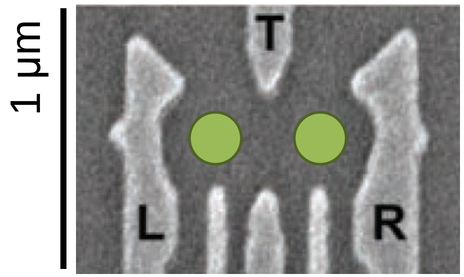

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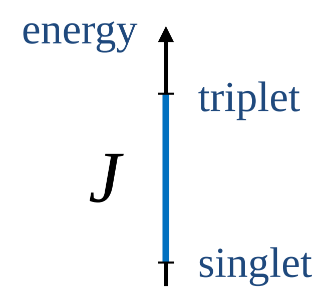

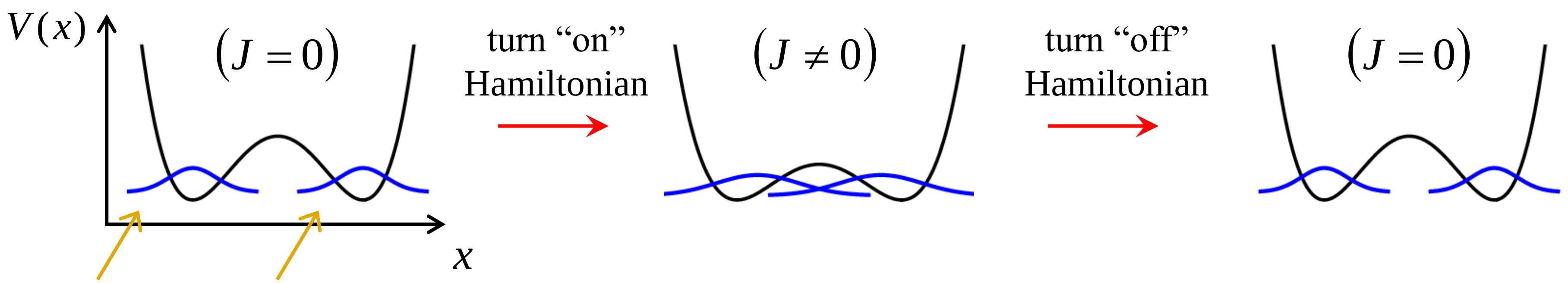

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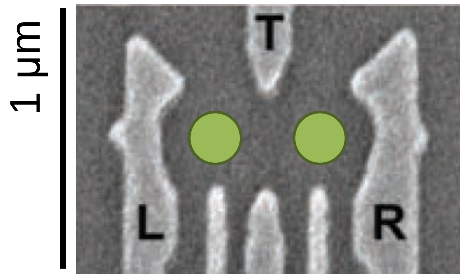

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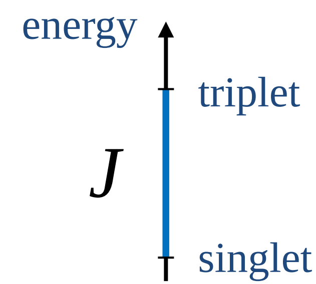
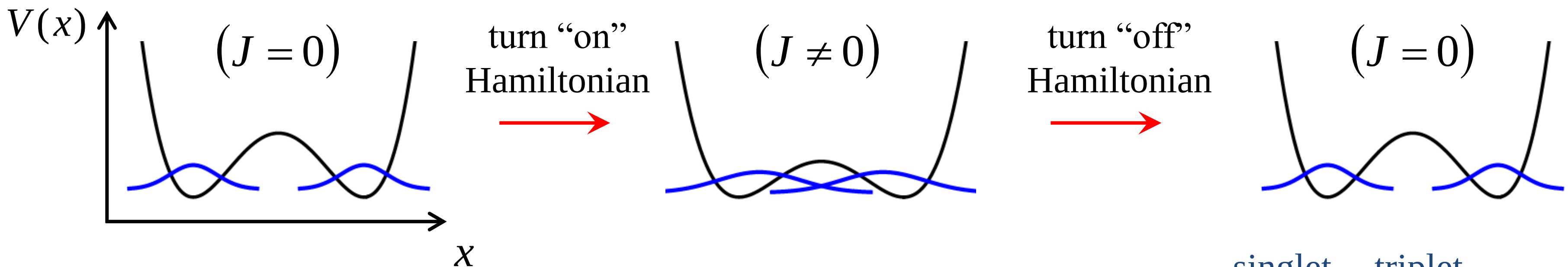
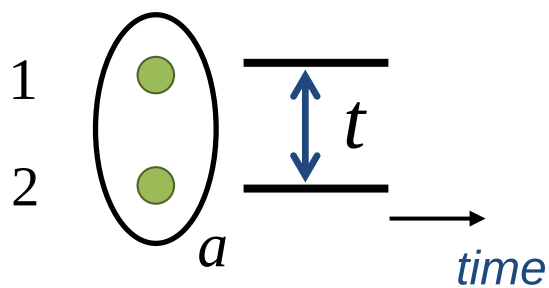




singlet triplet

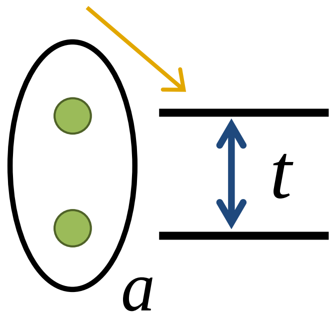
$a = \; 0 \quad 1$



$$\exp(-iHt) = \begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix} \qquad (J = 1)$$

Simple Exchange Pulses

exchange pulse of duration t

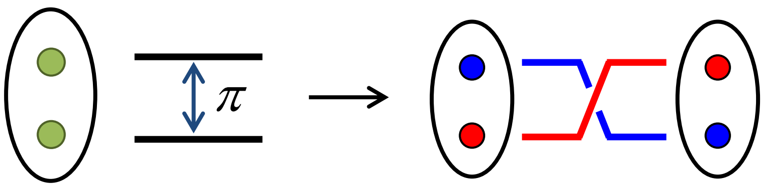


$$a = \quad 0 \qquad 1$$

$$\begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

- SWAP pulse

$$\boxed{t = \pi}$$

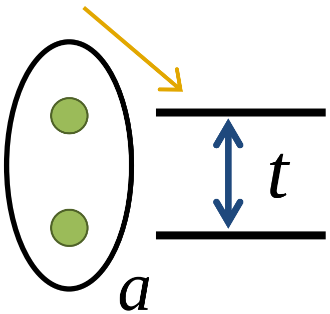


$$a = \quad 0 \qquad 1$$

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

Simple Exchange Pulses

exchange pulse of duration t

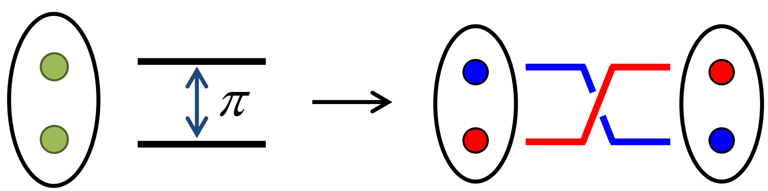


$$a = \quad 0 \quad 1$$

$$\begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

- SWAP pulse

$t = \pi$



$$a = \quad 0 \quad 1 \qquad\qquad 0 \quad 1$$

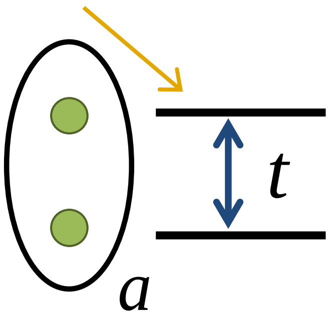
$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = -\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

singlet state $(a = 0)$ $= \left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle$

triplet states $(a = 1)$ $= \begin{cases} \left|\uparrow\uparrow\right\rangle \\ \left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle \\ \left|\downarrow\downarrow\right\rangle \end{cases}$

Simple Exchange Pulses

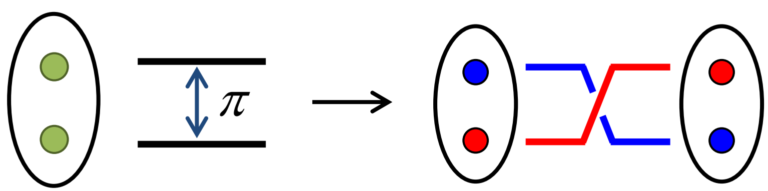
exchange pulse of duration t



$$a = \begin{matrix} 0 & 1 \end{matrix} \qquad \begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

- SWAP pulse

$t = \pi$

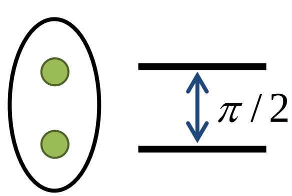


$$a = \begin{matrix} 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 \end{matrix}$$

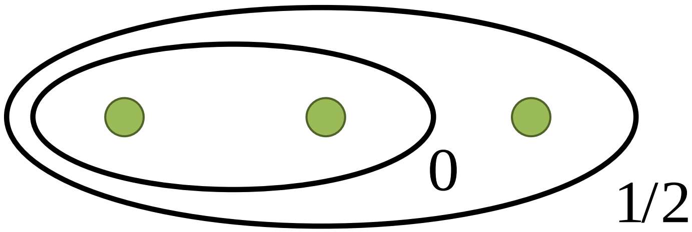
$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = -\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

- SWAP$^{1/2}$ pulse

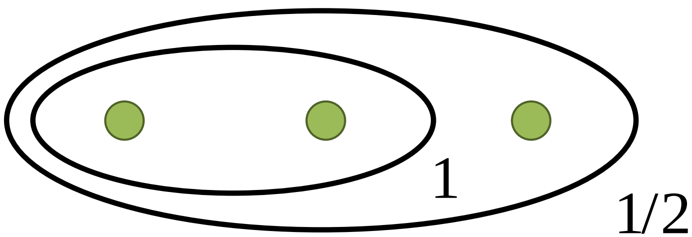
$t = \pi/2$



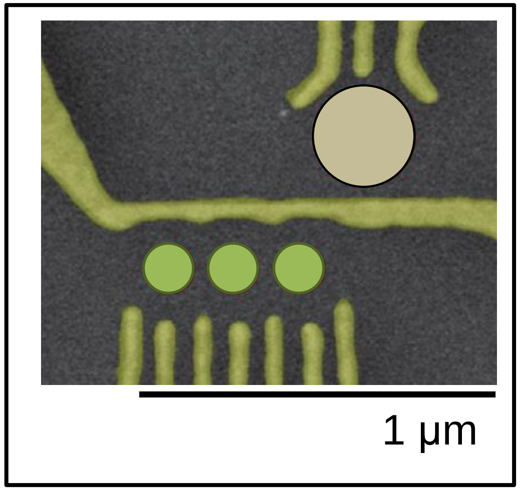
Three-Spin Qubit Encoding

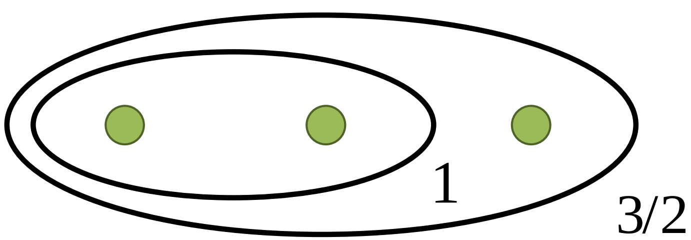


$= |0\rangle$



$= |1\rangle$





$= |\text{non-computational}\rangle$

DiVincenzo *et al.*, *Nature* (2000)

Medford *et al.*, *Nature Nanotechnology* (2013)

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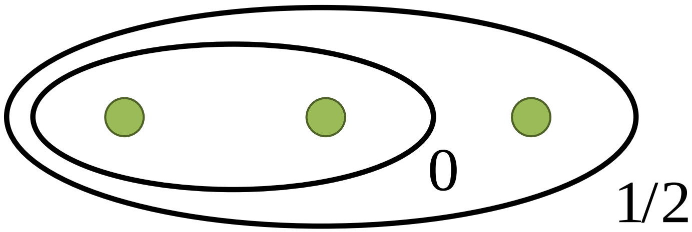
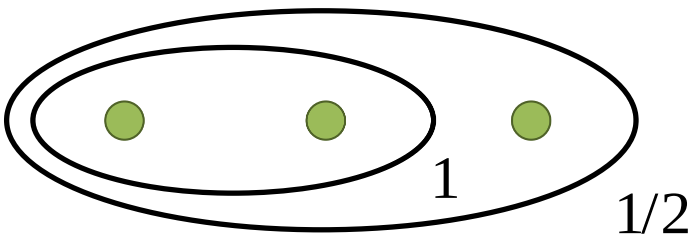


$= |0\rangle$



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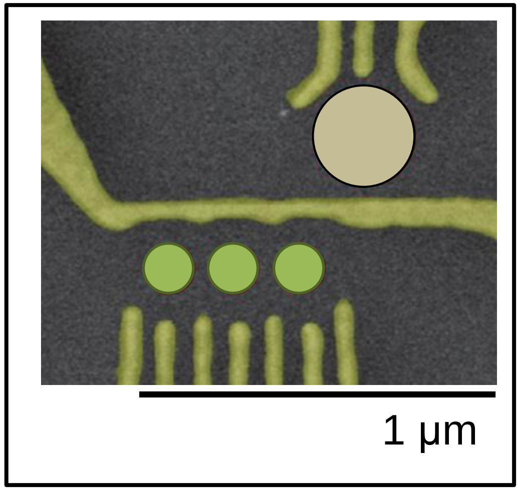
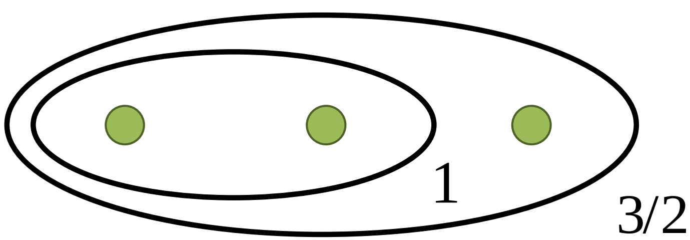




$= |\text{non–computational}\rangle$

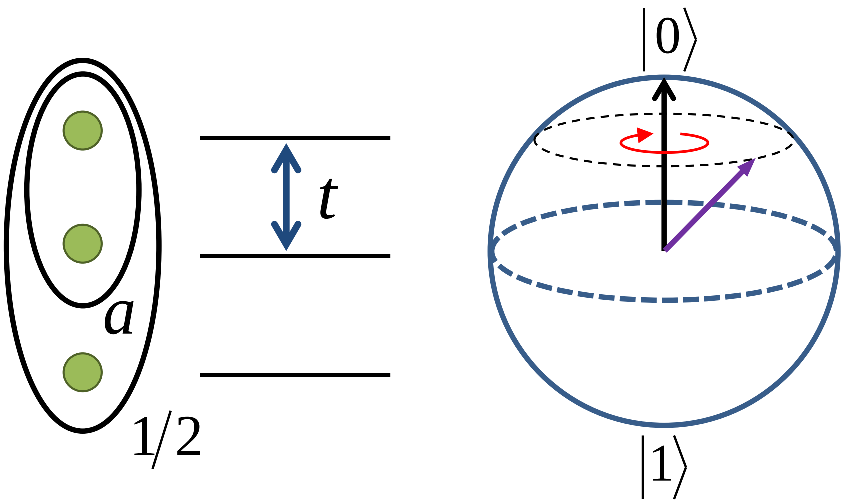
Transitions to this state are **leakage errors**.

DiVincenzo *et al.*, *Nature* (2000)

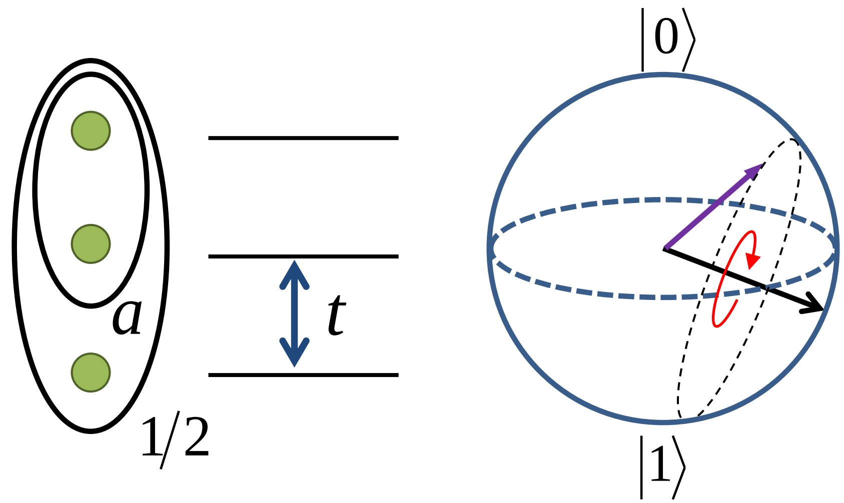
Medford *et al.*, *Nature Nanotechnology* (2013)

Single-Qubit Gates

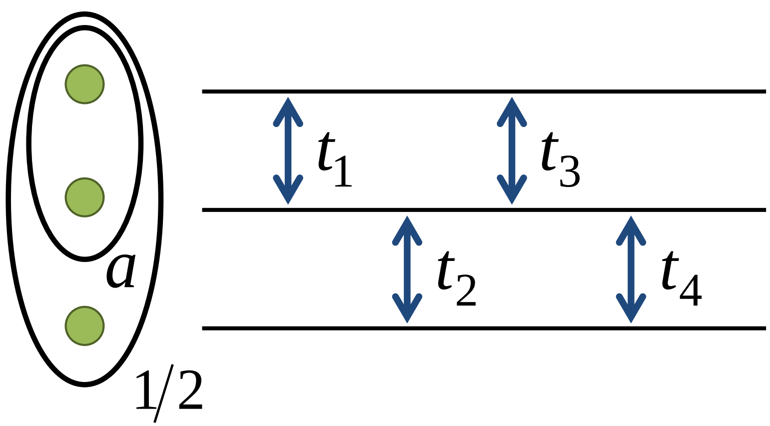
- Rotation about *z*-axis :



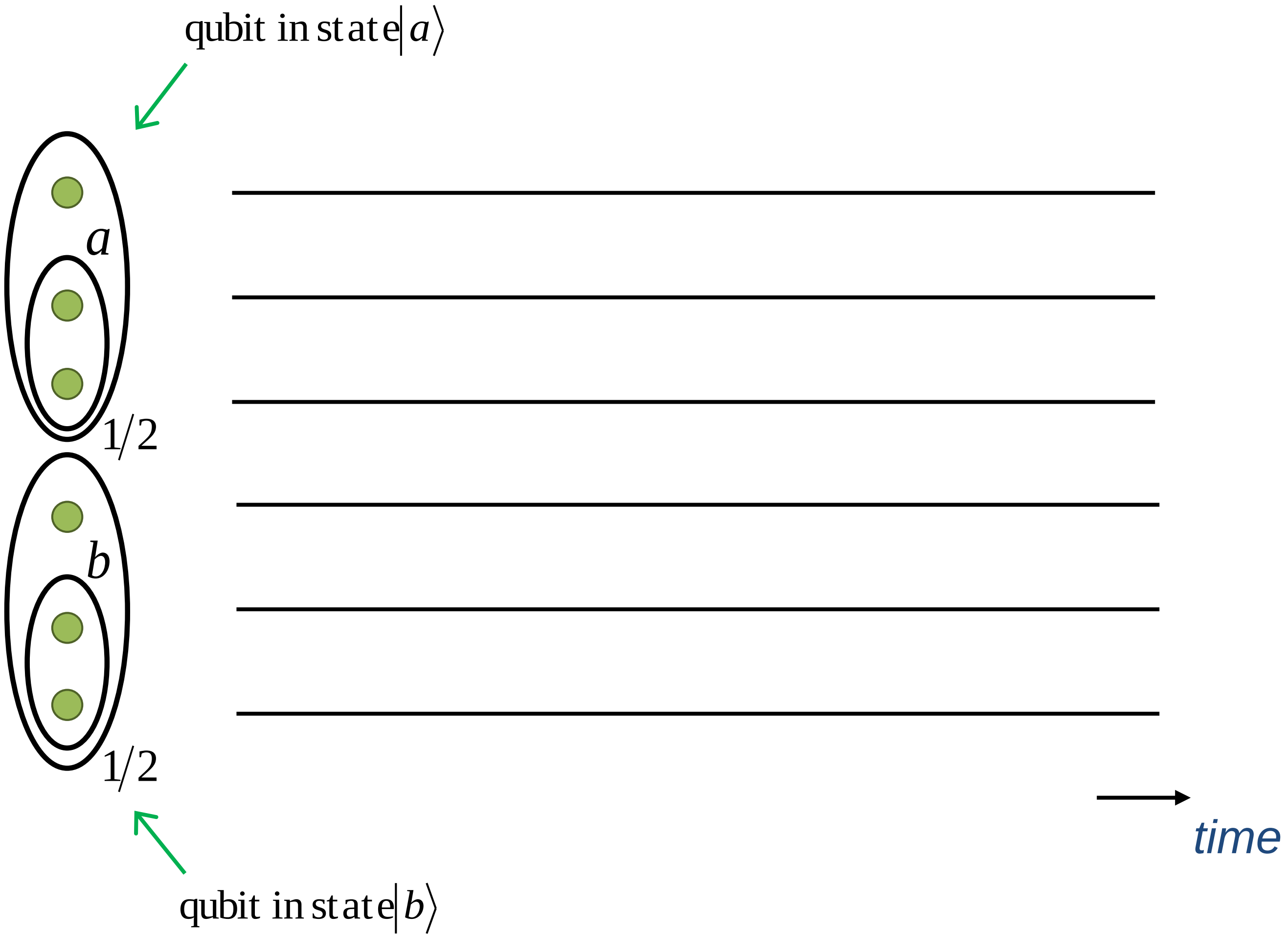
- Rotation about other axis:



arbitrary rotations
$\Rightarrow$



Two-Qubit Gates



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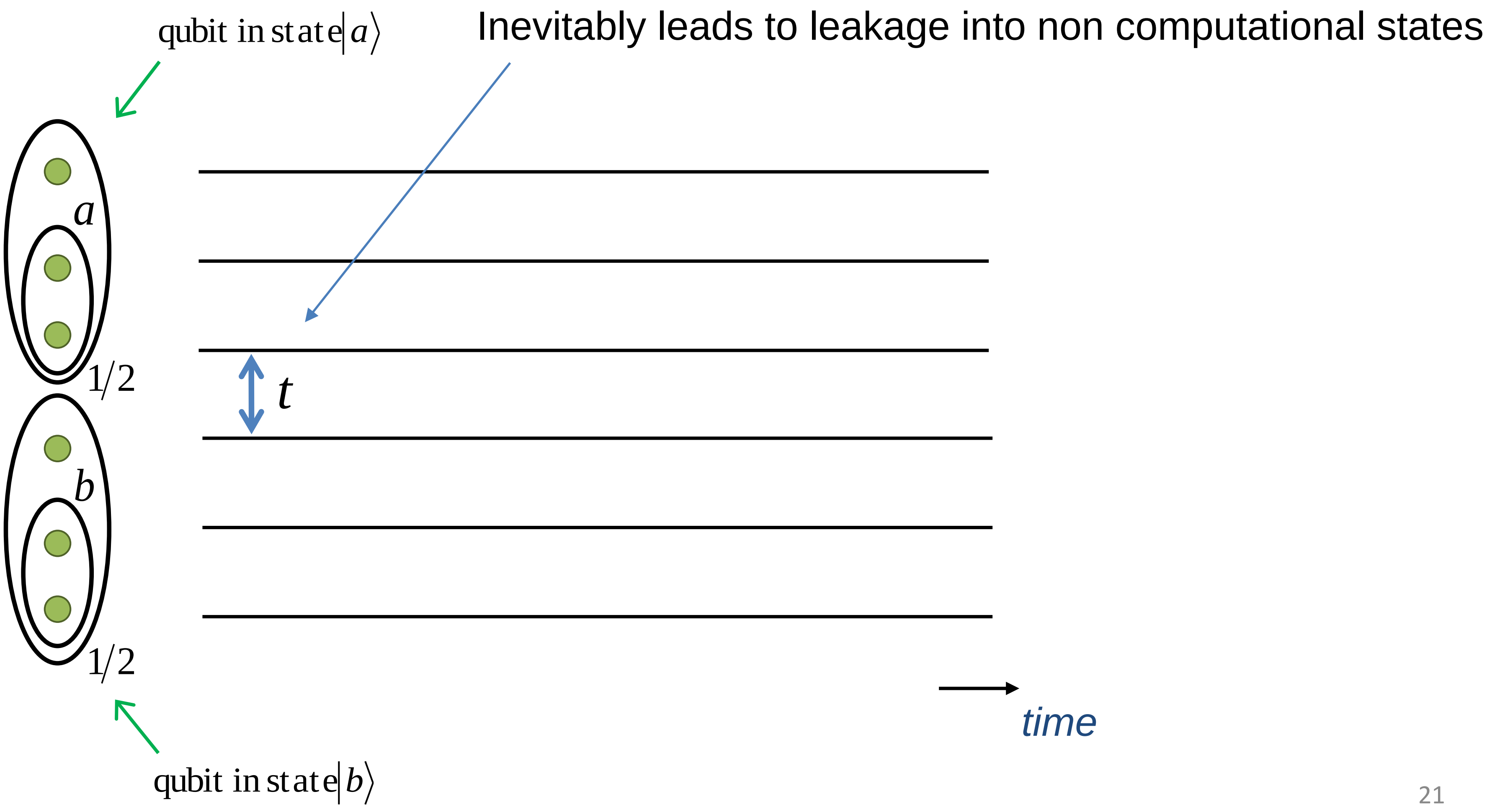
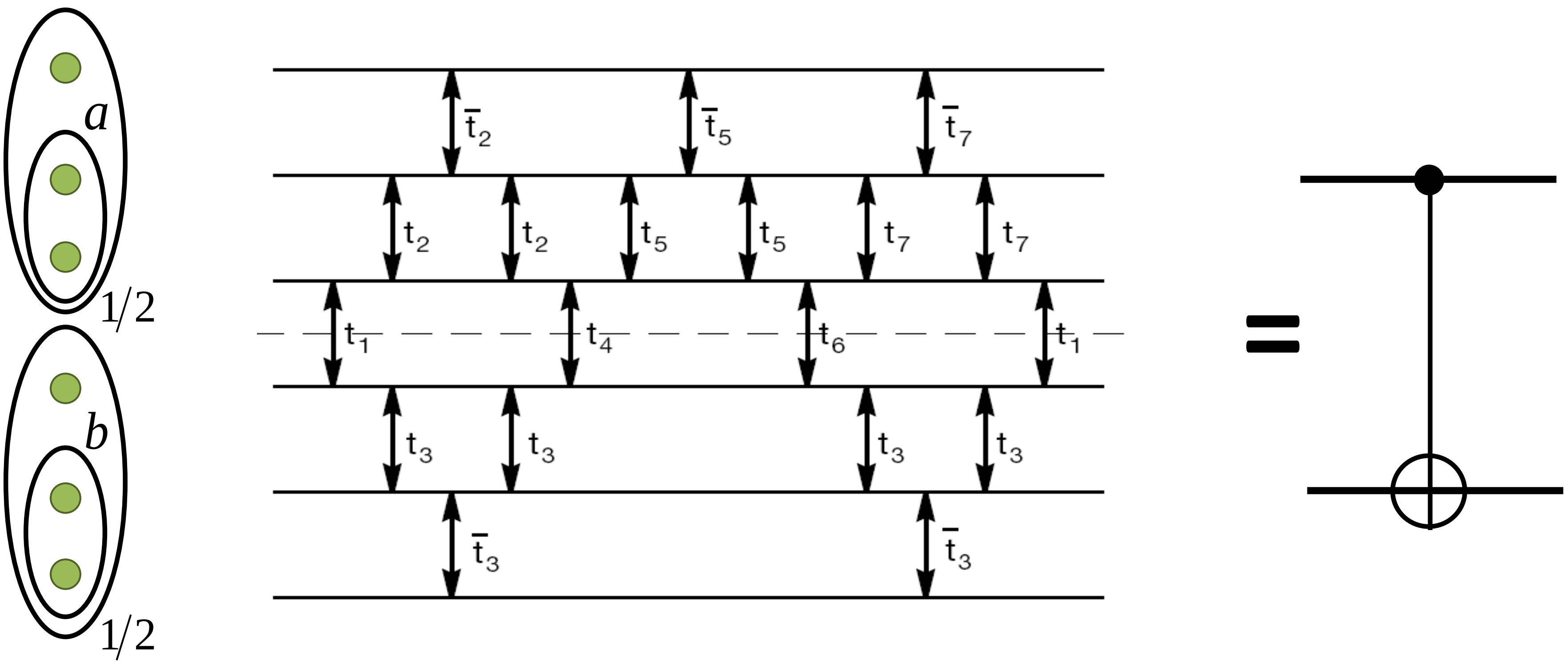


Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically

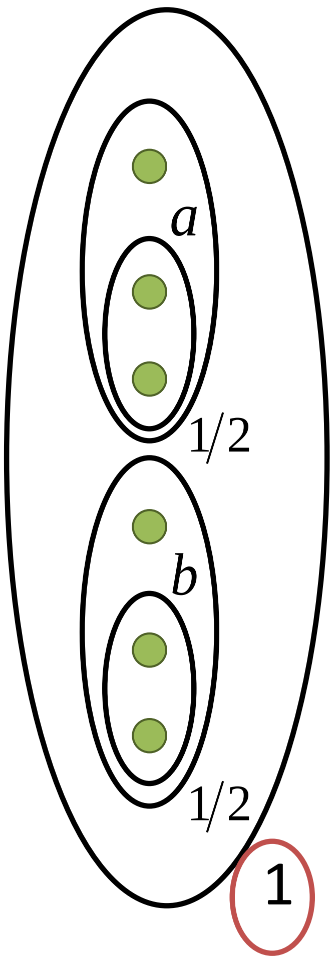


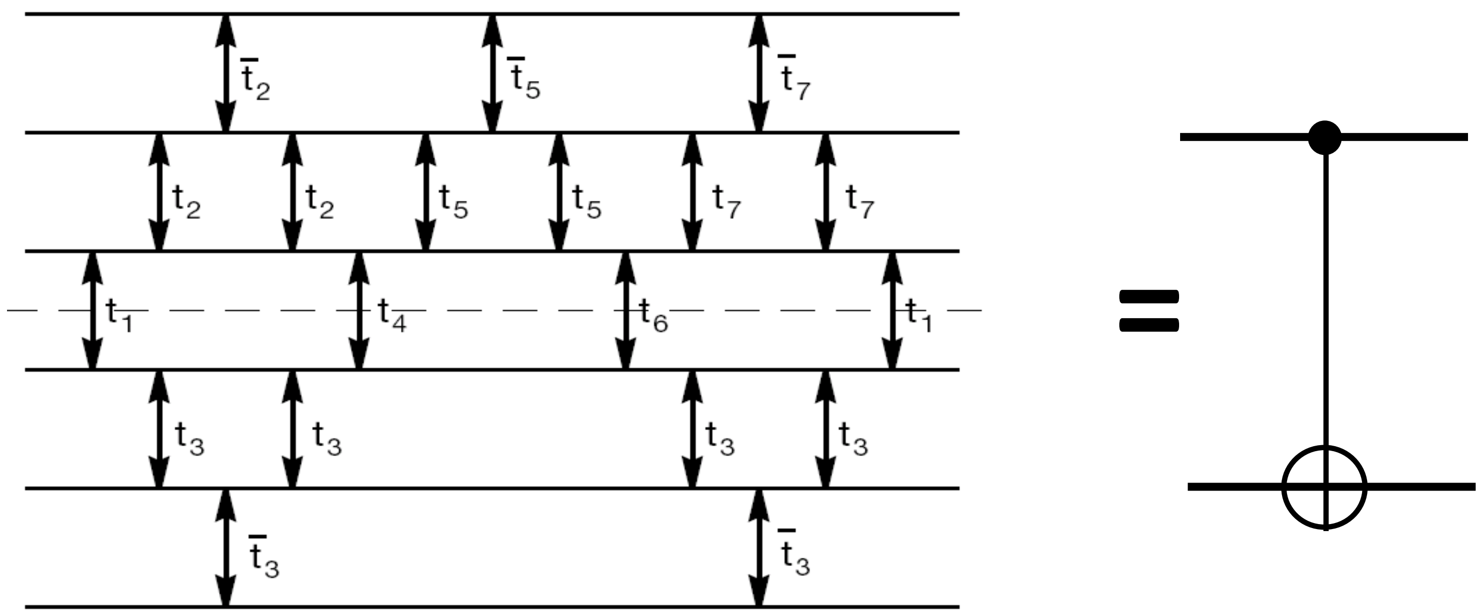
$$t_1 = 2.581.. , \quad t_2 = 1.303.. , \quad t_3 = 1.753.. , \quad \ldots$$

Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically



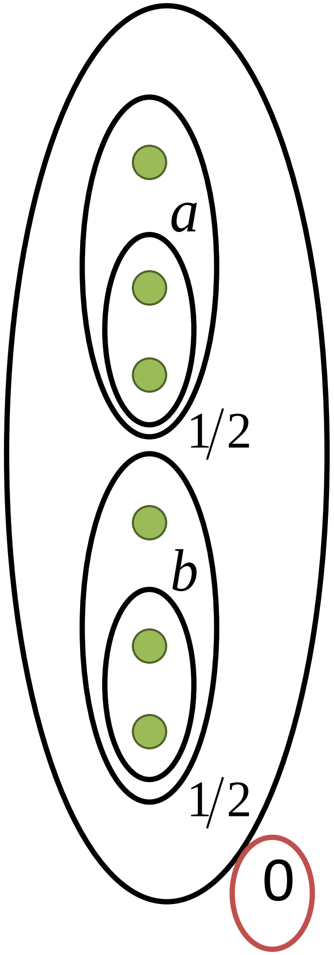


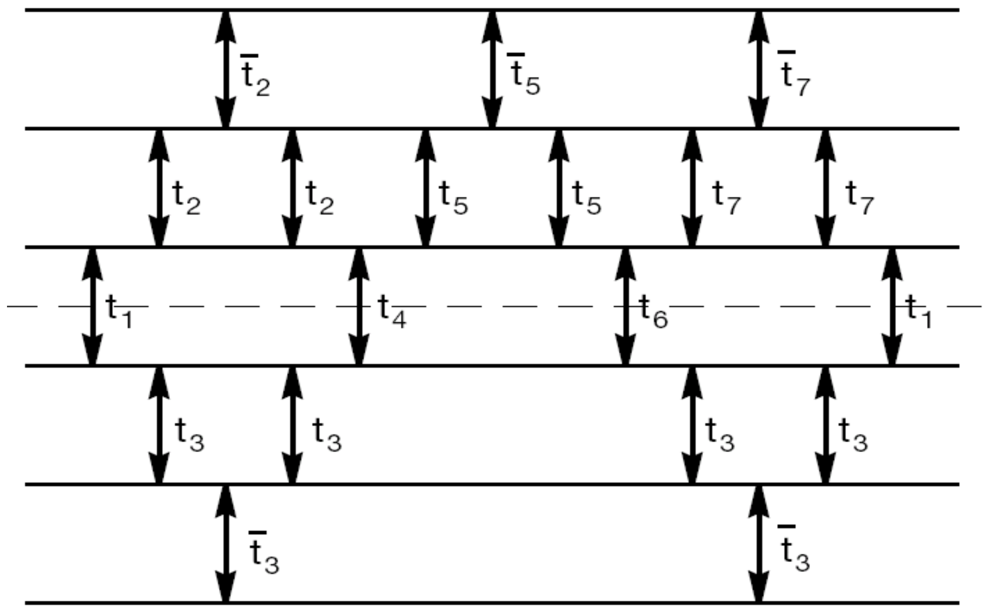
Gives a CNOT (up to single qubit operations) if the total spin is 1

Two-Qubit Gates

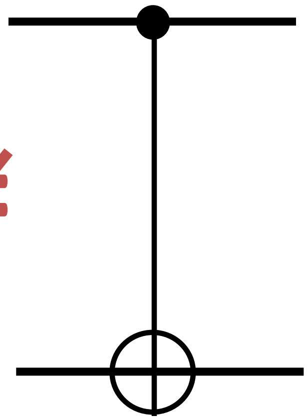
DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically





$\neq$

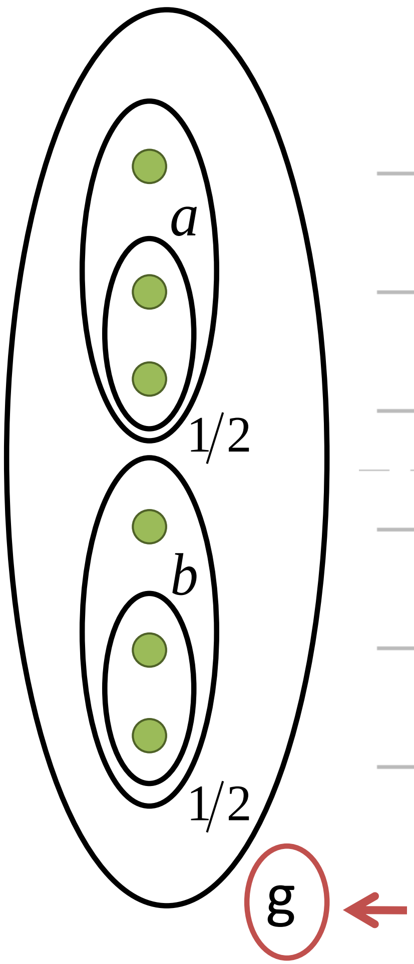


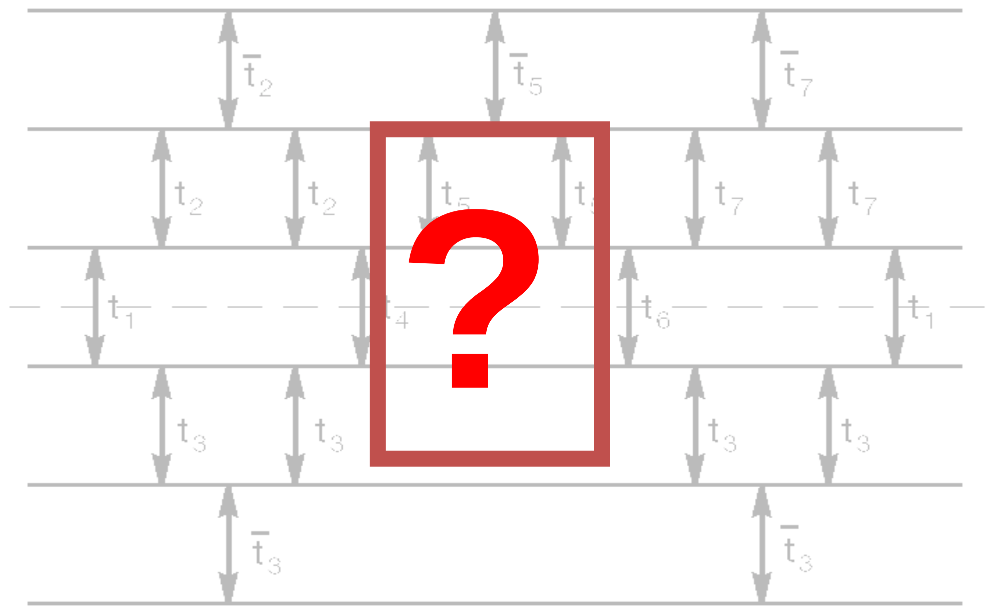
← Does not give a CNOT if the total spin is 0

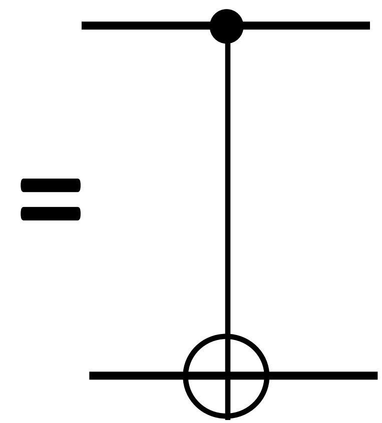
Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically





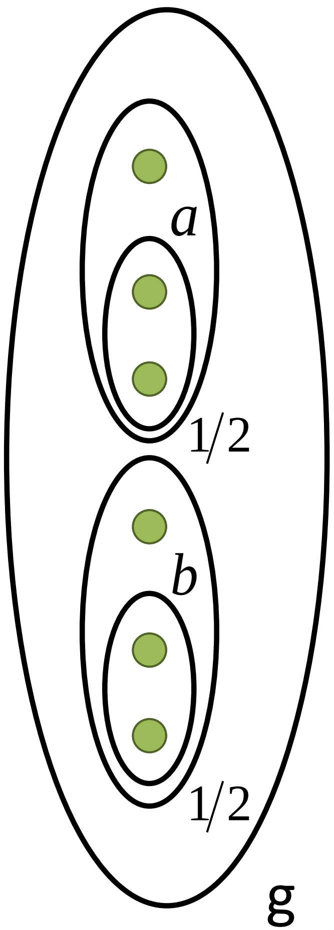


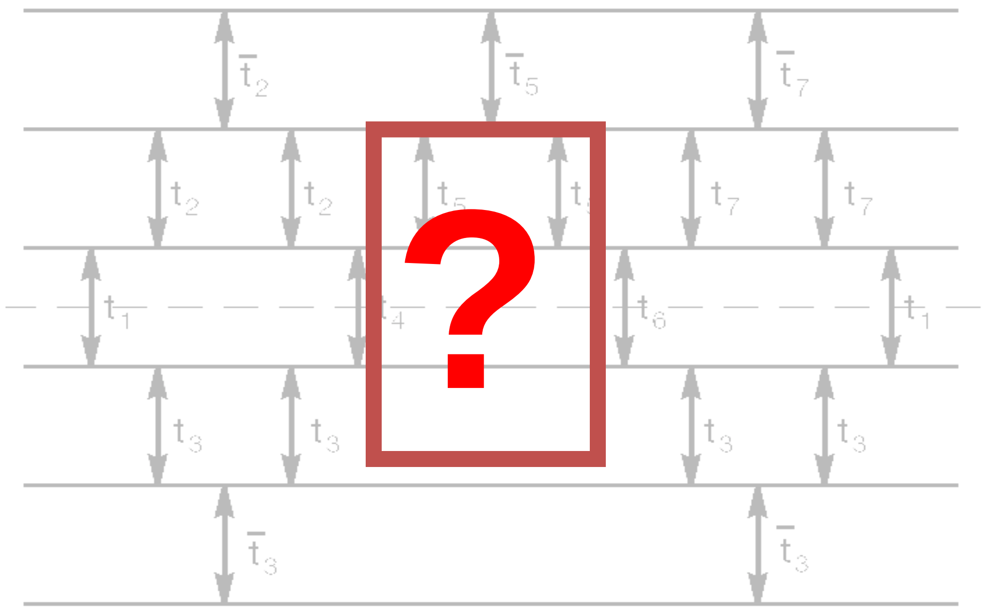
Question: Can we find a pulse sequence which gives identical entangling two qubit gates in the g=0 and g=1 sectors?

Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically





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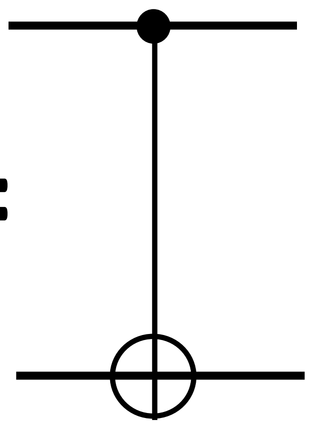
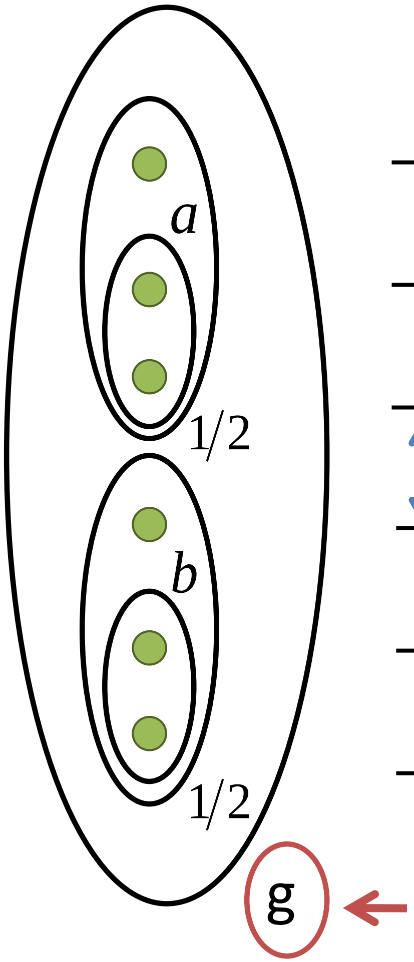


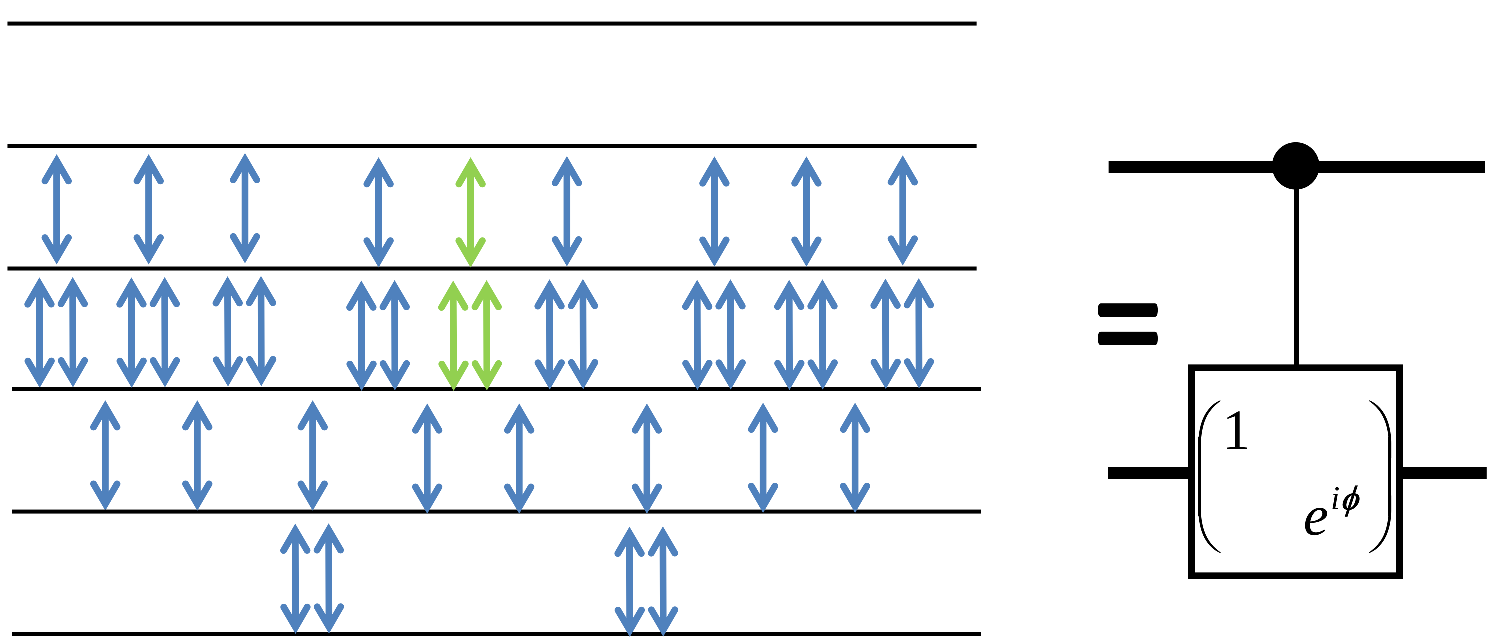
"… our numerical studies have failed to identify an implementation (even a good approximate one) for sequences of up to 36 exchanges…"

Answer: Yes

D. Zeuch, R. Cipri, NEB, *Phys. Rev. B* (2014)

39 pulse sequence found analytically



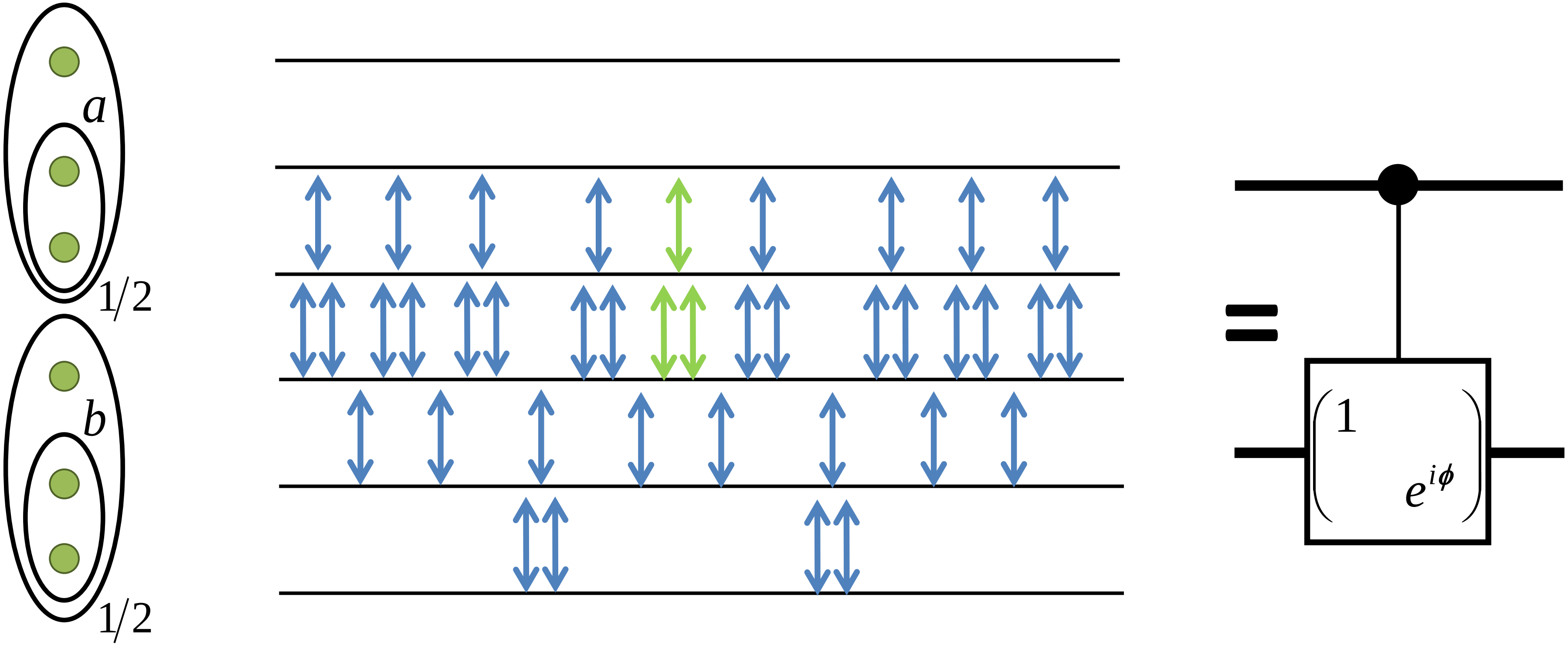


Give same gate for g=0 and 1

Answer: Yes

D. Zeuch, R. Cipri, NEB, *Phys. Rev. B* (2014)

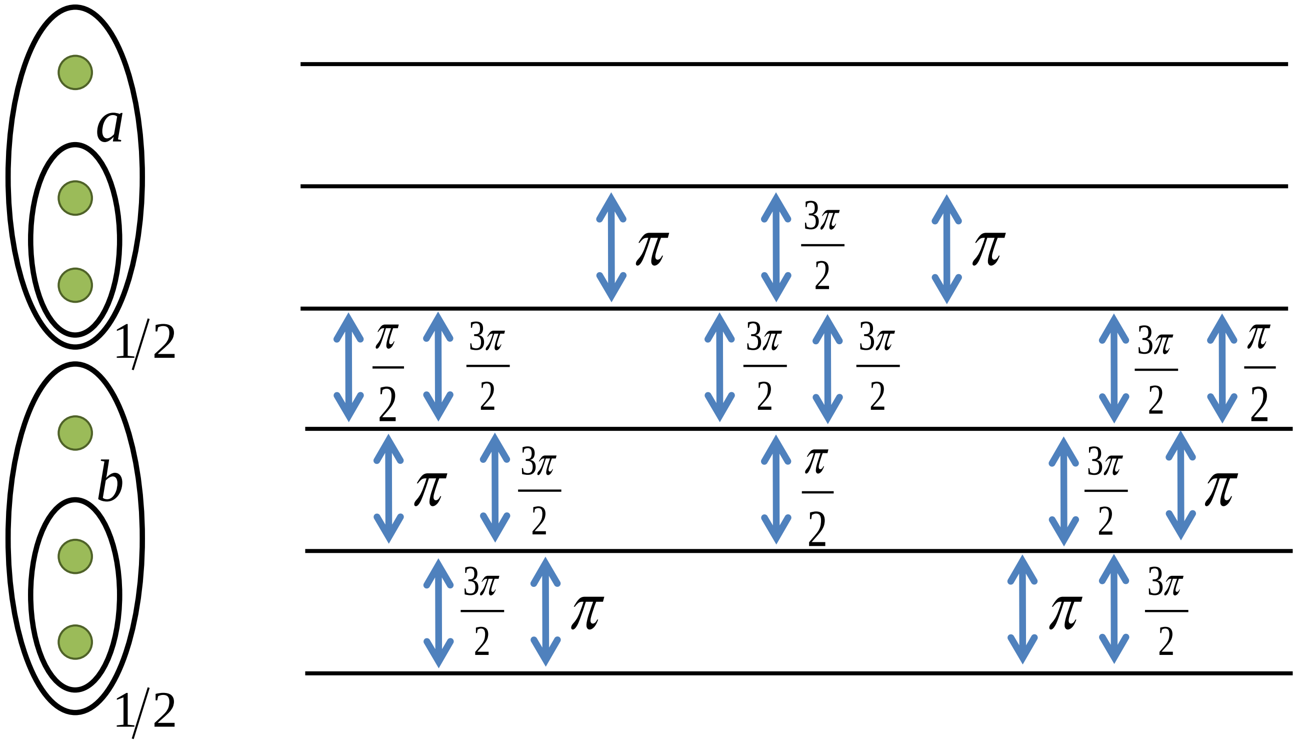
39 pulse sequence found analytically



Fong-Wandzura Sequence

Fong & Wandzura, *Quantum Information and Computation* (2011)

18 pulse sequence found numerically



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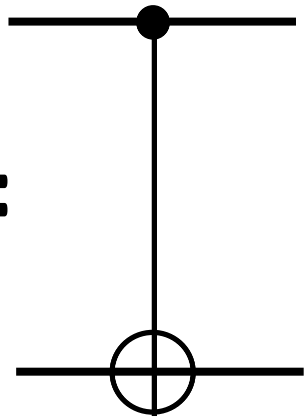
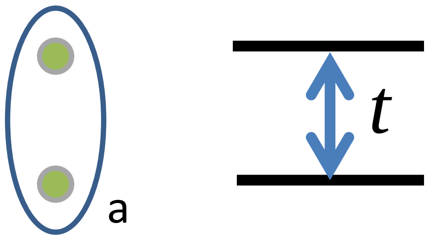


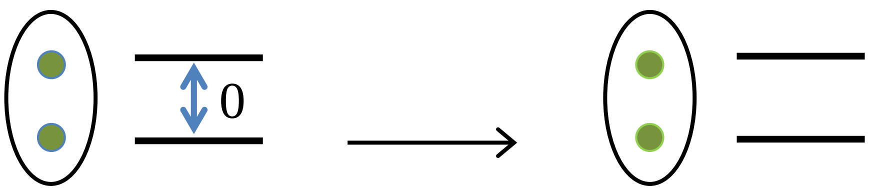
“Classical” Exchange Pulses



$a = 0 \quad 1$

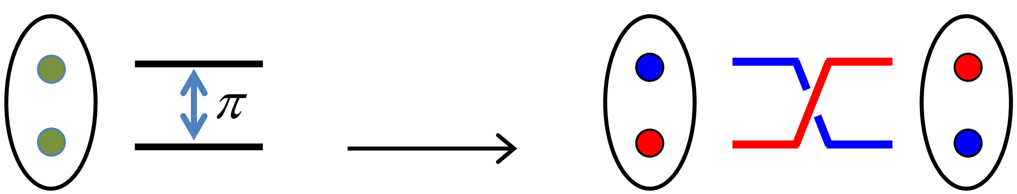
$$\begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

$t = 0$



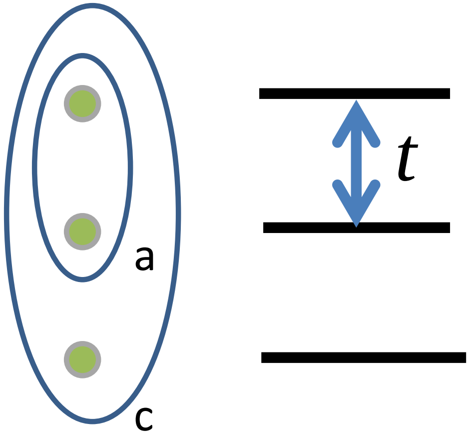
$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$t = \pi$



$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

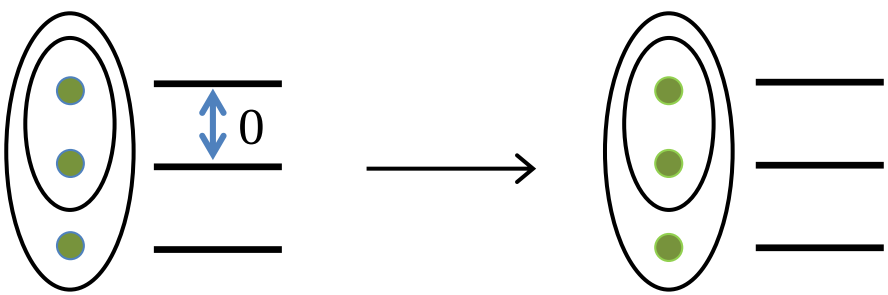
"Classical" Exchange Pulses



$$ac = \quad 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

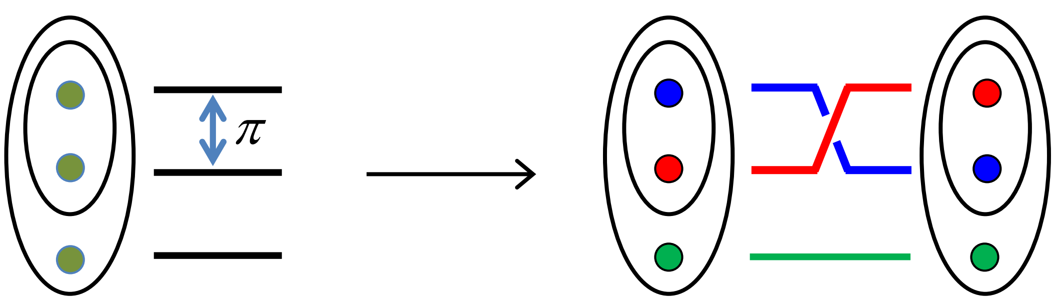
$$\left(\begin{array}{cc|c} 1 & & \\ & e^{-it} & \\ \hline & & e^{-it} \end{array}\right)$$

$t = 0$



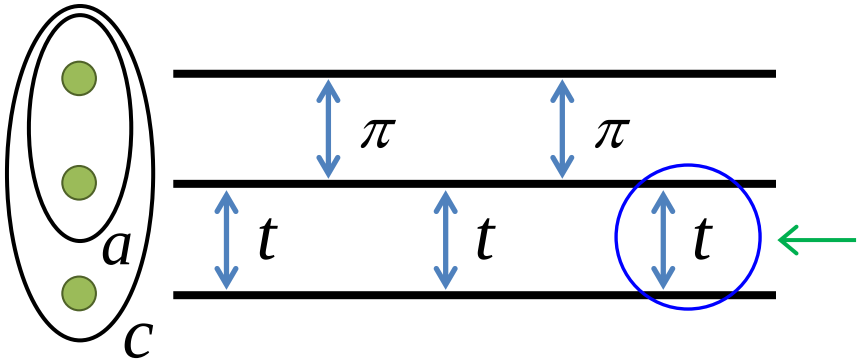
$$\left(\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & & 1 \end{array}\right)$$

$t = \pi$



$$\left(\begin{array}{cc|c} 1 & & \\ & -1 & \\ \hline & & -1 \end{array}\right)$$

A Simple Sequence

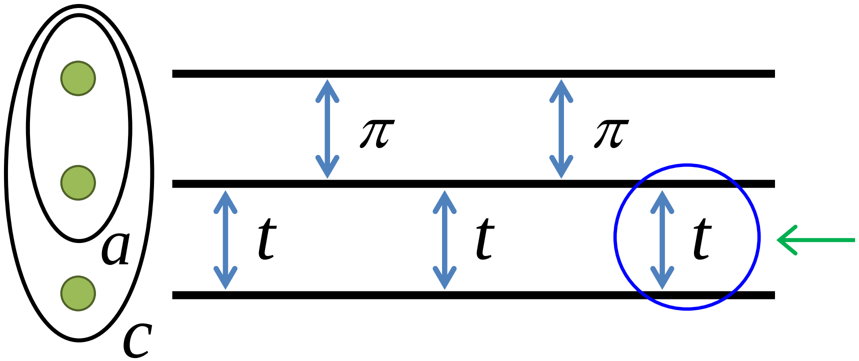


for $t = 0$ or π

$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

Zeuch, NEB, *Phys. Rev. A* (2016)

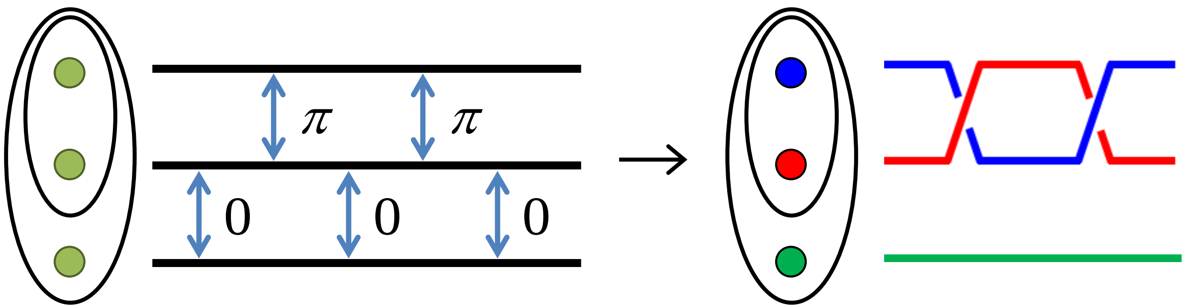
A Simple Sequence



for $t = 0$ or π

$$ac = 0\frac{1}{2} \quad 1\frac{1}{2} \quad 1\frac{3}{2}$$

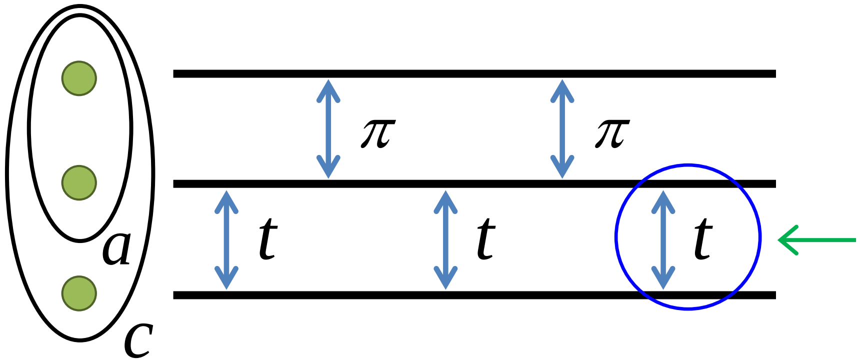
$t = 0$



Zeuch, NEB, *Phys. Rev. A* (2016)

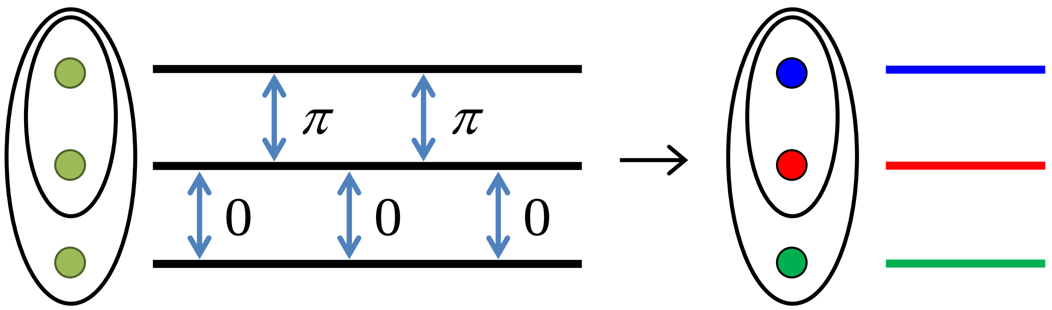
A Simple Sequence

$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$



for $t = 0$ or π

$t = 0$

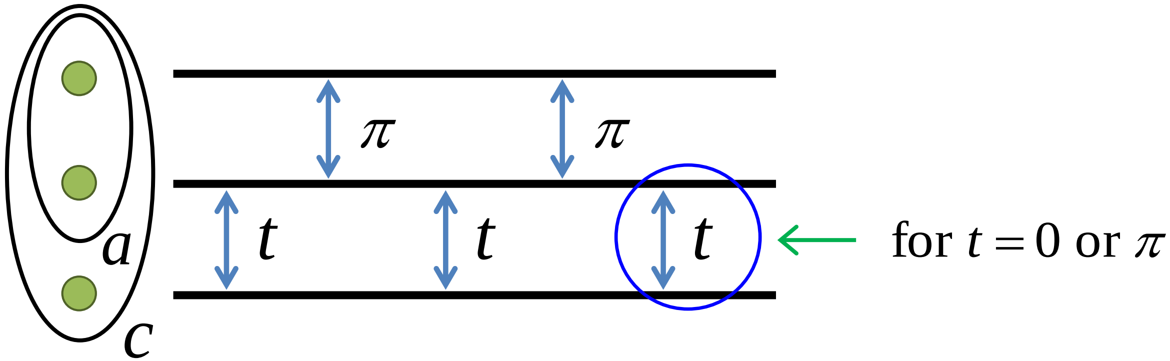


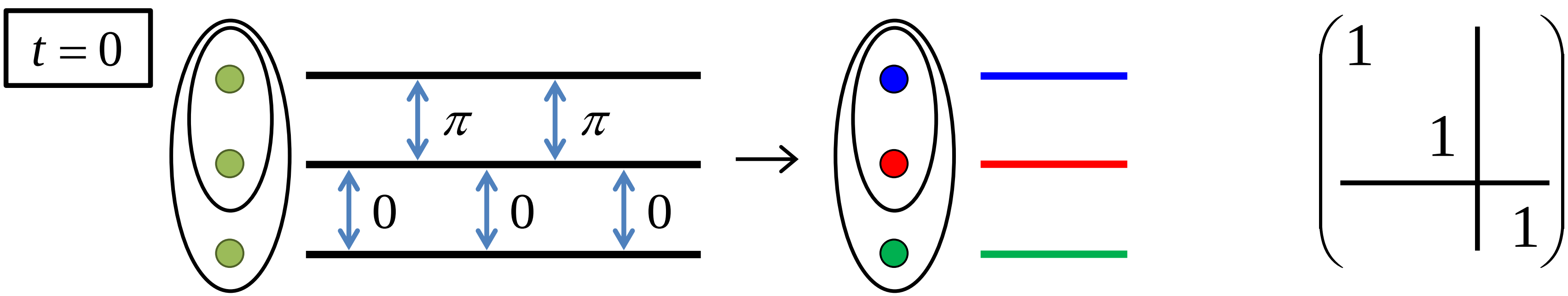
$$\left(\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & & 1 \end{array}\right)$$

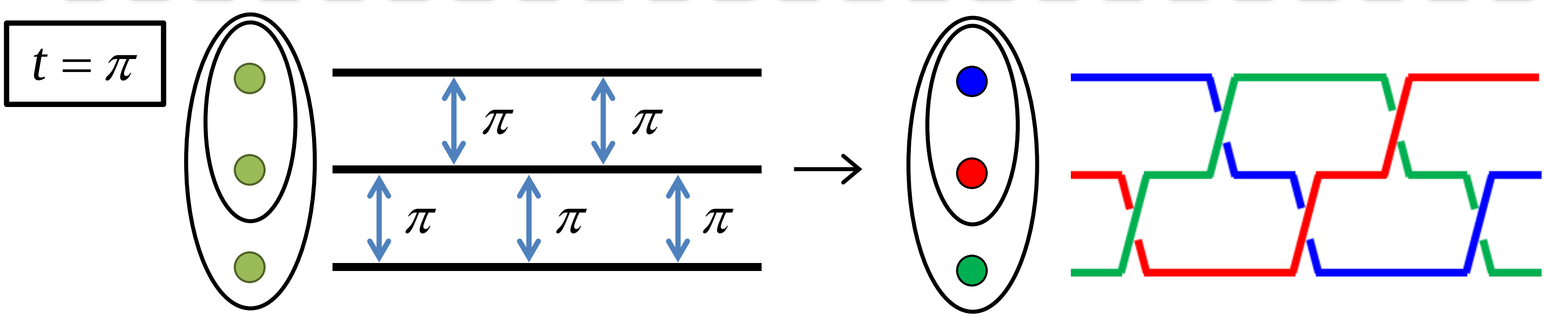
Zeuch, NEB, *Phys. Rev. A* (2016)

A Simple Sequence

$$ac = 0\frac{1}{2} \quad 1\frac{1}{2} \quad 1\frac{3}{2}$$

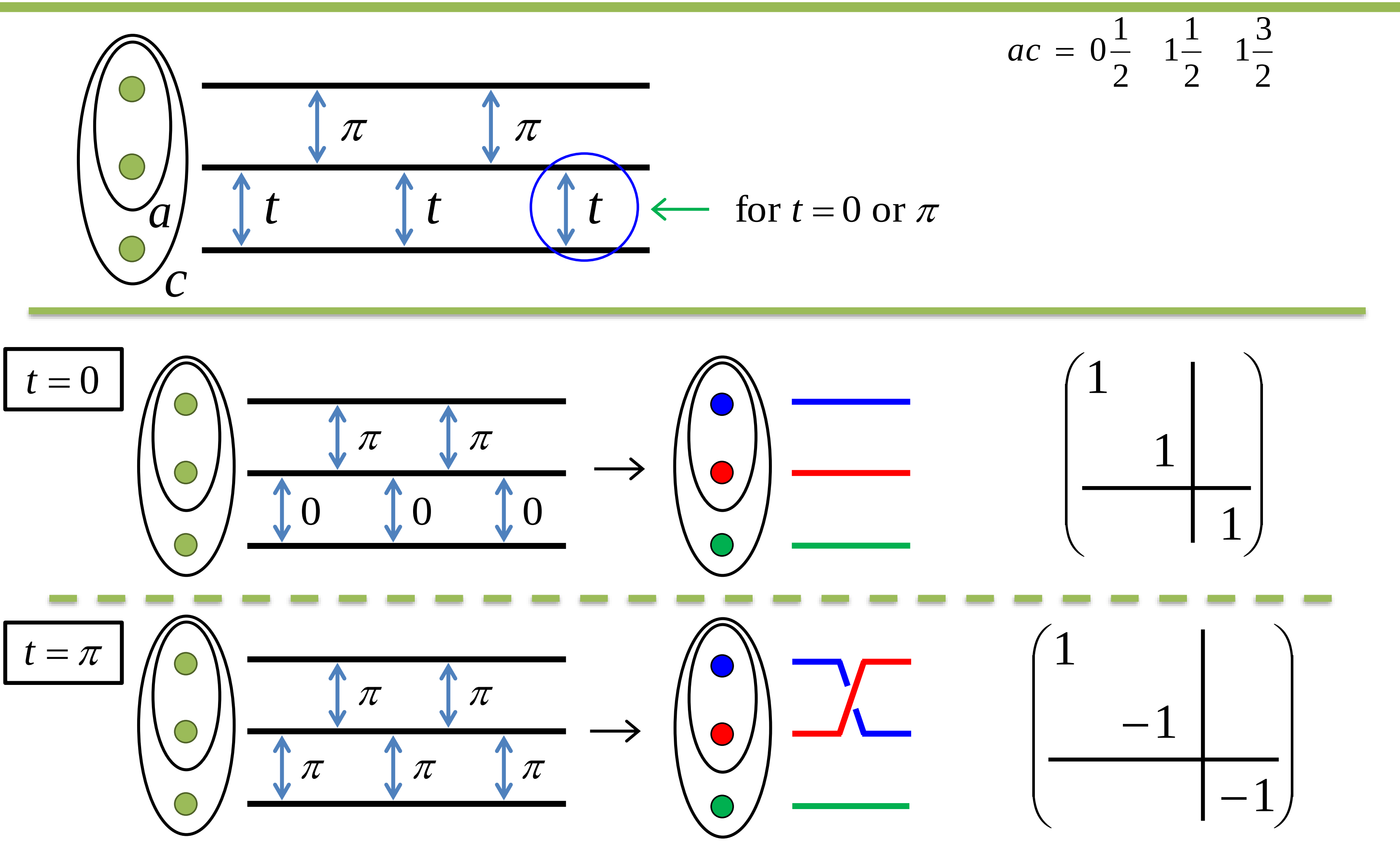






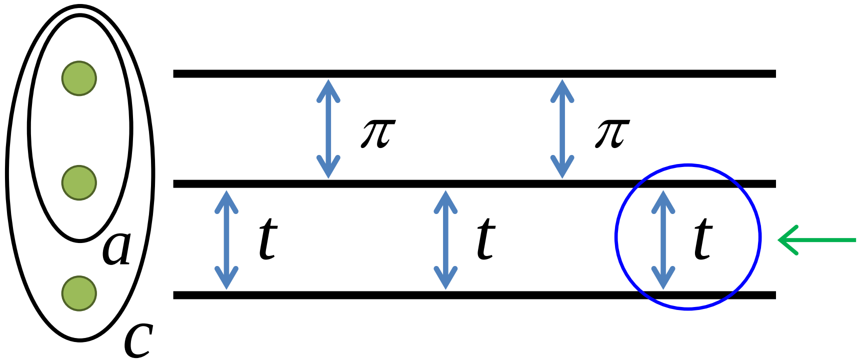
Zeuch, NEB, *Phys. Rev. A* (2016)

A Simple Sequence



Zeuch, NEB, *Phys. Rev. A* (2016)

A Simple Sequence



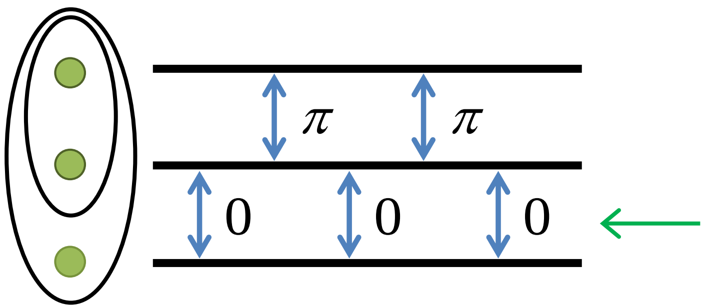
$$\begin{pmatrix} 1 & & \\ & m & \\ & & m \end{pmatrix}$$

where $m^2 = 1$

$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

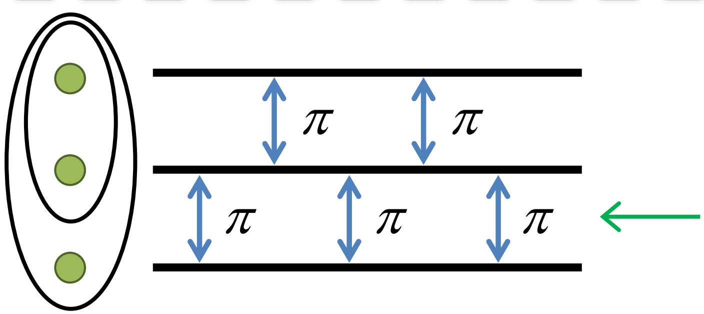
$$\begin{pmatrix} 1 & & \\ & m & \\ & & m \end{pmatrix}$$

$t = 0$



$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

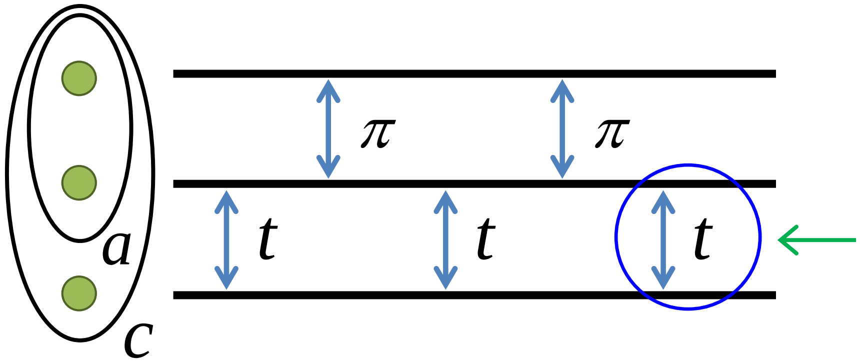
$t = \pi$



$$\begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \qquad \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

Zeuch, NEB, *Phys. Rev. A* (2016)

From Numbers to Matrices



$$\left(\begin{array}{cc|c} 1 & & \\ & m & \\ \hline & & m \end{array}\right)$$

where $m^2 = 1$

$$ac = 0\frac{1}{2} \quad 1\frac{1}{2} \quad 1\frac{3}{2}$$

$$\left(\begin{array}{cc|c} 1 & & \\ & m & \\ \hline & & m \end{array}\right)$$

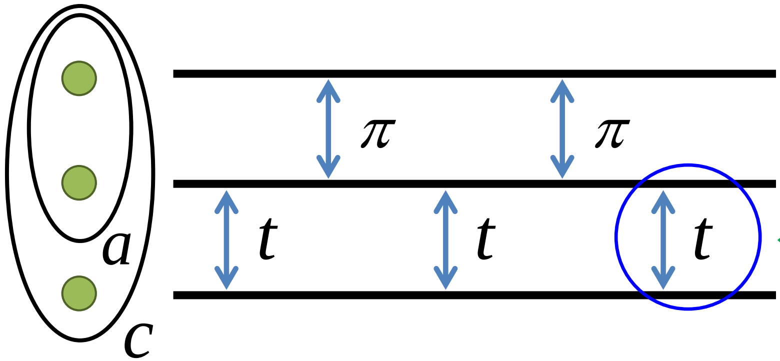
number → m —promote→ M ← 2x2 matrix

$$m^2 = 1 \qquad\qquad M^2 = I \text{ ← 2x2 identity}$$

$$\Rightarrow \quad m = \pm 1 \qquad\qquad \Rightarrow \quad M = \pm I$$

Zeuch, NEB, *Phys. Rev. A* (2016)

From Numbers to Matrices



$$\left(\begin{array}{cc|c} 1 & & \\ & m & \\ \hline & & m \end{array}\right)$$

where $m^2 = 1$

$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

$$\left(\begin{array}{cc|c} 1 & & \\ & m & \\ \hline & & m \end{array}\right)$$

number → m $\quad$ promote ⟶ $\quad$ M ← 2x2 matrix

$$m^2 = 1 \qquad\qquad M^2 = I$$

(2x2 identity → I)

$$\Rightarrow \quad m = \pm 1 \qquad\qquad \Rightarrow \quad M = \pm I, \quad M = \sigma_x$$

$$a = 0 \quad 1$$

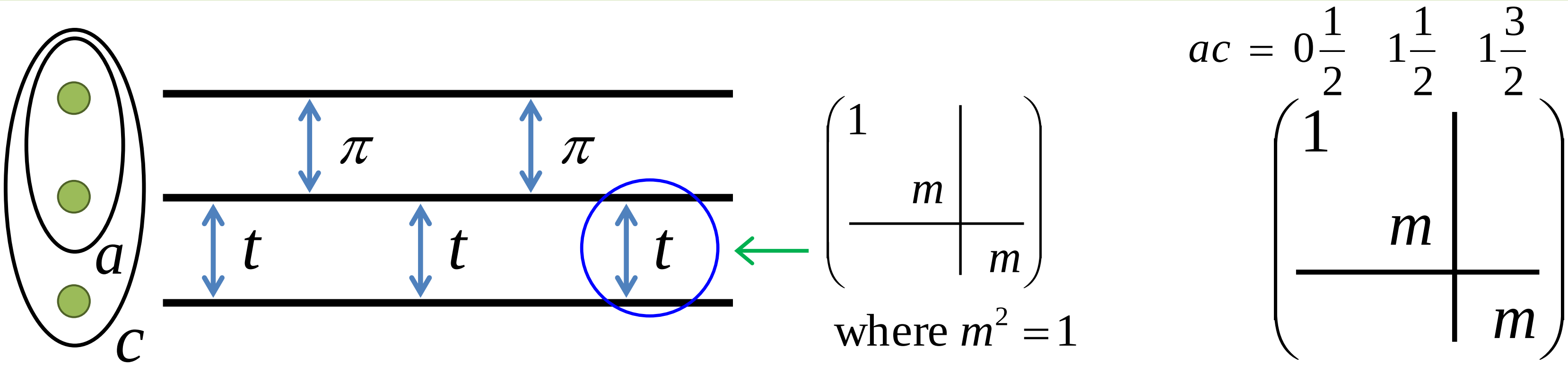
$$\begin{pmatrix} 1 & \\ & m \end{pmatrix}$$

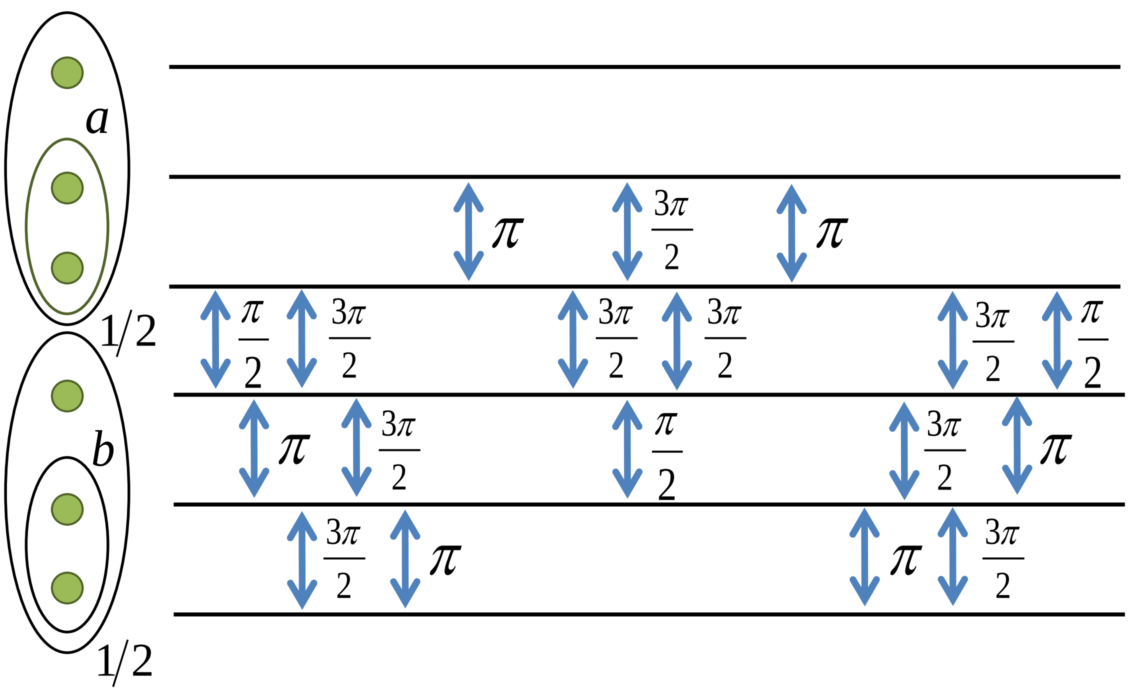
$$ab = 00 \ 01 \quad 10 \ 11$$

$$U_{CNOT} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{pmatrix}$$

Zeuch, NEB, *Phys. Rev. A* (2016)

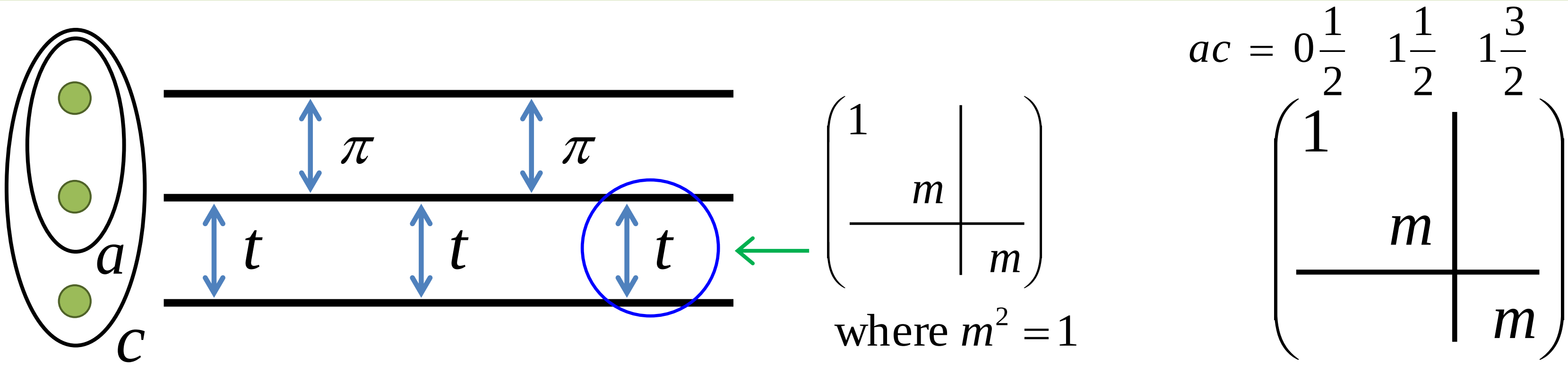
Sequence Elevation

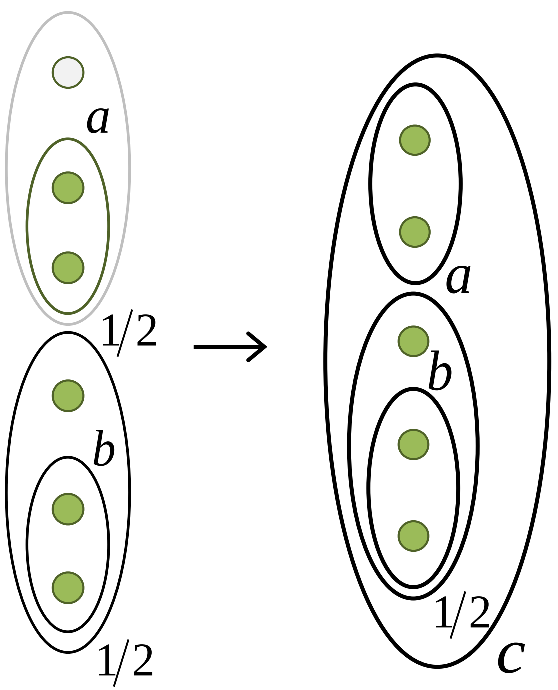




Zeuch, NEB, *Phys. Rev. A* (2016)

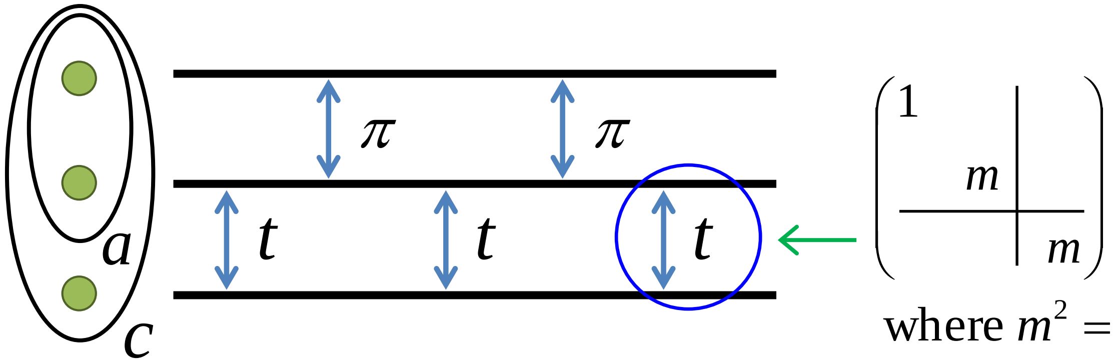
Sequence Elevation





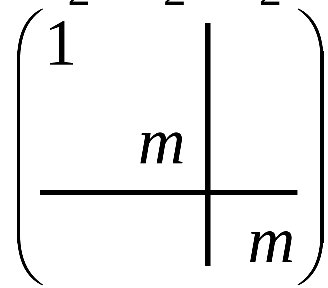
Zeuch, NEB, *Phys. Rev. A* (2016)

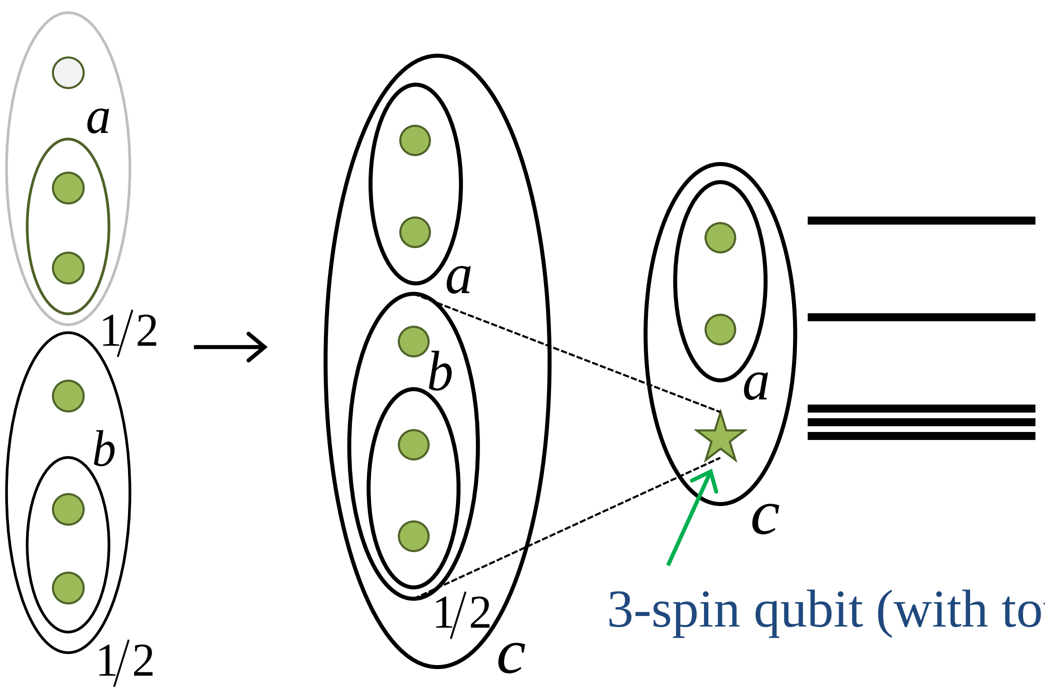
Sequence Elevation



where $m^2 = 1$

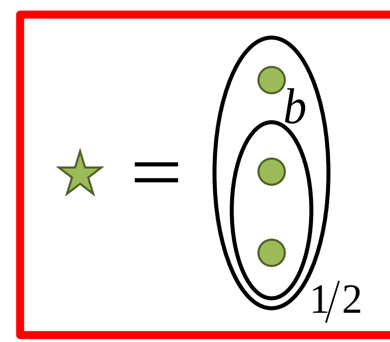
$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$





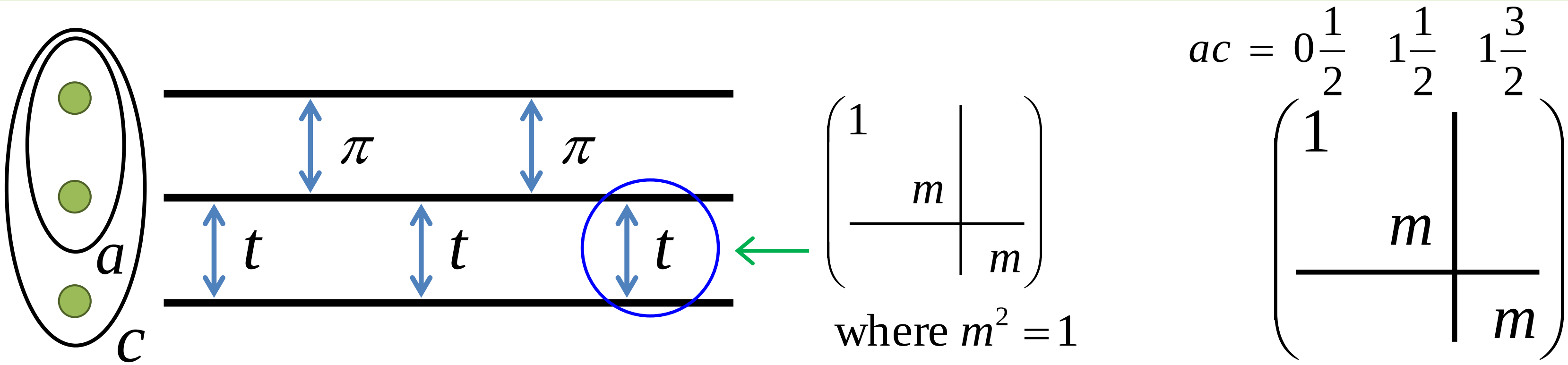
$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

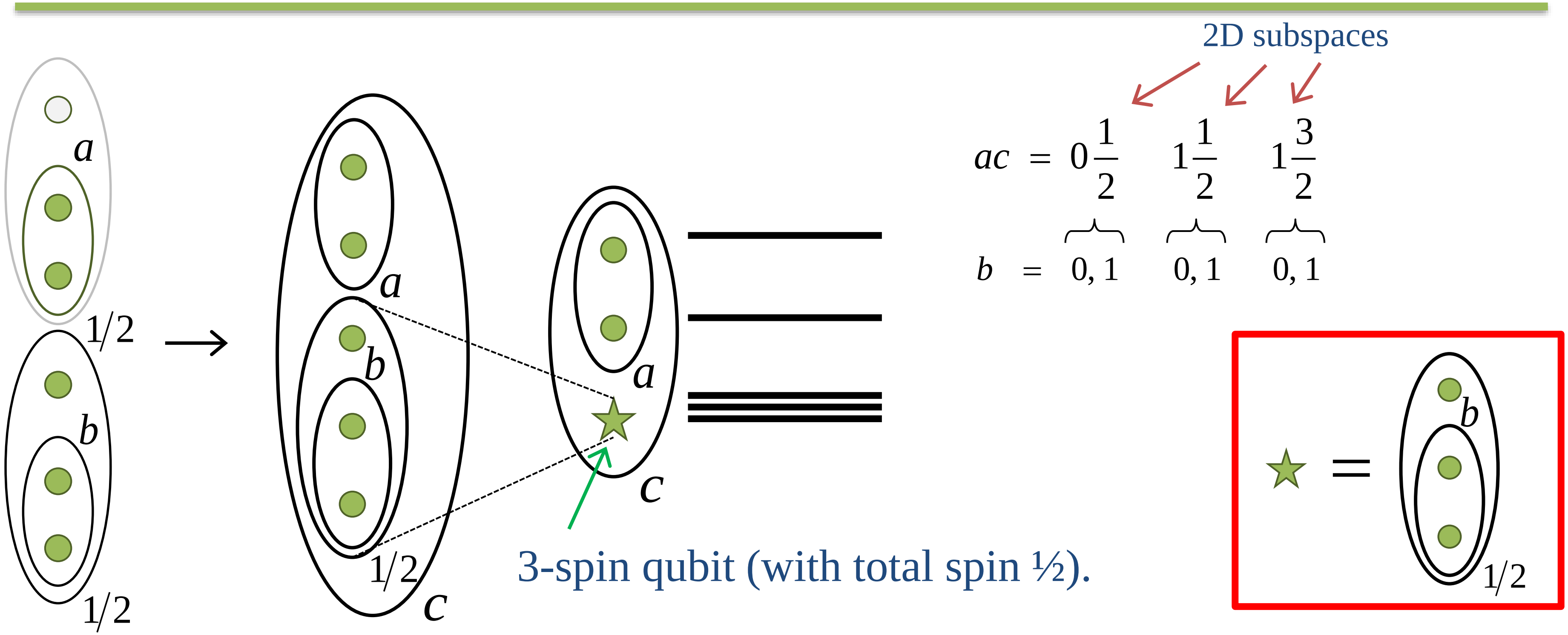
3-spin qubit (with total spin ½).



Zeuch, NEB, *Phys. Rev. A* (2016)

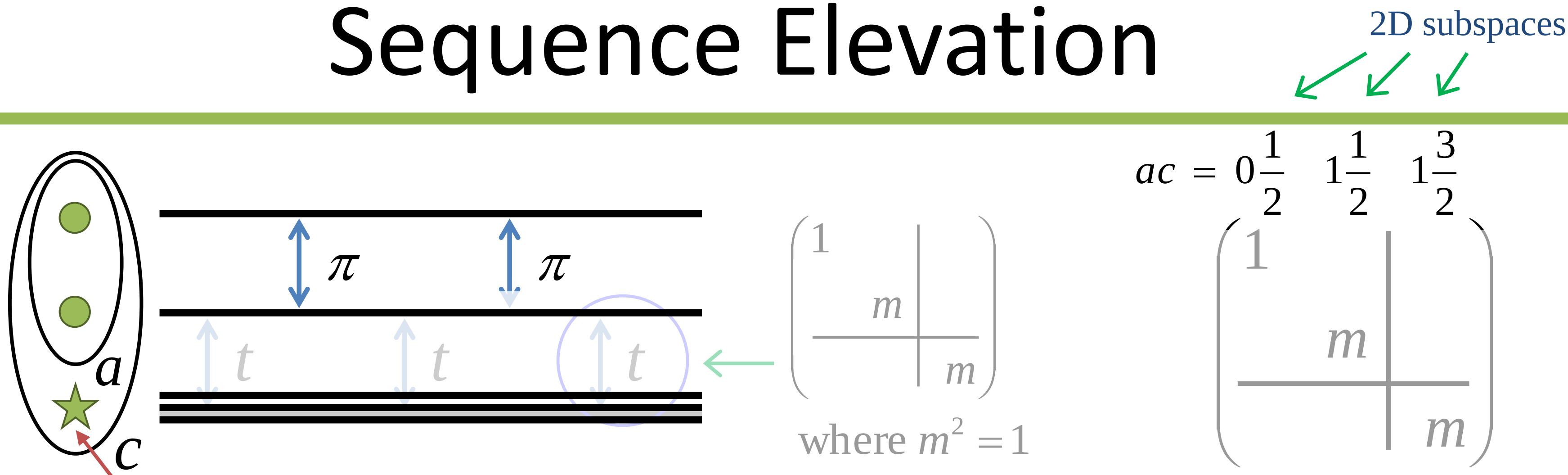
Sequence Elevation

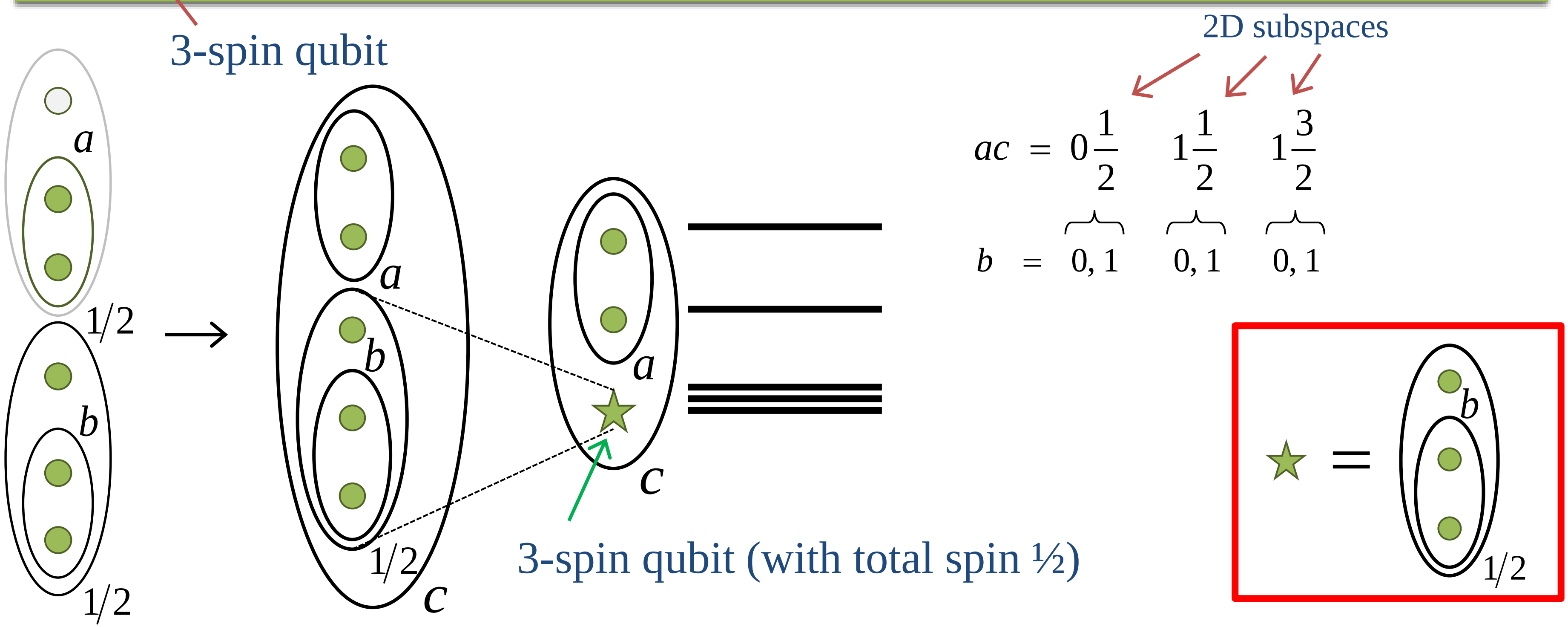




Zeuch, NEB, *Phys. Rev. A* (2016)

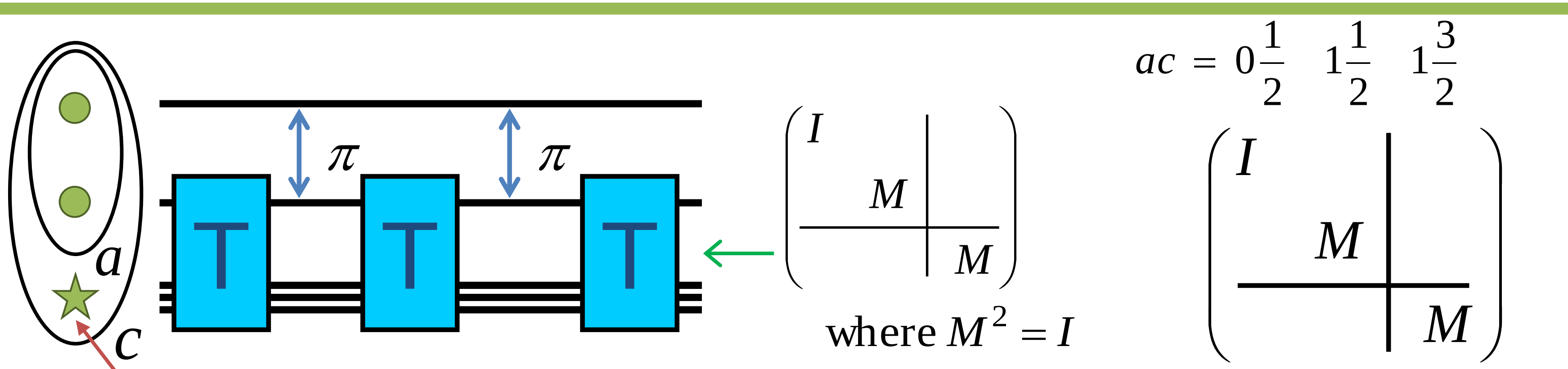
Sequence Elevation

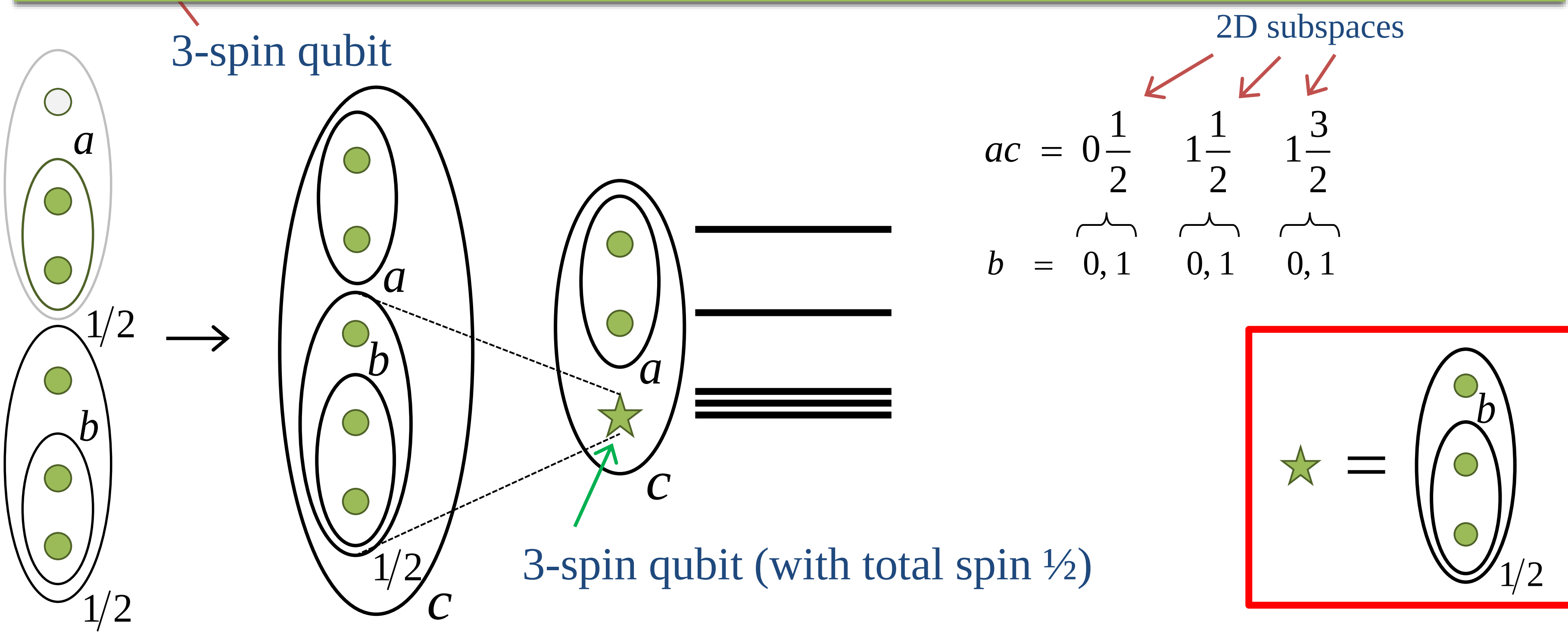




Zeuch, NEB, *Phys. Rev. A* (2016)

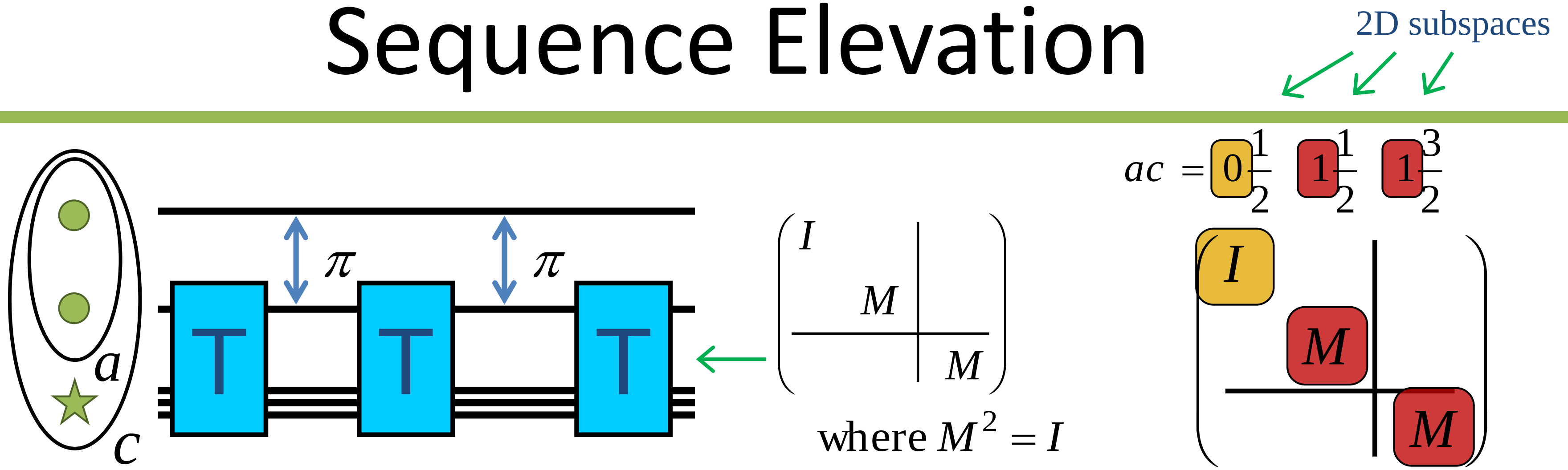
Sequence Elevation

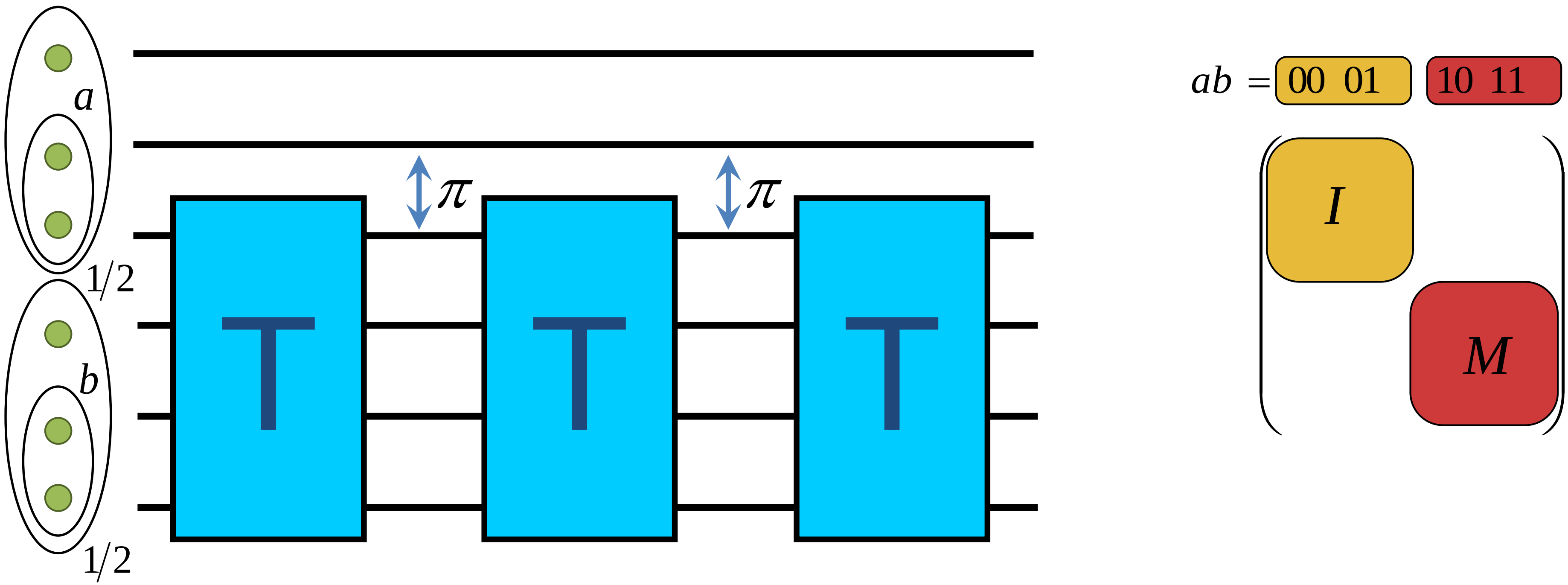




Zeuch, NEB, *Phys. Rev. A* (2016)

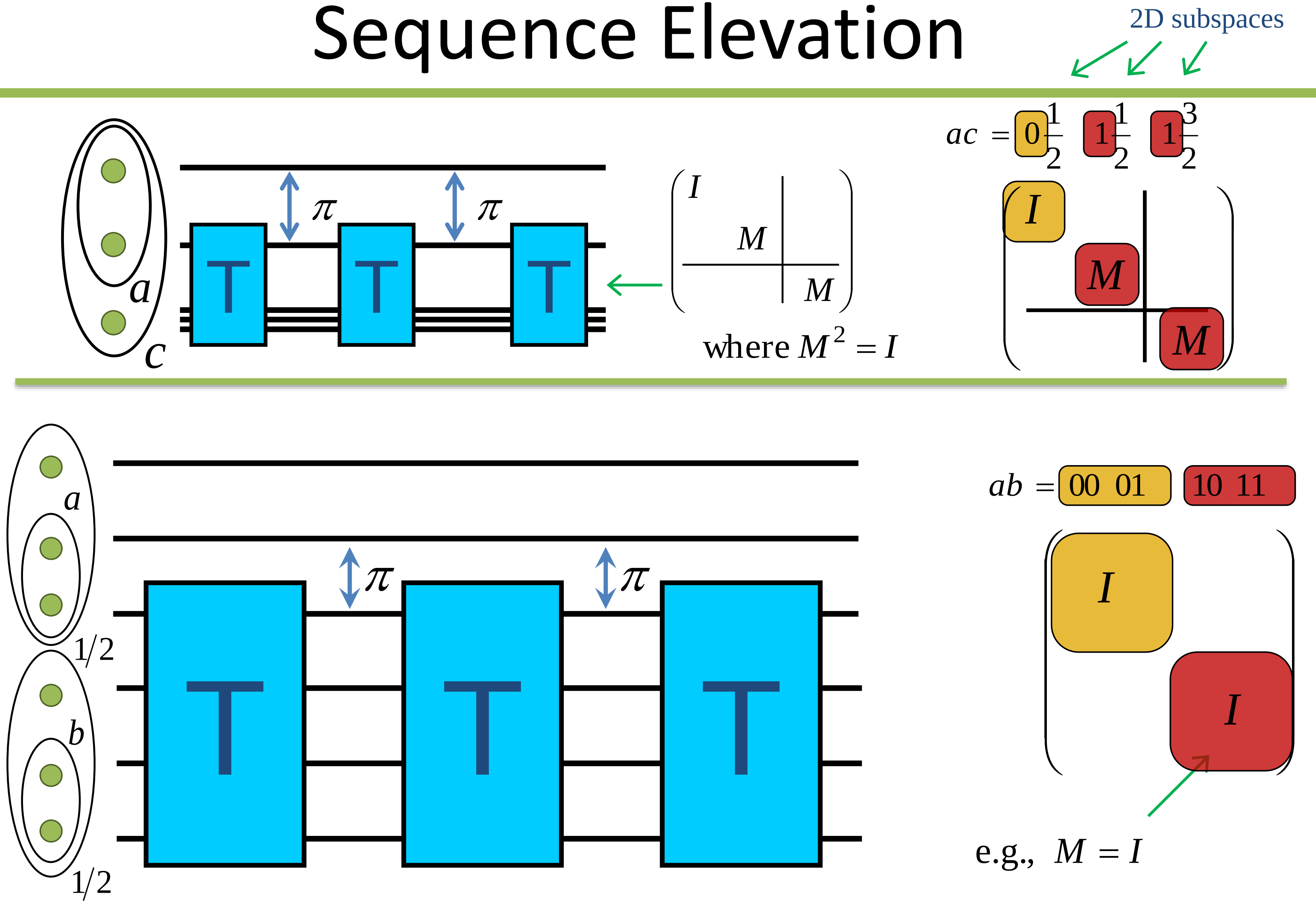
Sequence Elevation





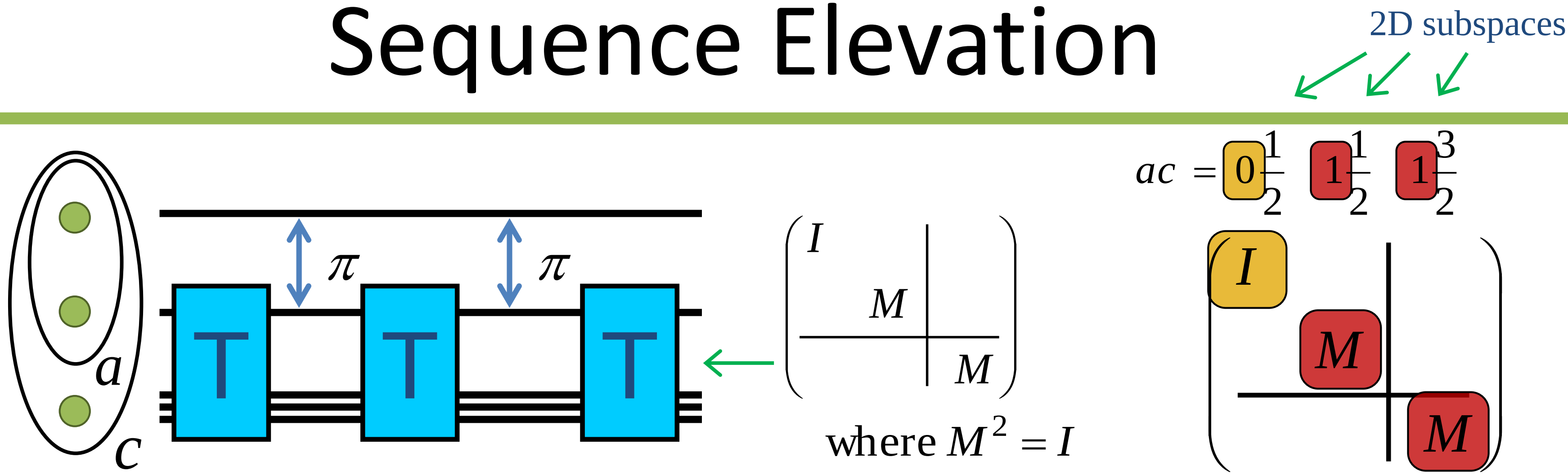
Zeuch, NEB, *Phys. Rev. A* (2016)

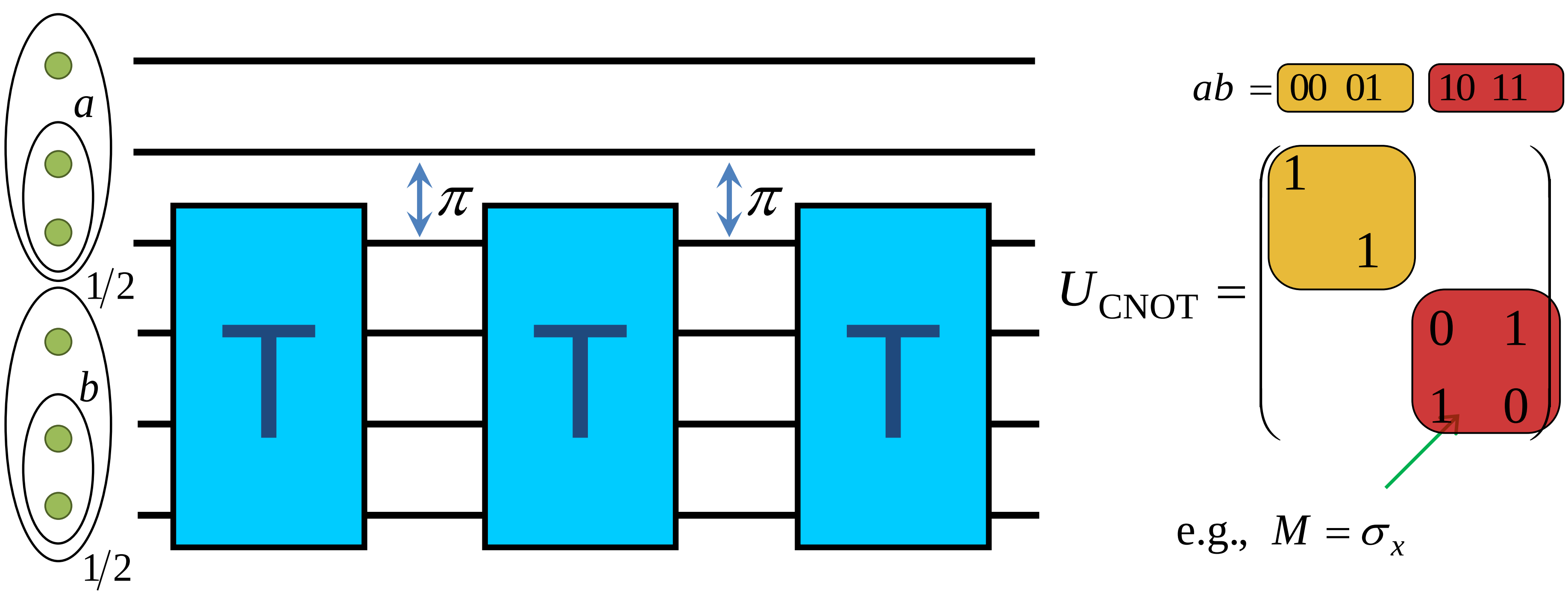
Sequence Elevation



Zeuch, NEB, *Phys. Rev. A* (2016)

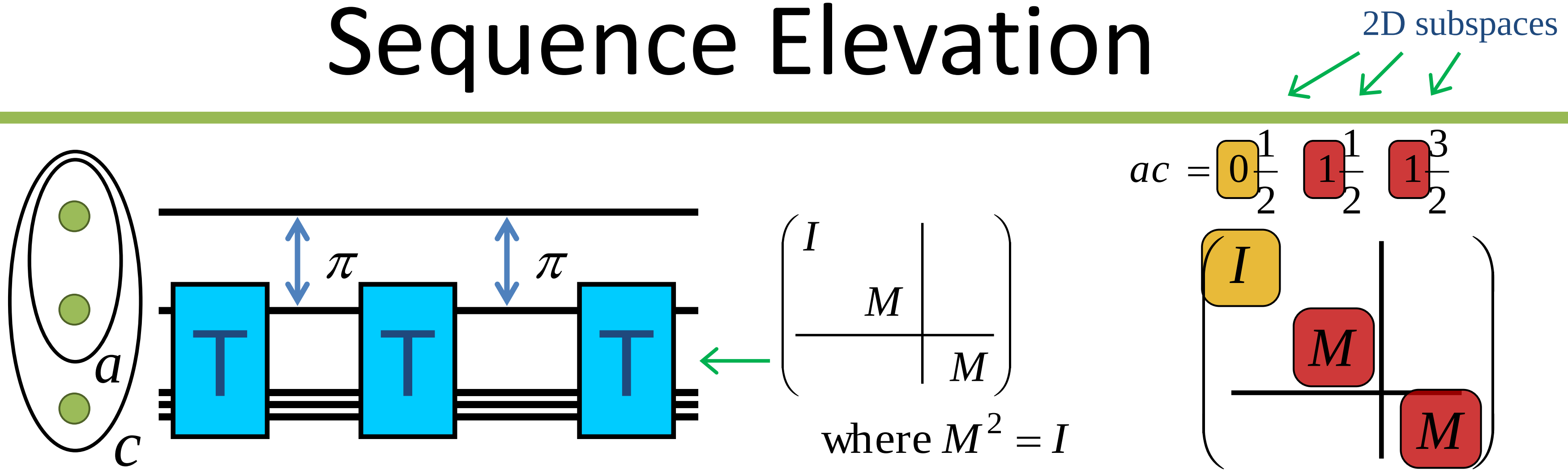
Sequence Elevation

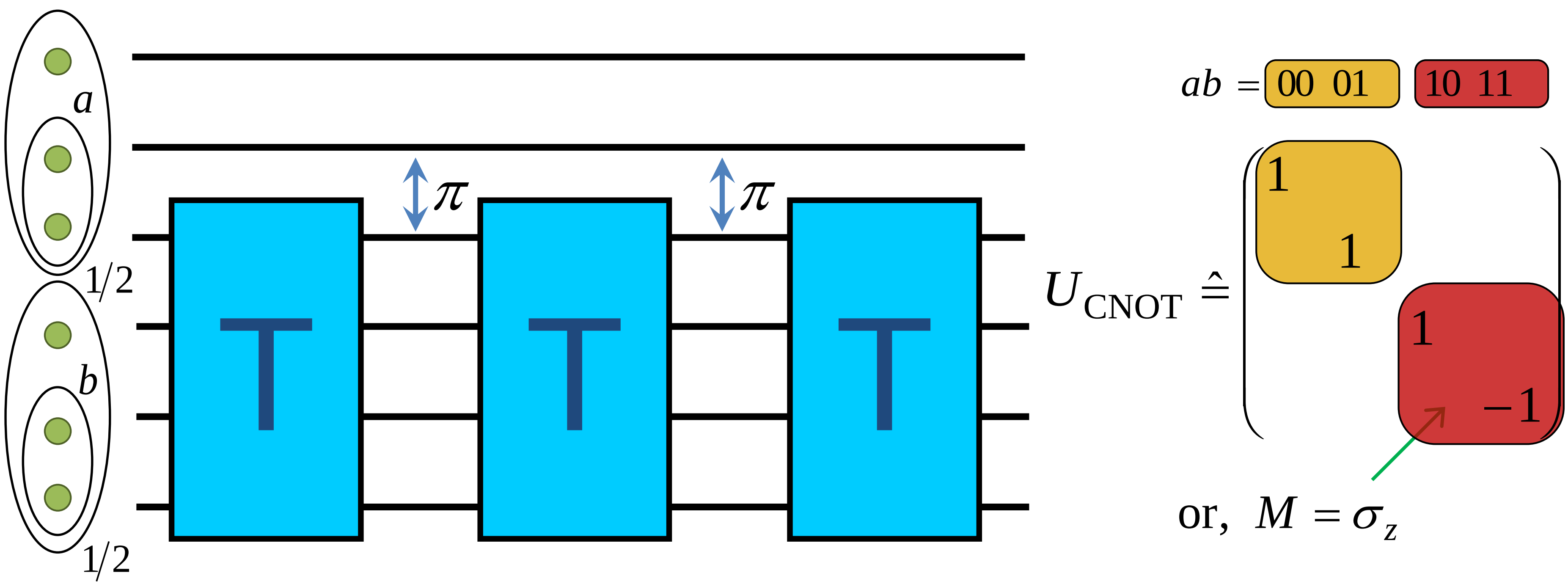




Zeuch, NEB, *Phys. Rev. A* (2016)

Sequence Elevation

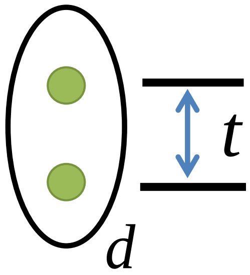




Zeuch, NEB, *Phys. Rev. A* (2016)

T Operation

- T generalizes an $m^2 = 1$ exchange pulse (consider: $M = \sigma_z$)

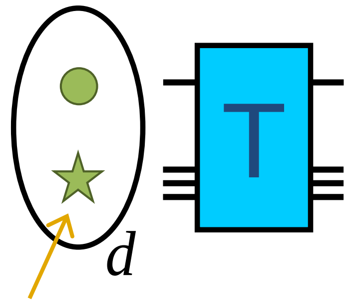


$$d = \quad 0 \quad 1$$

$$\begin{pmatrix} 1 & \\ & m \end{pmatrix}$$

where $m^2 = 1$

$$\longrightarrow$$



3-spin qubit

$$d = \quad 0 \quad 1$$

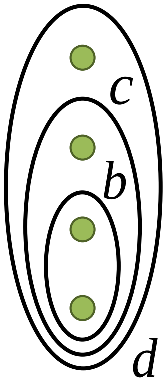
$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

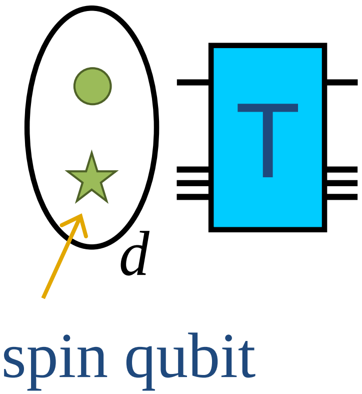
where $M^2 = 1$

Zeuch, NEB, *Phys. Rev. A* (2016)

T Operation

$$c = \underbrace{\tfrac{1}{2}}_{d=0} \quad \underbrace{\tfrac{1}{2} \quad \tfrac{3}{2}}_{d=1}$$



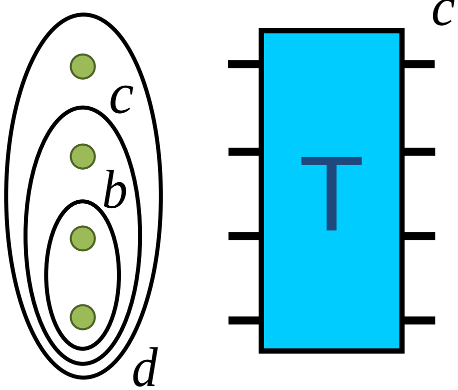


$$d = \quad 0 \quad 1$$

$$\begin{pmatrix} I & \\ & \sigma_z \end{pmatrix}$$

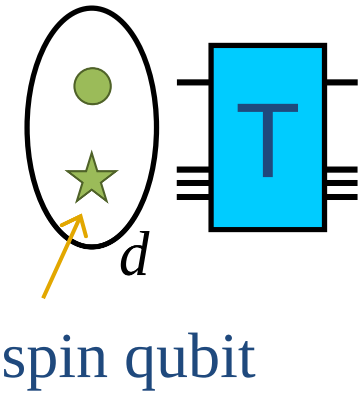
Zeuch, NEB, *Phys. Rev. A* (2016)

T Operation



$$\begin{array}{c} \\ c = \end{array} \begin{array}{ccc} \overbrace{\tfrac{1}{2}}^{d=0} & \overbrace{\tfrac{1}{2} \quad \tfrac{3}{2}}^{d=1} \end{array}$$

$$\left(\begin{array}{c|cc} I & & \\ \hline & \sigma_z & \\ & & \alpha \end{array}\right)$$



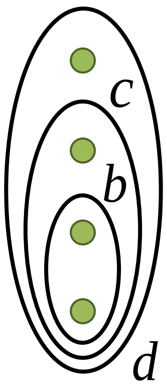
$$d = \quad 0 \qquad 1$$

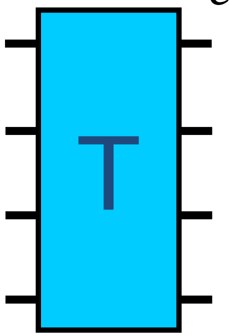
$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

where $M^2 = 1$

Zeuch, NEB, *Phys. Rev. A* (2016)

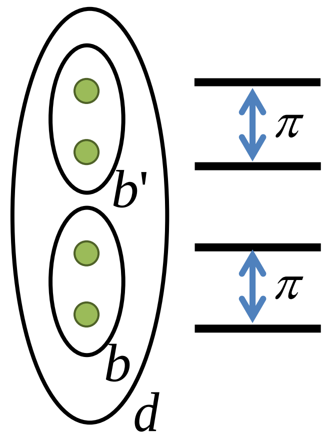
T Operation





$$
c = \overbrace{\tfrac{1}{2}}^{d=0} \quad \overbrace{\tfrac{1}{2} \quad \tfrac{3}{2}}^{d=1}
\qquad
\left(\begin{array}{c|cc} I & & \\ \hline & \sigma_z & \\ & & \alpha \end{array}\right)
$$

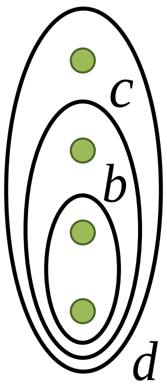
- Consider ..

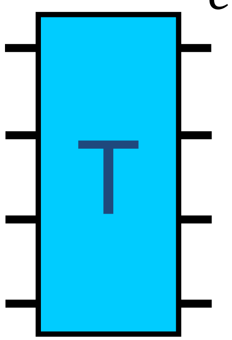


$$
bb' = \overbrace{00 \;\; 11}^{d=0} \quad \overbrace{01 \;\; 10 \;\; 11}^{d=1}
\qquad
\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)
$$

Zeuch, NEB, *Phys. Rev. A* (2016)

T Operation

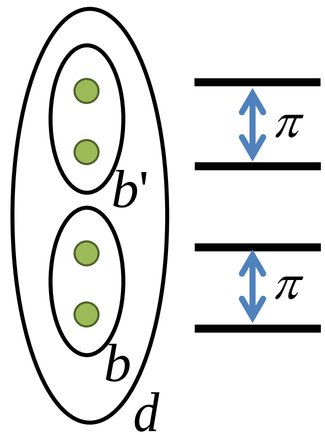




$$c = \underbrace{\tfrac{1}{2}}_{d=0} \quad \underbrace{\tfrac{1}{2} \quad \tfrac{3}{2}}_{d=1}$$

$$\left(\begin{array}{c|cc} I & & \\ \hline & \sigma_z & \\ & & \alpha \end{array}\right)$$

- Consider ..



$$bb' = \underbrace{00 \;\; 11}_{d=0} \quad \underbrace{01 \;\; 10 \;\; 11}_{d=1}$$

$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)$$

$$\cong \sigma_z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

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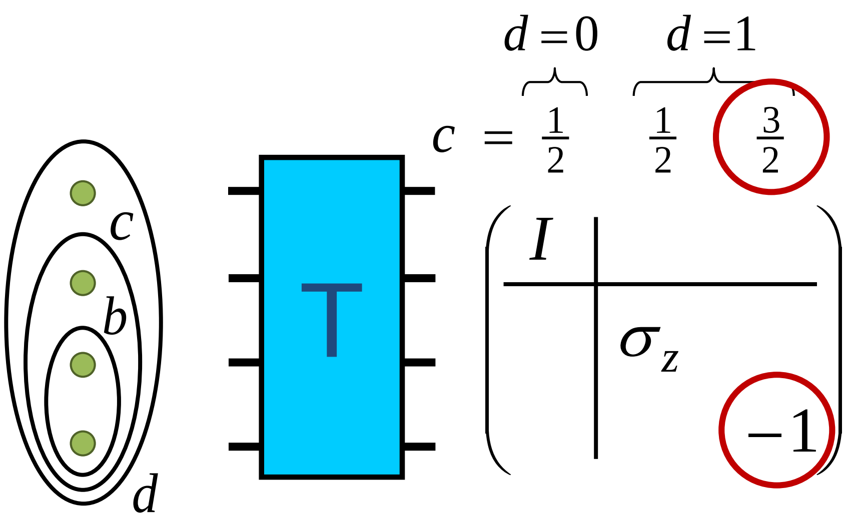
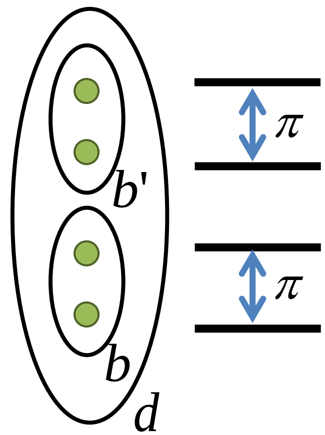
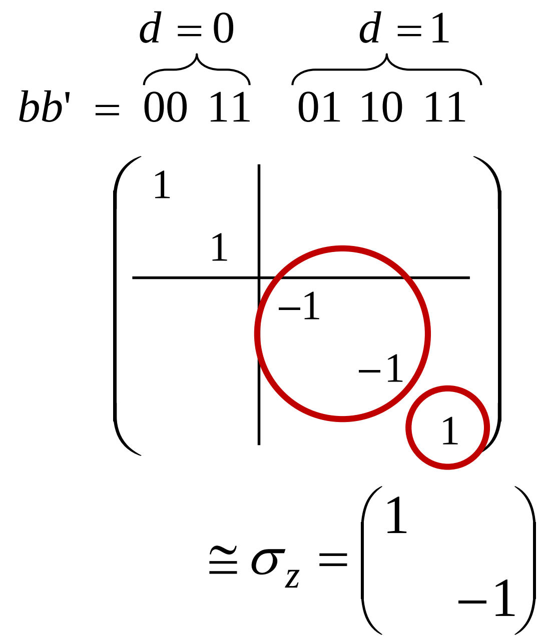
T Operation

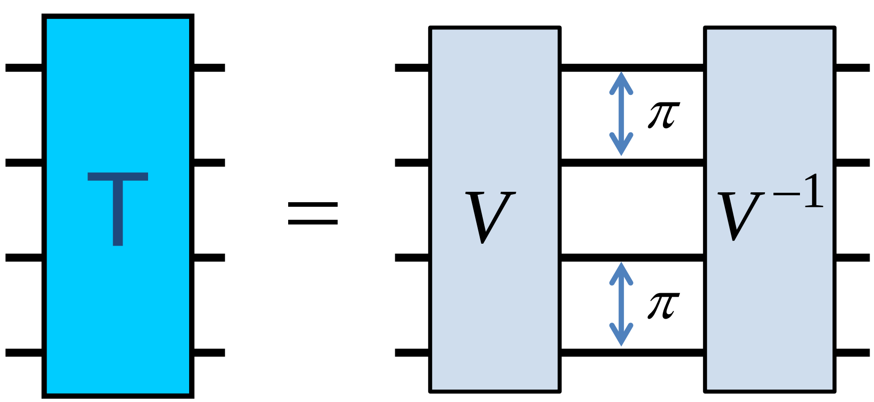


- Consider ..





- New sequence ***V***



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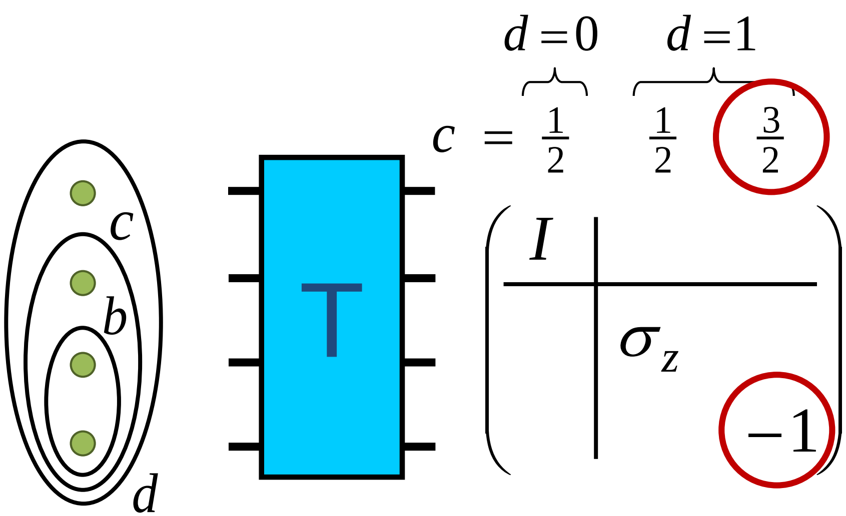
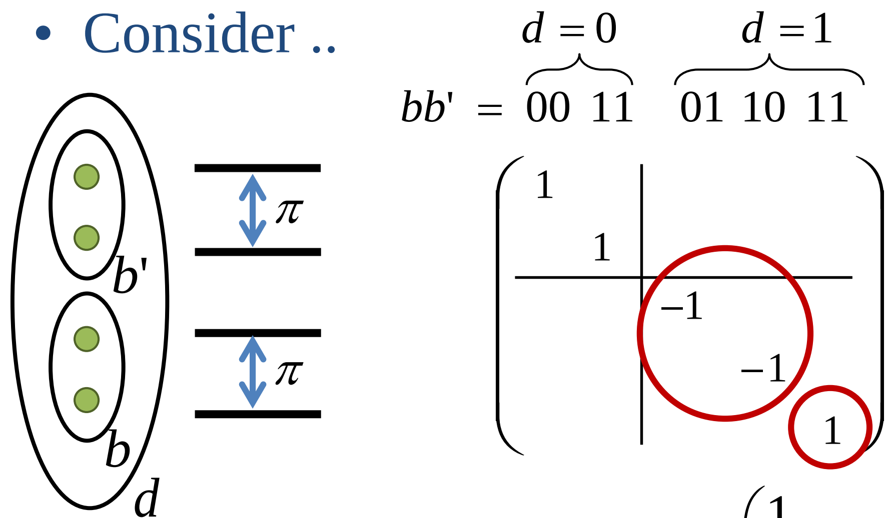
T Operation

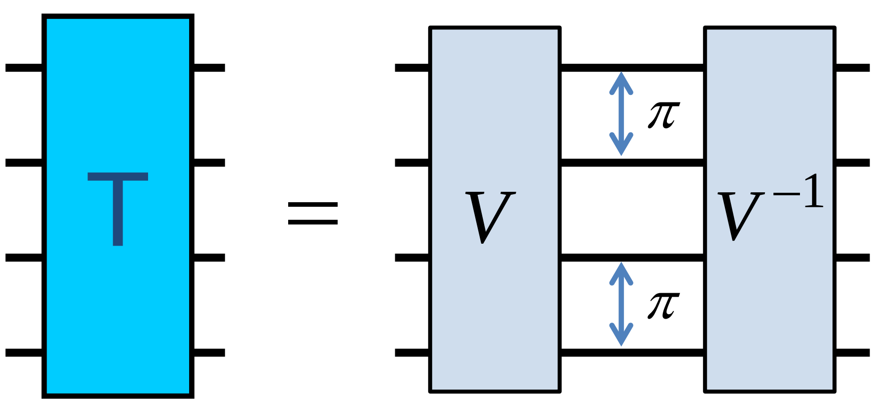


- Consider ..

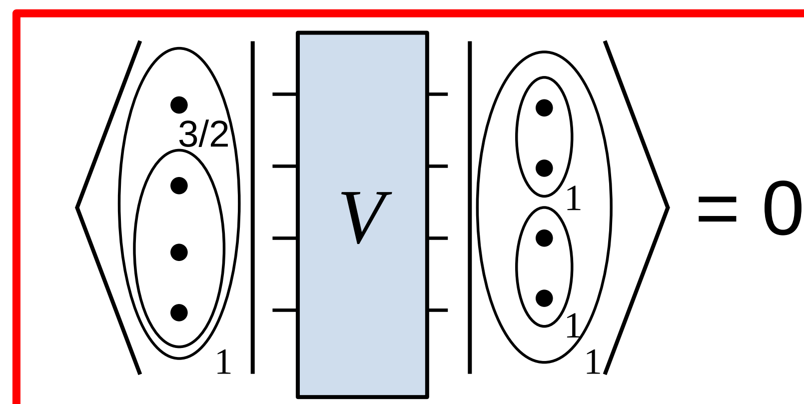


$$\cong \sigma_z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

- New sequence ***V***



Constraint:



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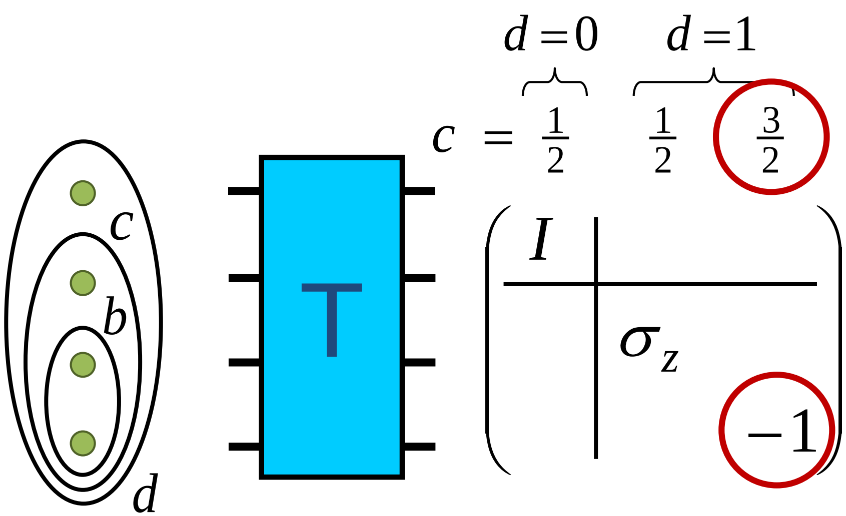
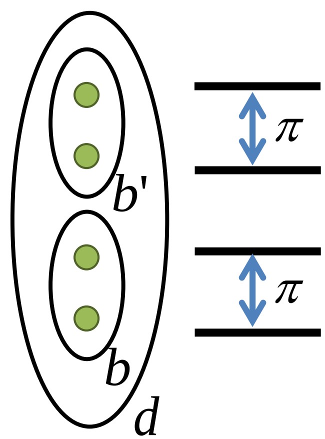
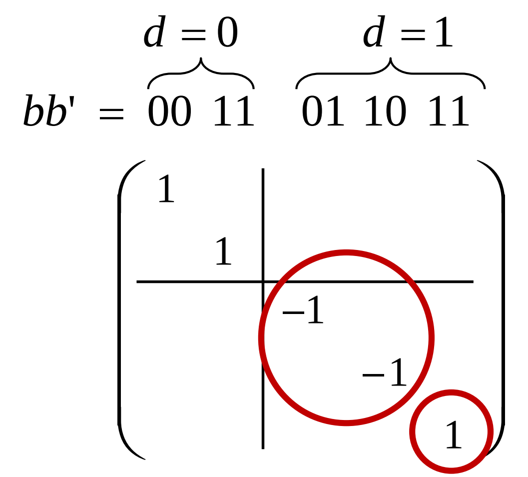
T Operation



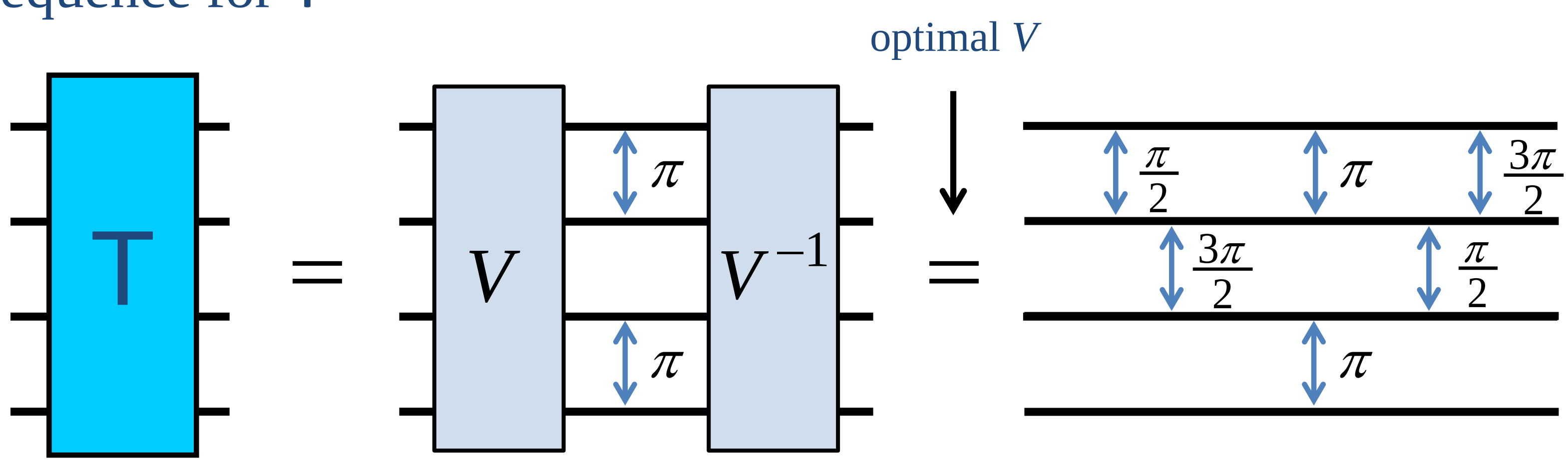
- Consider ..





$$\cong \sigma_z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

- Sequence for T



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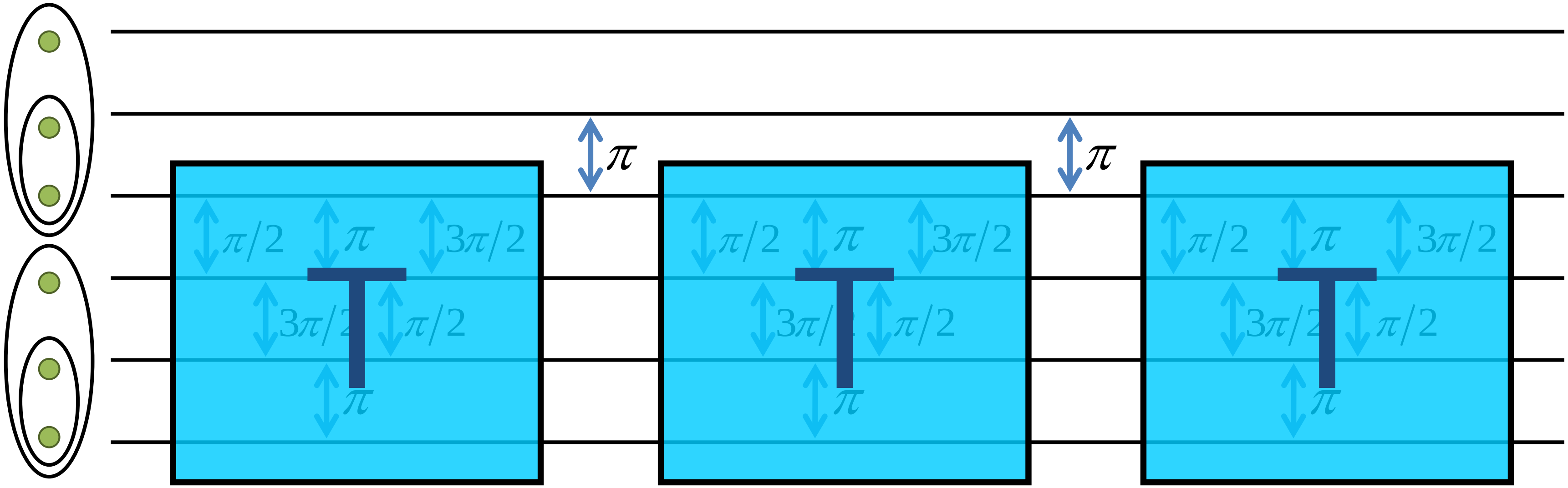
Sequence Comparison



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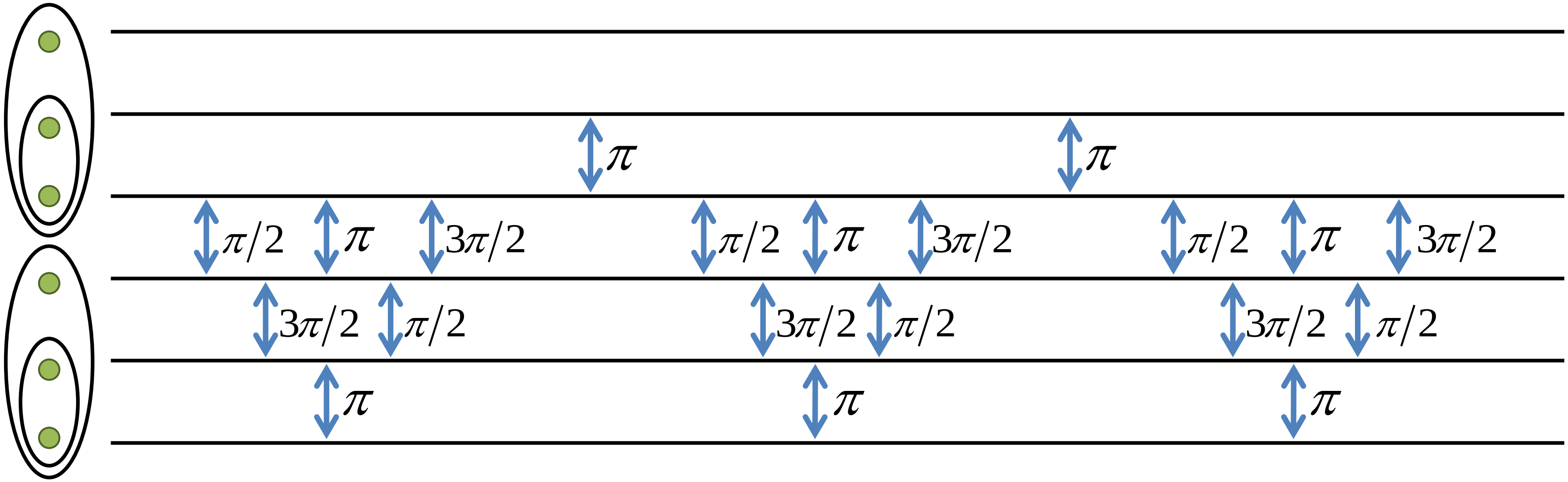
Sequence Comparison



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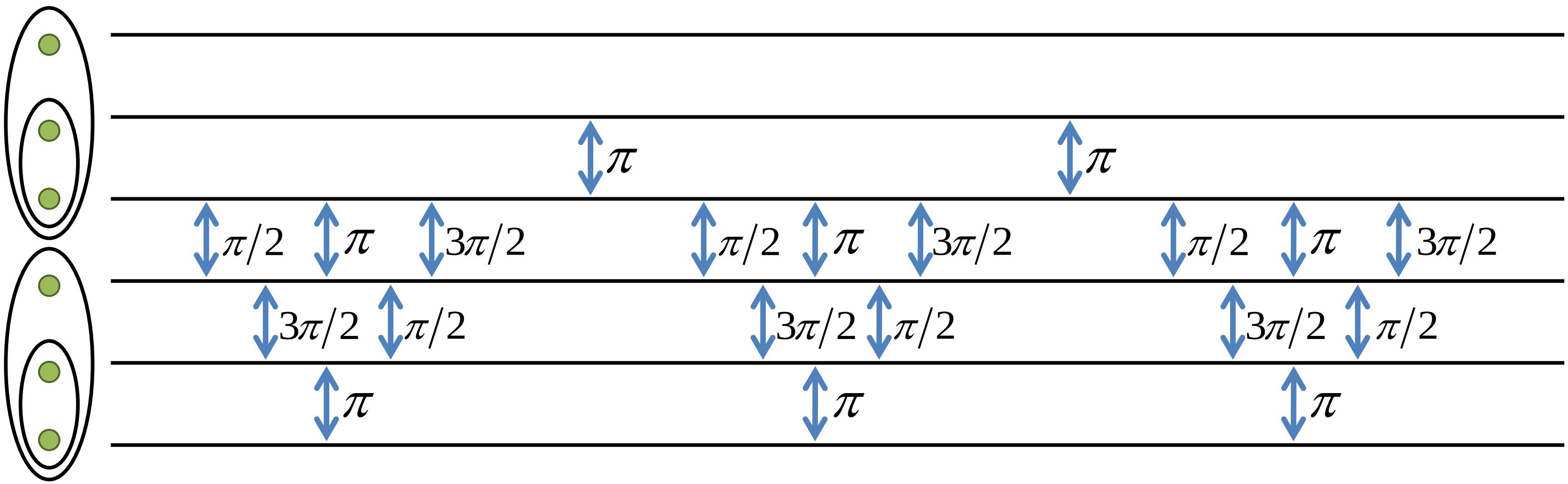
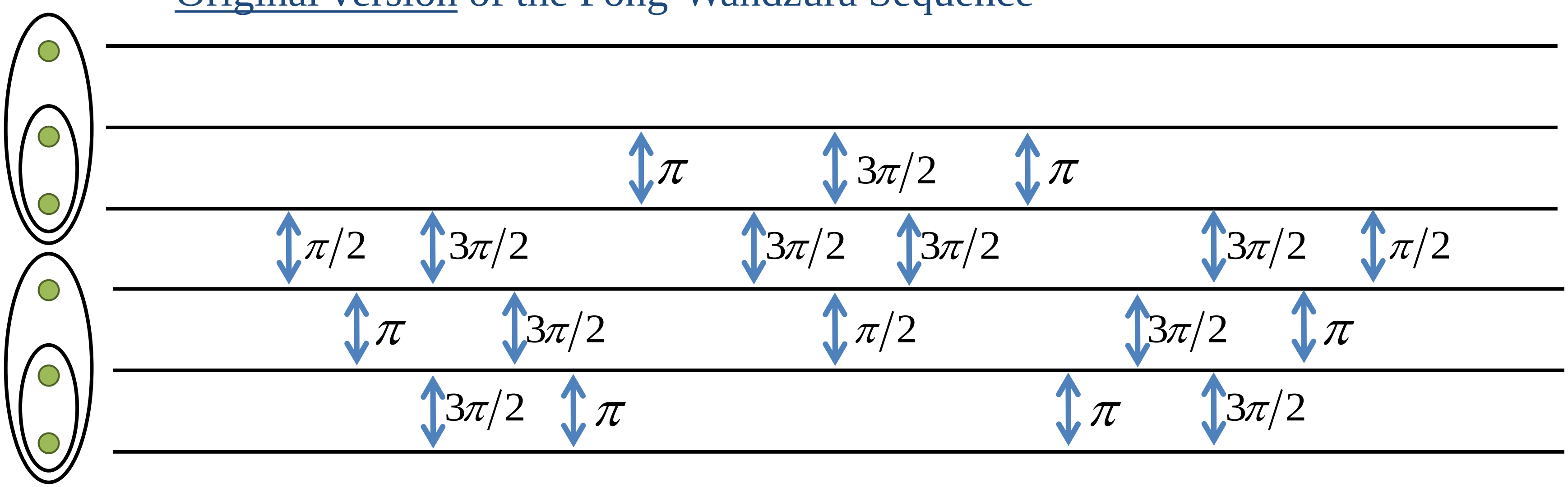
Sequence Comparison



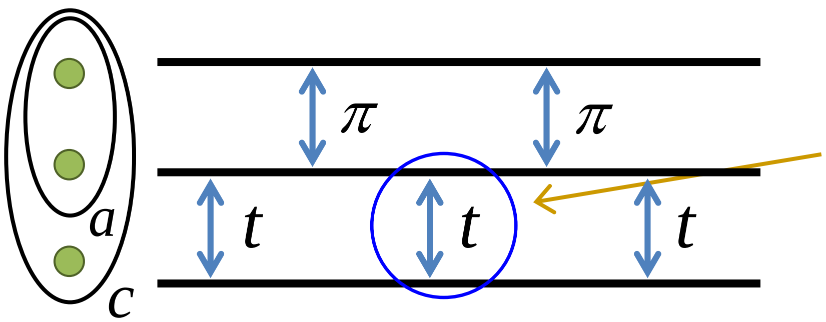
- Original version of the Fong-Wandzura Sequence



New Sequences?

- Three-spin sequence

$$m^2 = 1$$



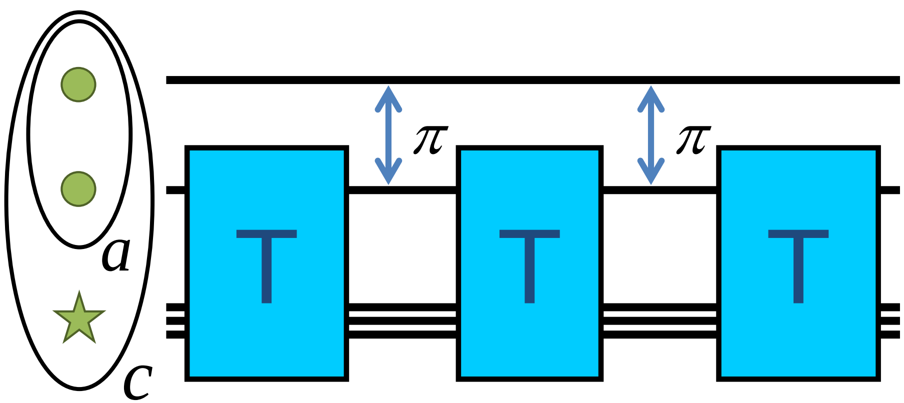
$$\begin{pmatrix} 1 & & \\ & m & \\ & & m \end{pmatrix}$$

$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

$$\begin{pmatrix} 1 & & \\ & m & \\ & & m \end{pmatrix}$$

- Elevated sequence = Fong-Wandzura

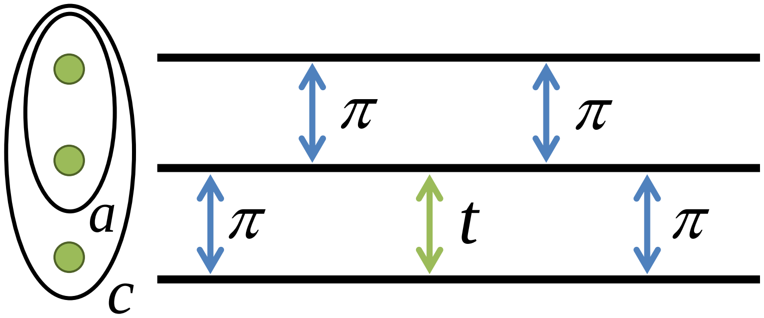
$$M^2 = I$$



$$\begin{pmatrix} I & & \\ & M & \\ & & M \end{pmatrix}$$

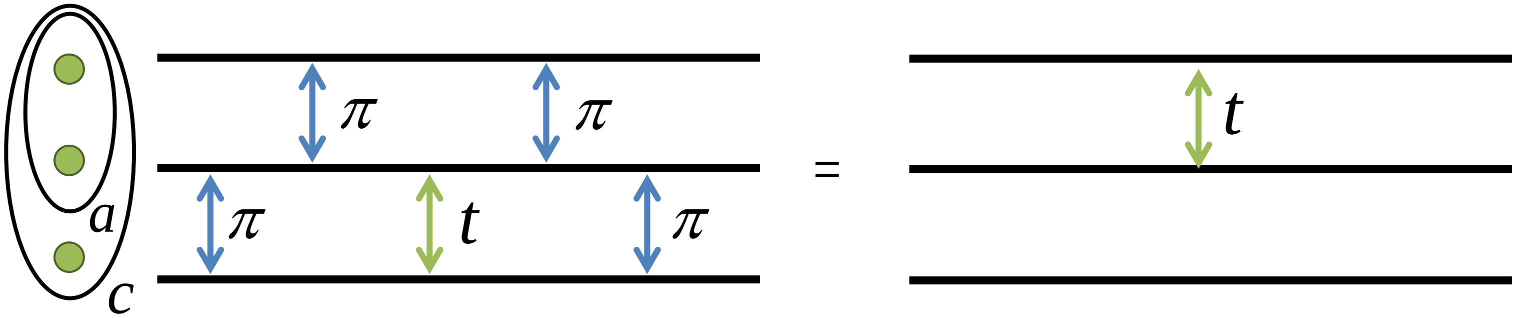
New Sequences?

- Another three-spin sequence



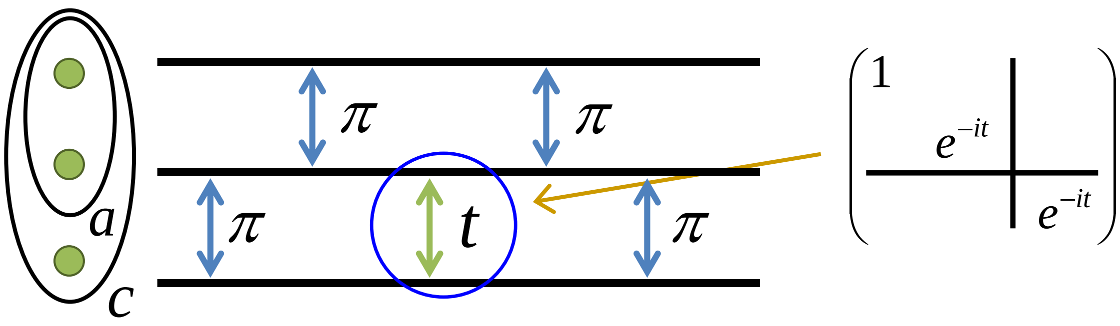
New Sequences?

- Another three-spin sequence



New Sequences?

- Another three-spin sequence



$$ac = 0\frac{1}{2} \quad 1\frac{1}{2} \quad 1\frac{3}{2}$$

$$\left(\begin{array}{cc|c} 1 & & \\ & e^{-it} & \\ \hline & & e^{-it} \end{array}\right)$$

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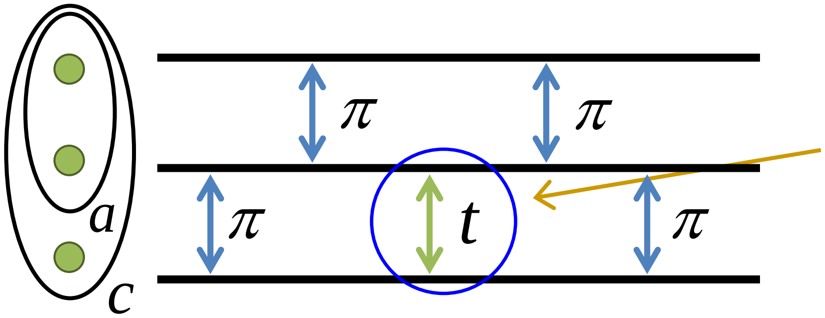
New Sequences?

- Another three-spin sequence

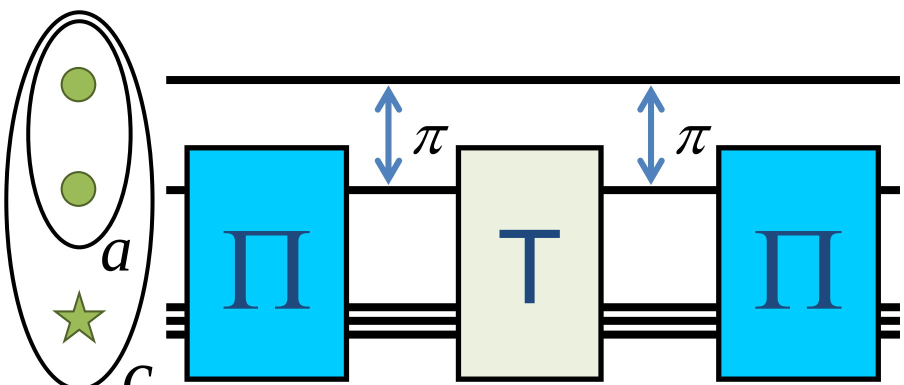


$$\begin{pmatrix} \begin{matrix} 1 & \\ & e^{-it} \end{matrix} & \vline & \\ \hline & \vline & e^{-it} \end{pmatrix}$$

$$ac = 0\tfrac{1}{2} \quad 1\tfrac{1}{2} \quad 1\tfrac{3}{2}$$

$$\begin{pmatrix} \begin{matrix} 1 & \\ & e^{-it} \end{matrix} & \vline & \\ \hline & \vline & e^{-it} \end{pmatrix}$$

- Elevated sequence



$$\begin{pmatrix} \begin{matrix} I & \\ & M \end{matrix} & \vline & \\ \hline & \vline & M \end{pmatrix}$$

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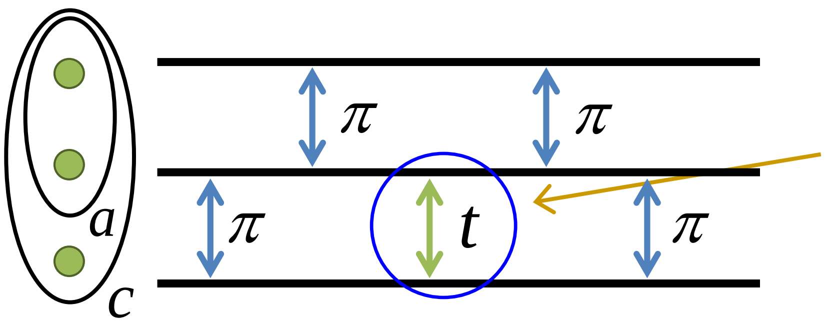
New Sequences?

- Another three-spin sequence

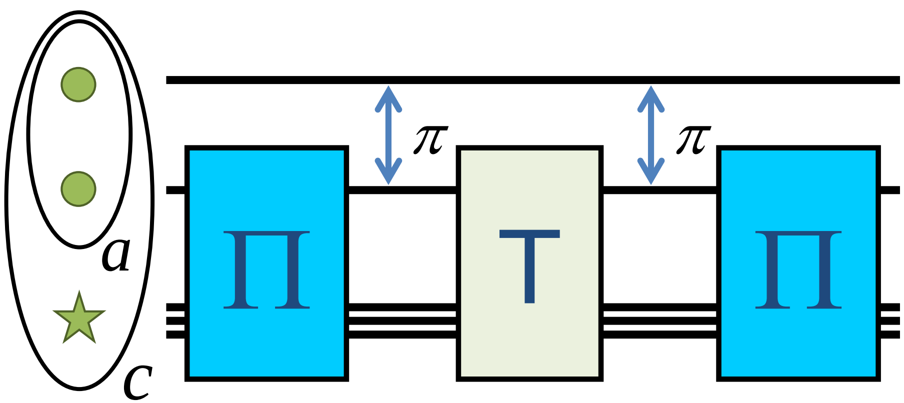


$$\left(\begin{array}{cc|c} 1 & & \\ & e^{-it} & \\ \hline & & e^{-it} \end{array}\right)$$

$$ac = 0\frac{1}{2} \quad 1\frac{1}{2} \quad 1\frac{3}{2}$$

$$\left(\begin{array}{cc|c} 1 & & \\ & e^{-it} & \\ \hline & & e^{-it} \end{array}\right)$$

- Elevated sequence



No longer require $M^2 = I$

$$\left(\begin{array}{cc|c} I & & \\ & M & \\ \hline & & M \end{array}\right)$$

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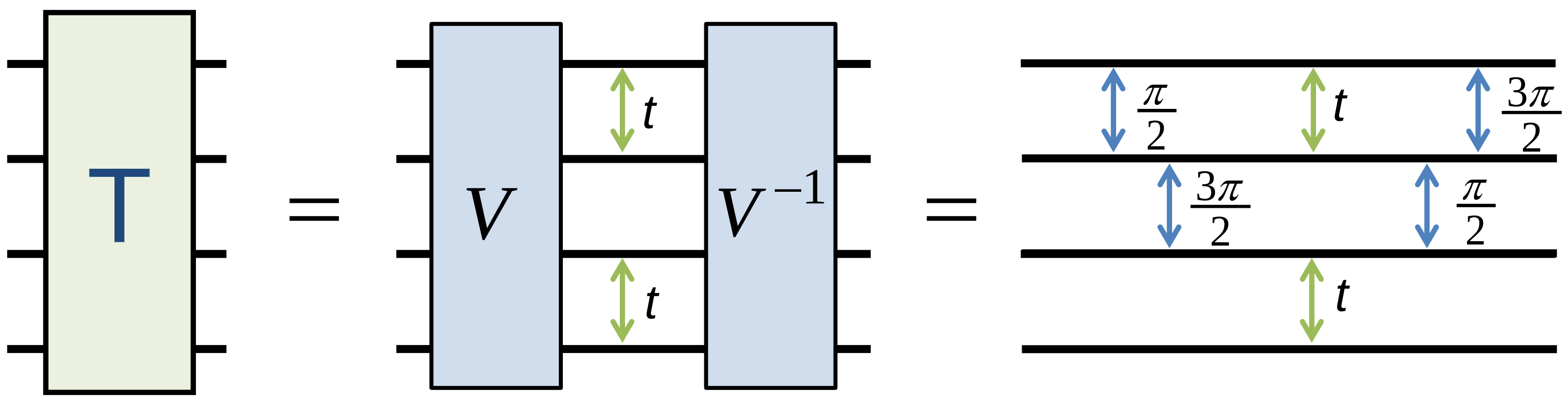
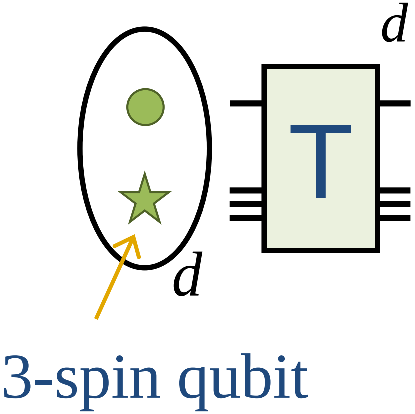
T Operation





$$d = \begin{matrix} 0 & 1 \end{matrix}$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

$$\mathbb{M}(\phi) = e^{i\xi(t)} e^{i\phi(t)\hat{\mathbf{n}}(t)\cdot\boldsymbol{\sigma}/2}$$

$$\phi(t) = 2\arccos((5\cos(\pi t/2) + 3\cos(3\pi t/2))/8)$$

$$\xi(t) = -\pi t/2$$

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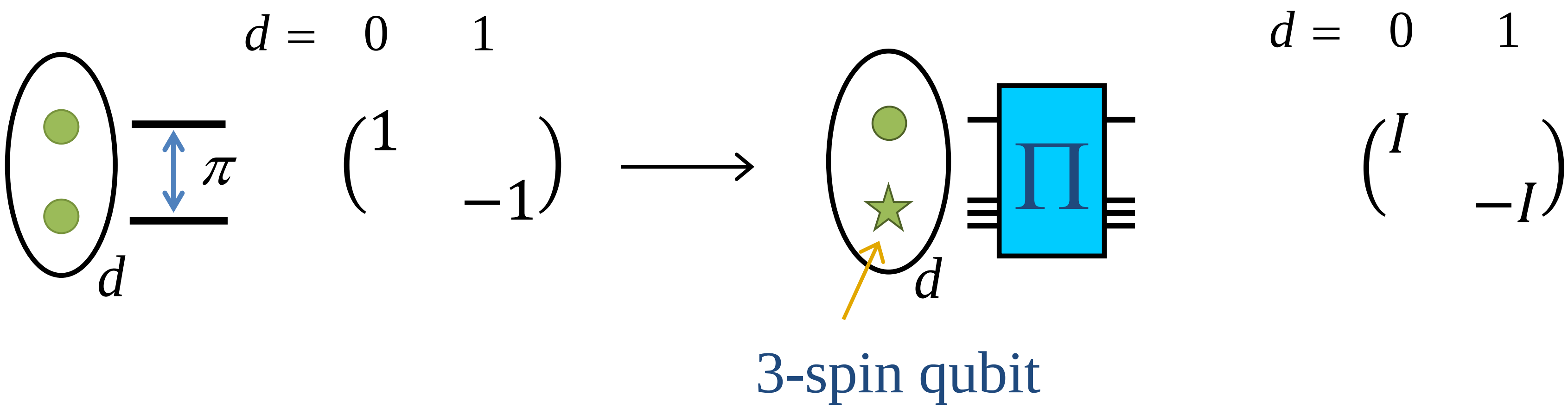
Π Operation

- Π generalizes a SWAP



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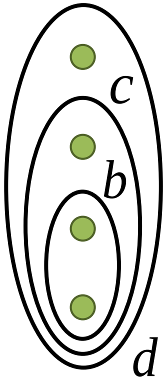
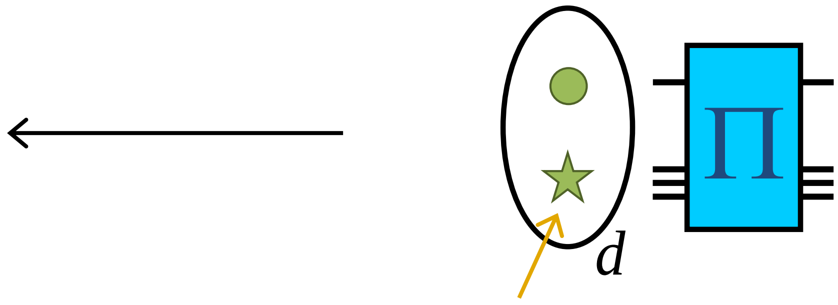
Π Operation

$$\overbrace{c = \tfrac{1}{2}}^{d=0} \quad \overbrace{\tfrac{1}{2} \quad \tfrac{3}{2}}^{d=1}$$





$$d = \quad 0 \quad 1$$

$$\begin{pmatrix} I & \\ & -I \end{pmatrix}$$

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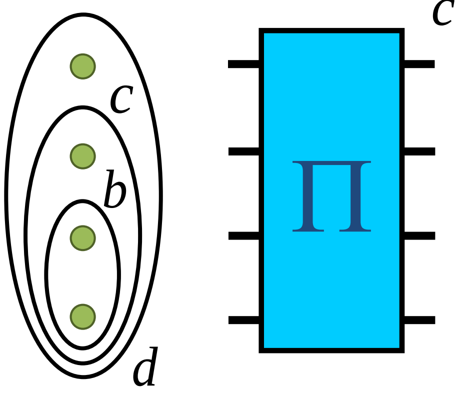
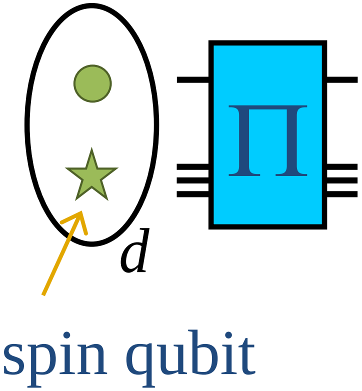
Π Operation



$$\overbrace{}^{d=0}\quad \overbrace{\qquad\qquad}^{d=1}$$

$$c = \tfrac{1}{2} \qquad \tfrac{1}{2} \qquad \tfrac{3}{2}$$

$$\left(\begin{array}{c|cc} I & & \\ \hline & -I & \\ & & \alpha \end{array}\right)$$



$$d = \quad 0 \qquad 1$$

$$\begin{pmatrix} I & \\ & -I \end{pmatrix}$$

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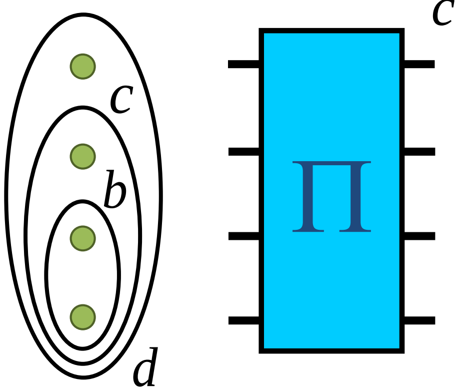
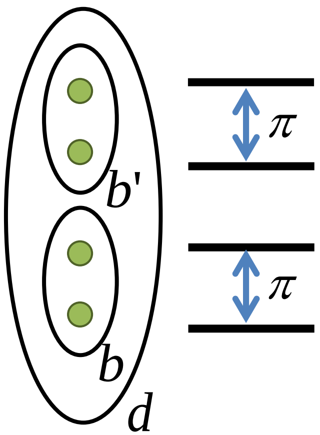
Π Operation



$$c = \underbrace{\tfrac{1}{2}}_{d=0}\quad \underbrace{\tfrac{1}{2}\qquad \tfrac{3}{2}}_{d=1}$$

$$\left(\begin{array}{c|cc} I & & \\ \hline & -I & \\ & & \alpha \end{array}\right)$$

- Consider ..



$$bb' = \underbrace{00\ \ 11}_{d=0}\quad \underbrace{01\ \ 10\ \ 11}_{d=1}$$

$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)$$

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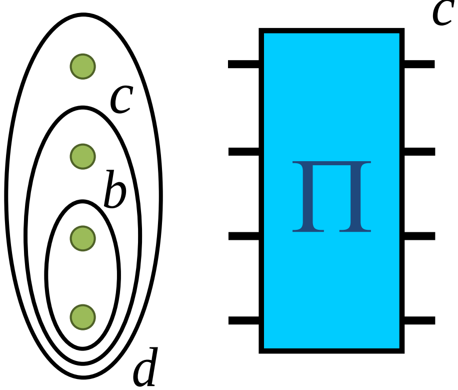
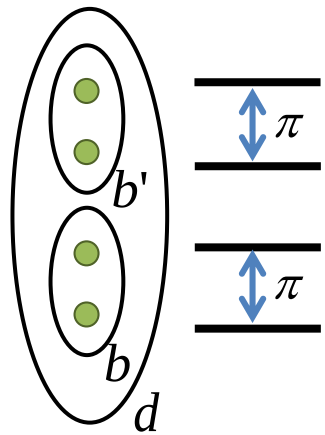
Π Operation



$$c = \overbrace{\tfrac{1}{2}}^{d=0} \quad \overbrace{\tfrac{1}{2} \qquad \tfrac{3}{2}}^{d=1}$$

$$\left(\begin{array}{c|cc} I & & \\ \hline & -I & \\ & & \alpha \end{array}\right)$$

- Consider ..



$$bb' = \overbrace{00 \;\; 11}^{d=0} \quad \overbrace{01 \;\; 10 \;\; 11}^{d=1}$$

$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)$$

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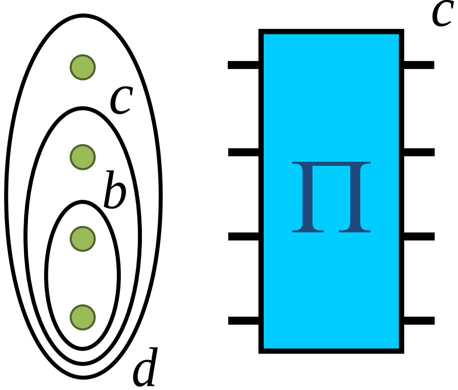
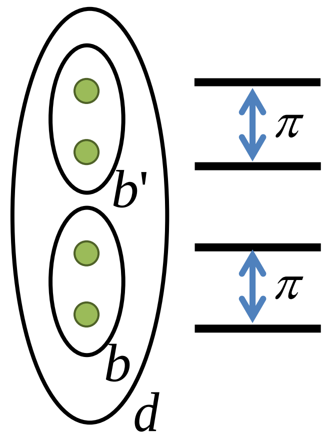
Π Operation



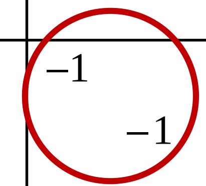
$$c = \overbrace{\tfrac{1}{2}}^{d=0} \quad \overbrace{\tfrac{1}{2} \qquad \tfrac{3}{2}}^{d=1}$$

$$\left(\begin{array}{c|cc} I & & \\ \hline & -I & \\ & & \alpha \end{array}\right)$$

- Consider ..

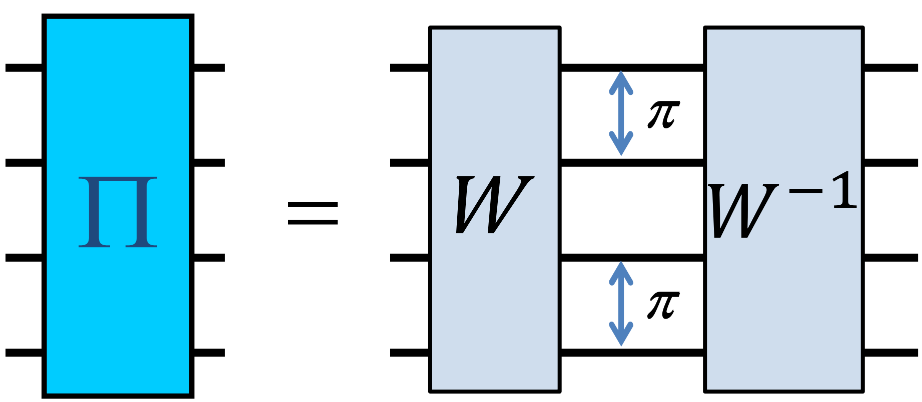


$$bb' = \overbrace{00 \;\; 11}^{d=0} \quad \overbrace{01 \;\; 10 \;\; 11}^{d=1}$$



$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)$$

- New sequence ***W***



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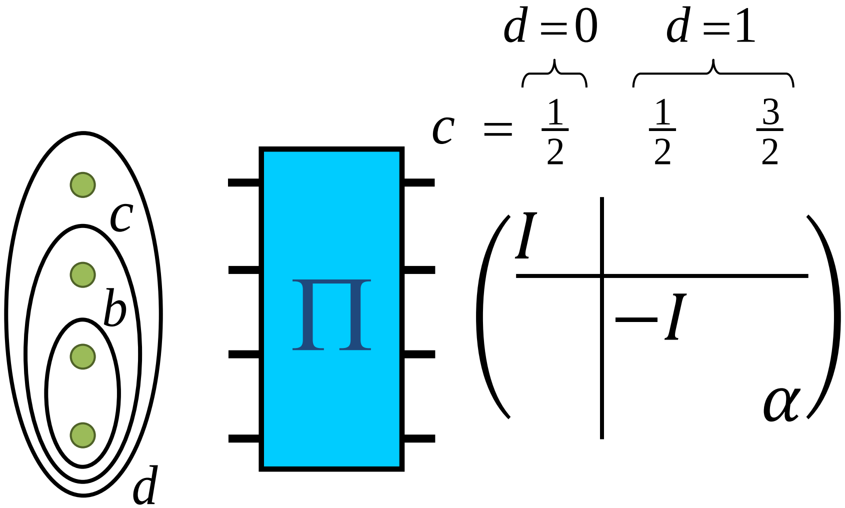
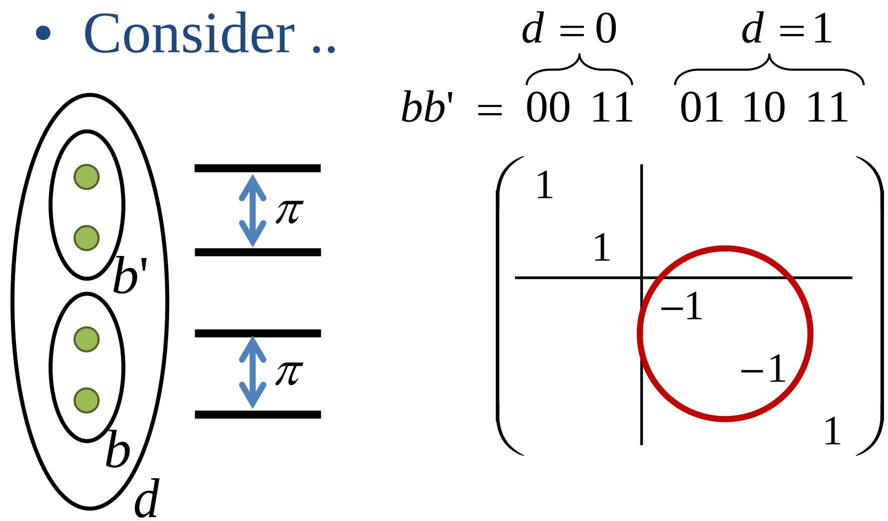
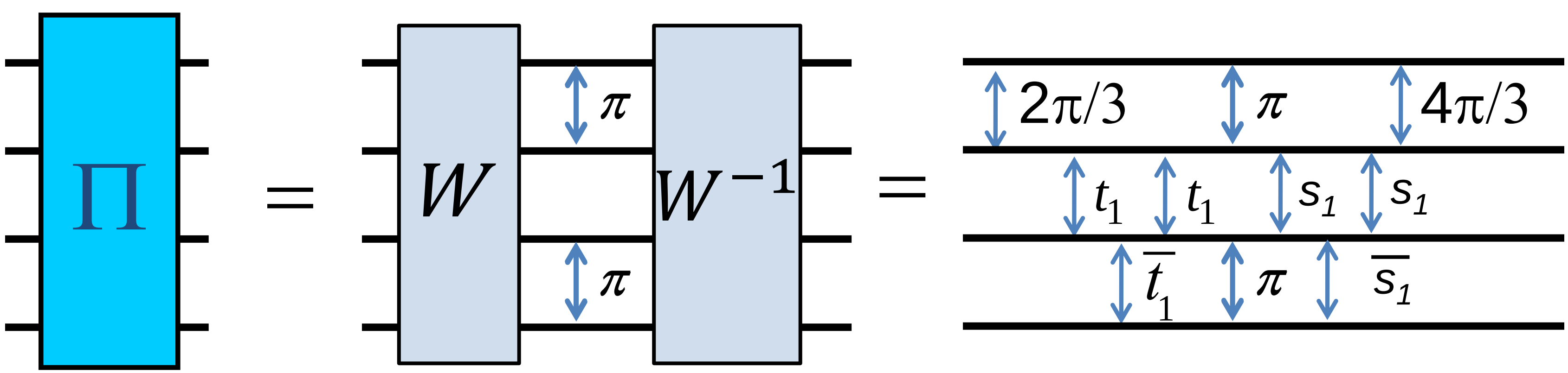
Π Operation



- Consider ..



- Sequence for Π



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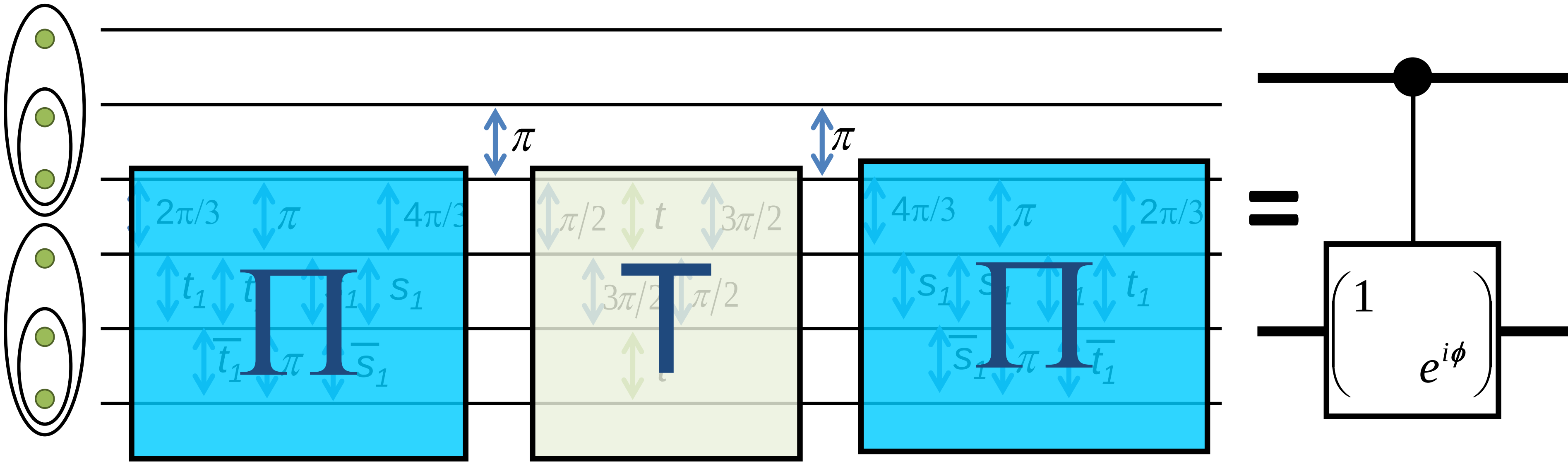
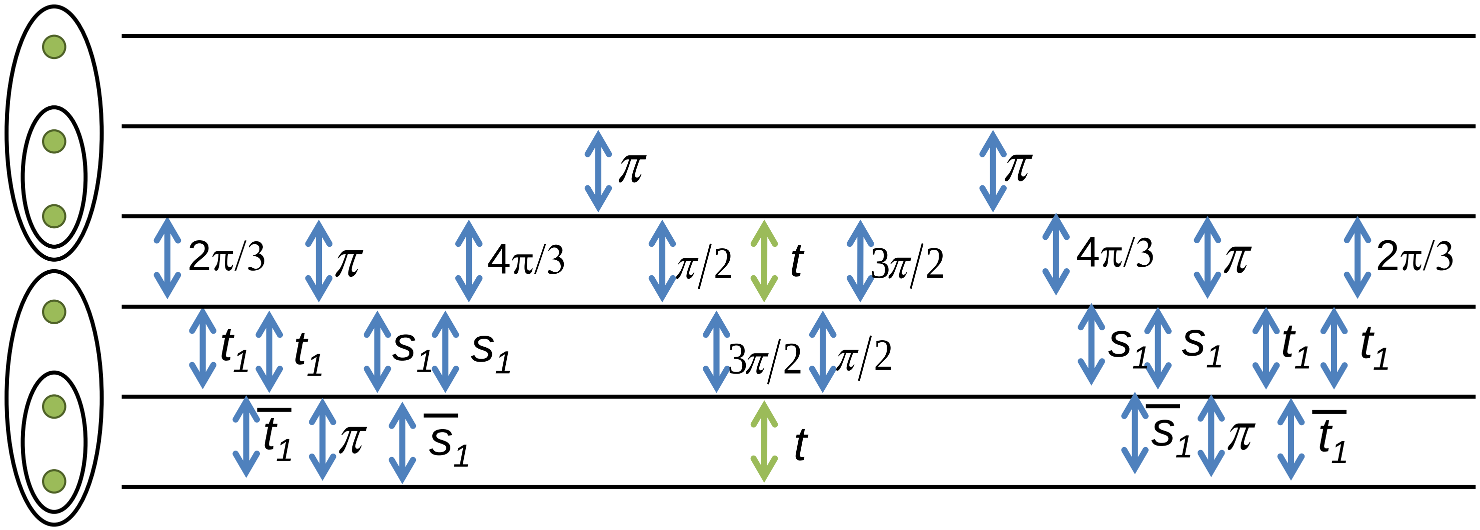
Full Sequence

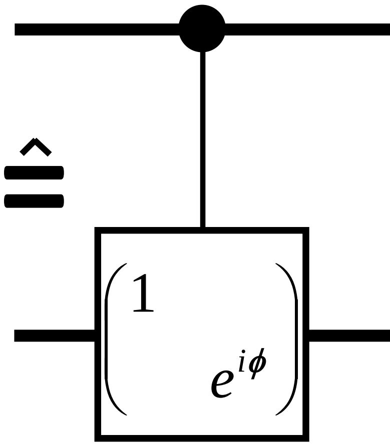


Zeuch, NEB, *In preparation*

Full Sequence



$\hat{=}$



t_1=1.34004…,s_1= 2π–$t_{1,\ldots}$

$$\phi(t) = 2\arccos((5\cos(\pi t/2) + 3\cos(3\pi t/2))/8)$$

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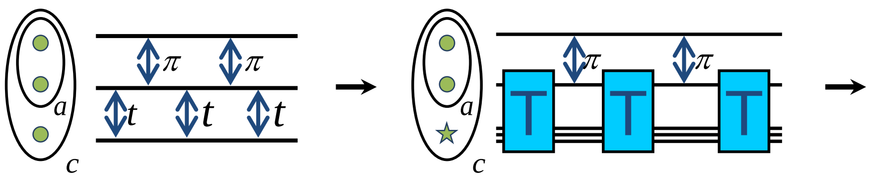
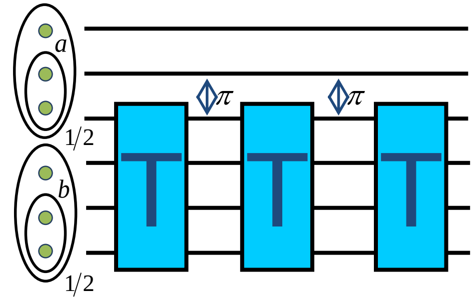
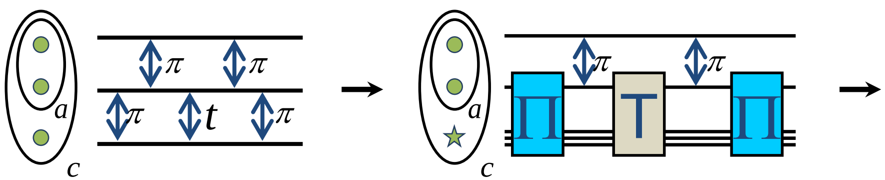
Summary

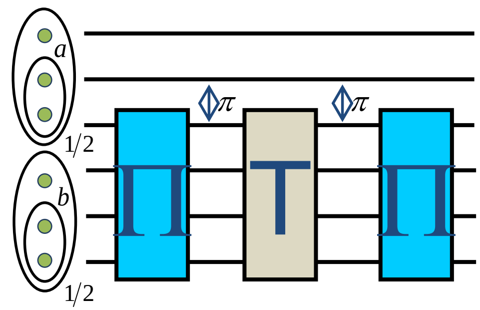
Analytic Derivation of Fong-Wandzura CNOT sequence:





Generalization leading to new sequences:





Open questions:

1) Can we prove Fong-Wandzura sequence is truly optimal?
2) More efficient general gate constructions?
3) Can these tools be used to construct more “robust” sequences?

Zeuch Cipri, NEB. *Analytic Pulse Sequence Construction for Exchange-Only Quantum Computation*, Phys. Rev. B **90**, 045306 (2014)
Zeuch, NEB, *A Simple Derivation of the Fong-Wandzura Pulse Sequence*, Phys. Rev. A 93, 010303(R) (2016)
Zeuch, NEB*, Efficient Two-Qubit Pulse Sequences Beyond CNOT*, In preparation.