Designing Two-Qubit Gates for Exchange-Only Quantum Computation

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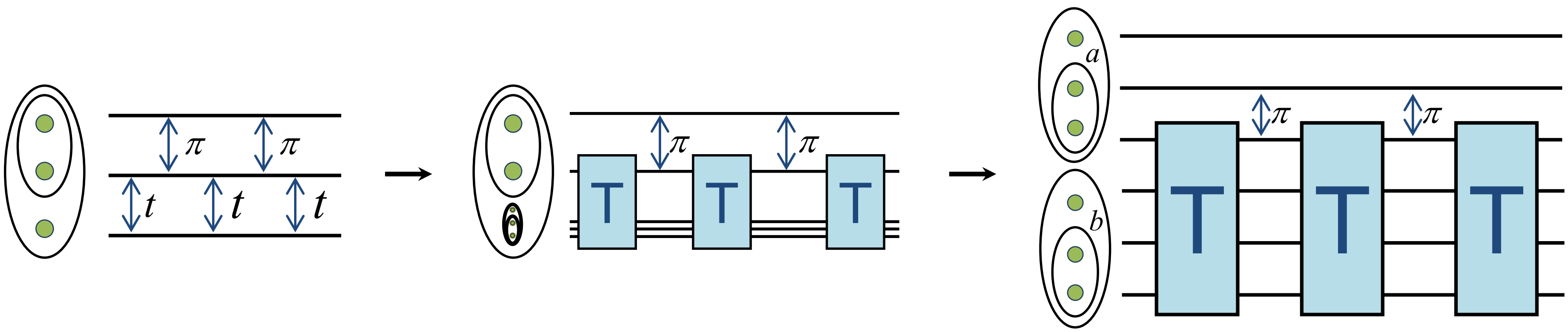


Work done in collaboration with:

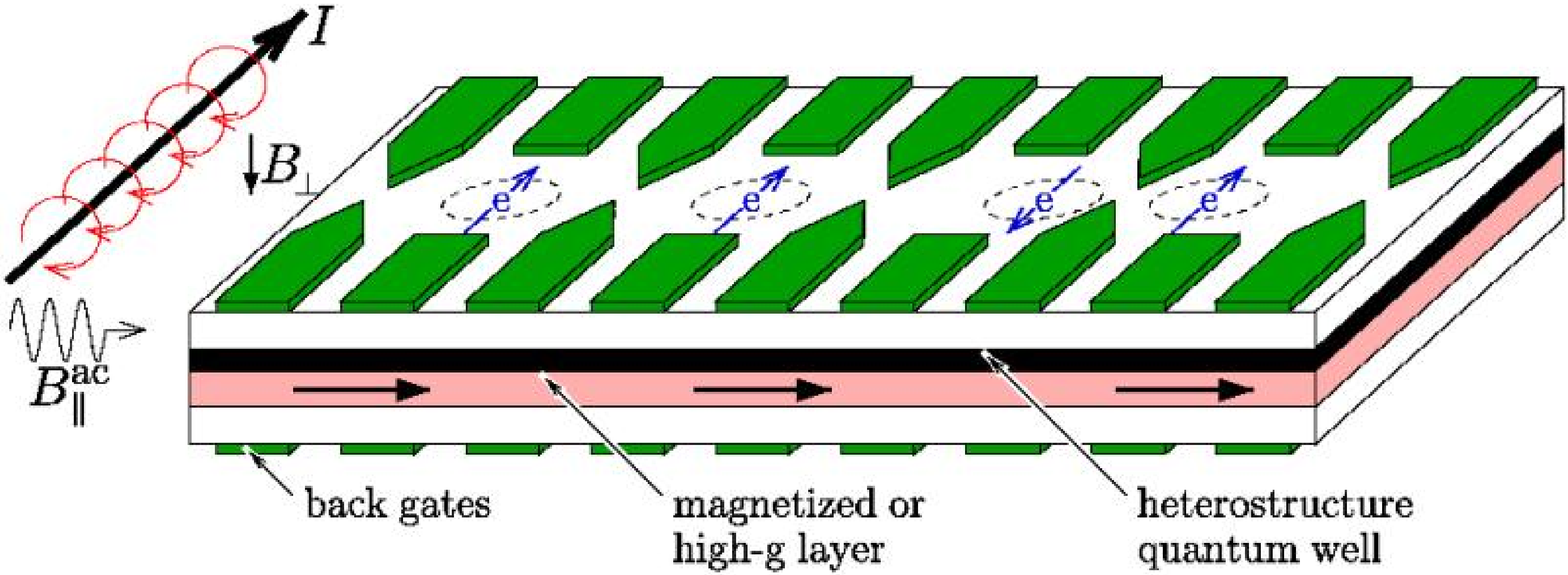
Daniel Zeuch, FSU → Peter Gruenberg Institut, Research Center Juelich

Zeuch, NEB, Phys. Rev. A **98**, 010303 (2016)
Zeuch, NEB, Phys. Rev. B **102**, 075311 (2020)

Cal State Los Angeles, Colloquium, Dec. 2, 2021

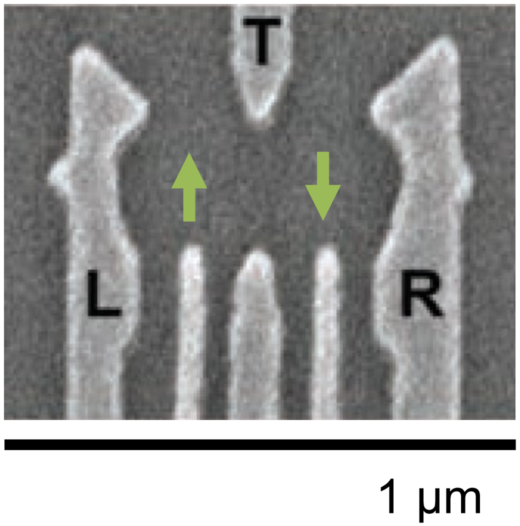
Early Vision of a Solid State Quantum Computer

Loss & DiVincenzo, Phys. Rev. B (1998)

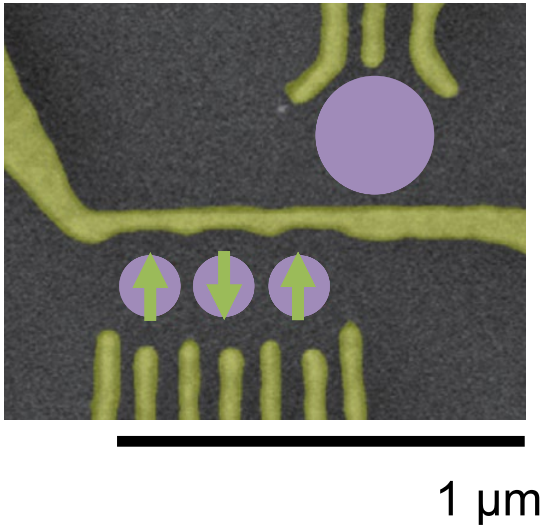


Decades of Slow Steady Progress

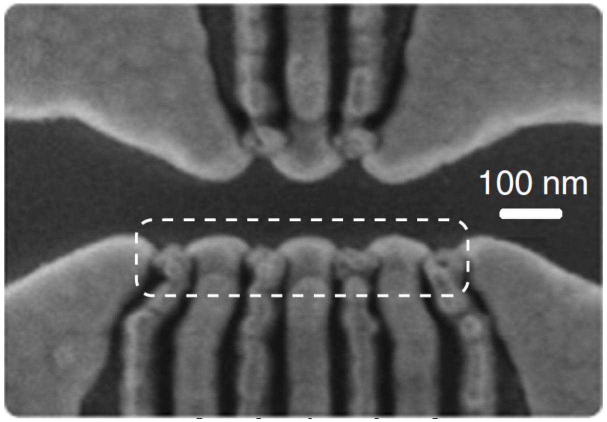
Petta *et al*., *Science* (2005)



Medford *et al*., *Nature Nanotechnology* (2013)

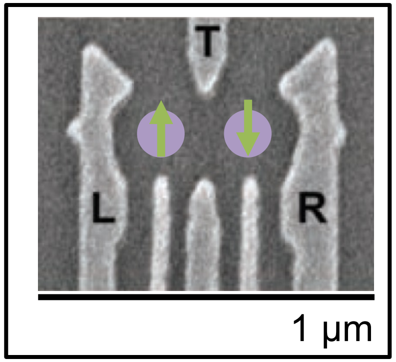


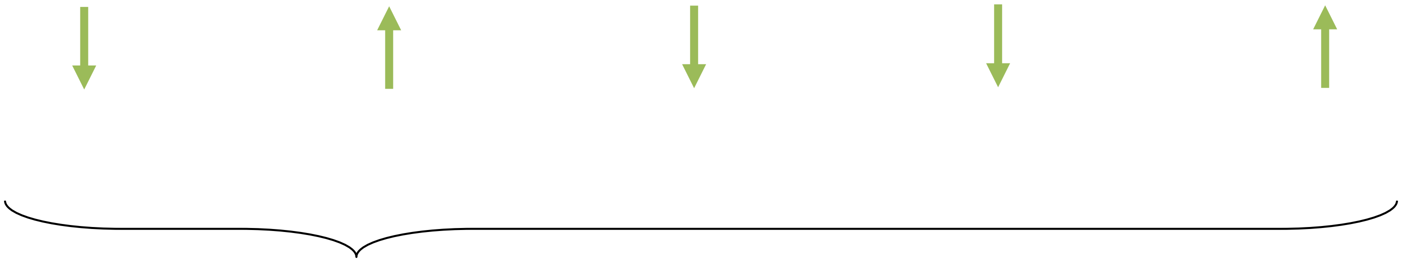
Andrews *et al*., *Nature Nanotechnology* (2019)



Basic Idea

- Use electron spins as qubits



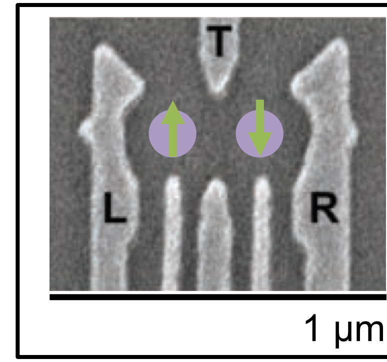


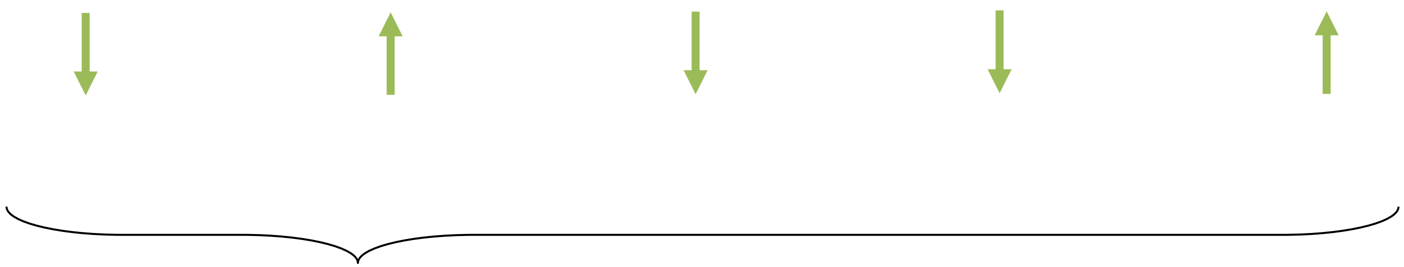
spin-1/2 chain: electrons
in quantum dots

Exchange-Based QC

- Quantum gates through spin exchange

$$H_i = J\, \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$





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Bacon, Kempe, Lidar & Whaley, *Phys. Rev. Lett.* (2000)

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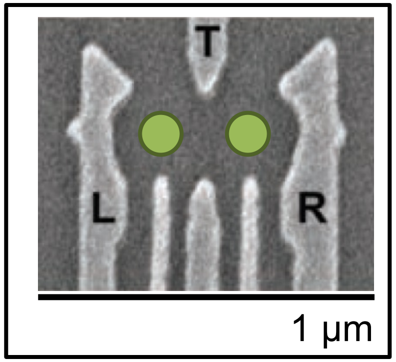

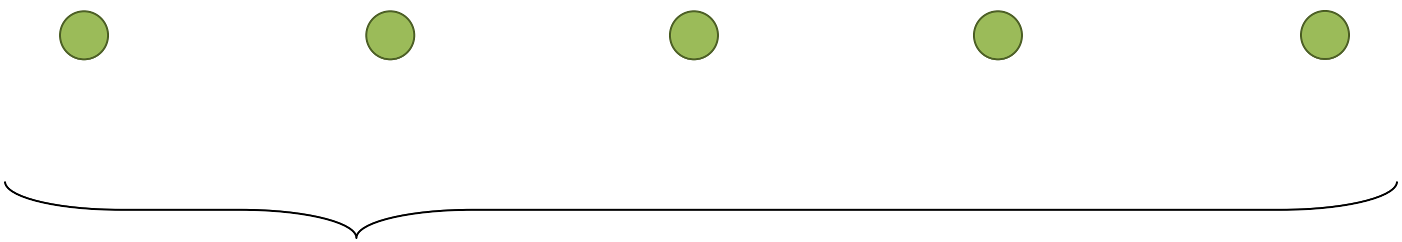

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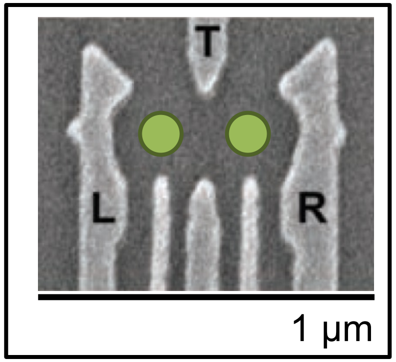

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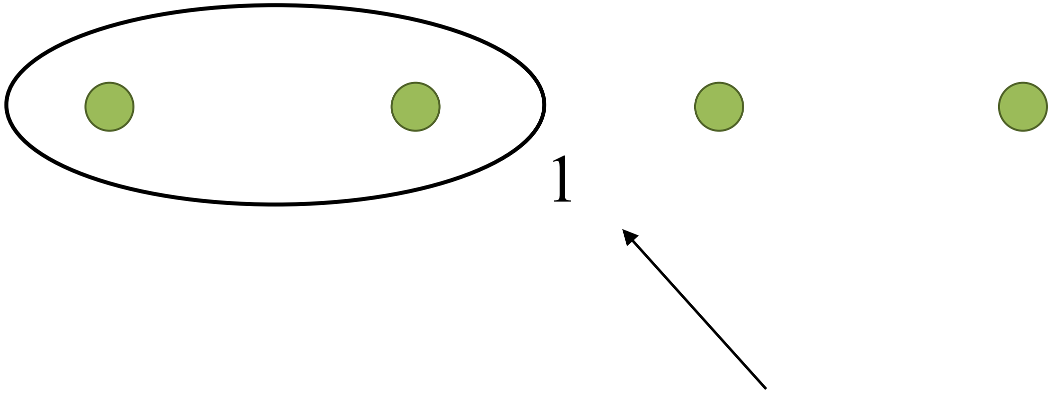

total spin of the oval

Bacon, Kempe, Lidar & Whaley, *Phys. Rev. Lett.* (2000)

Exchange-Based QC

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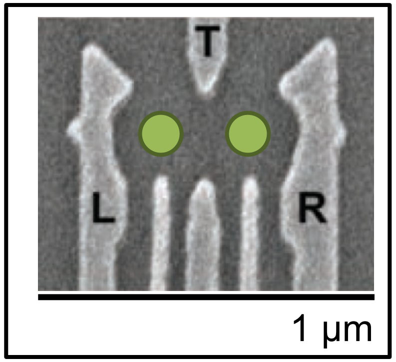

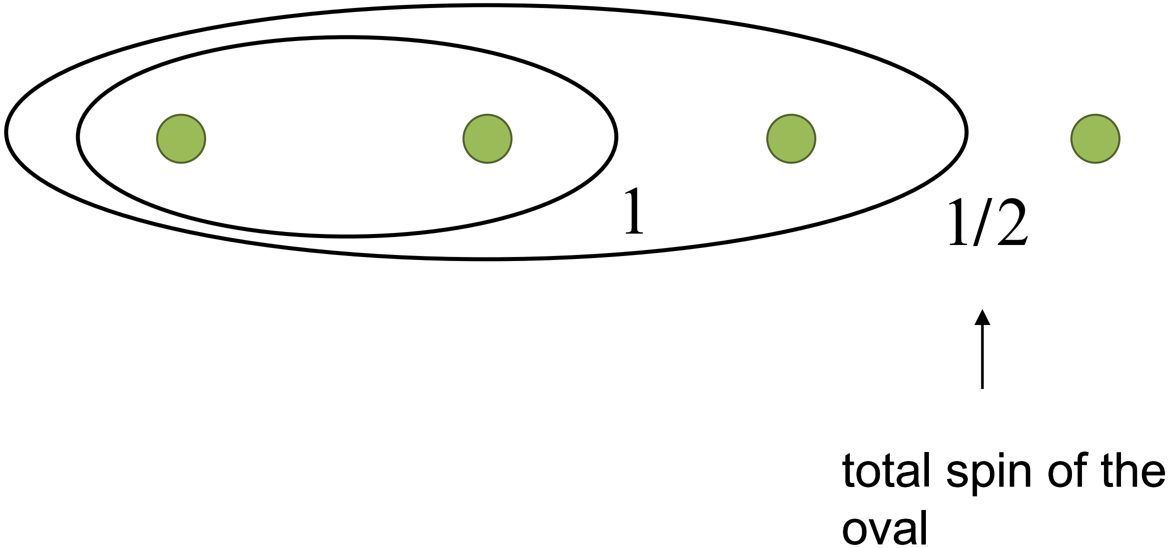

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Exchange-Based QC

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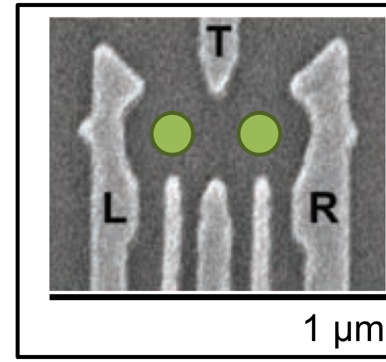
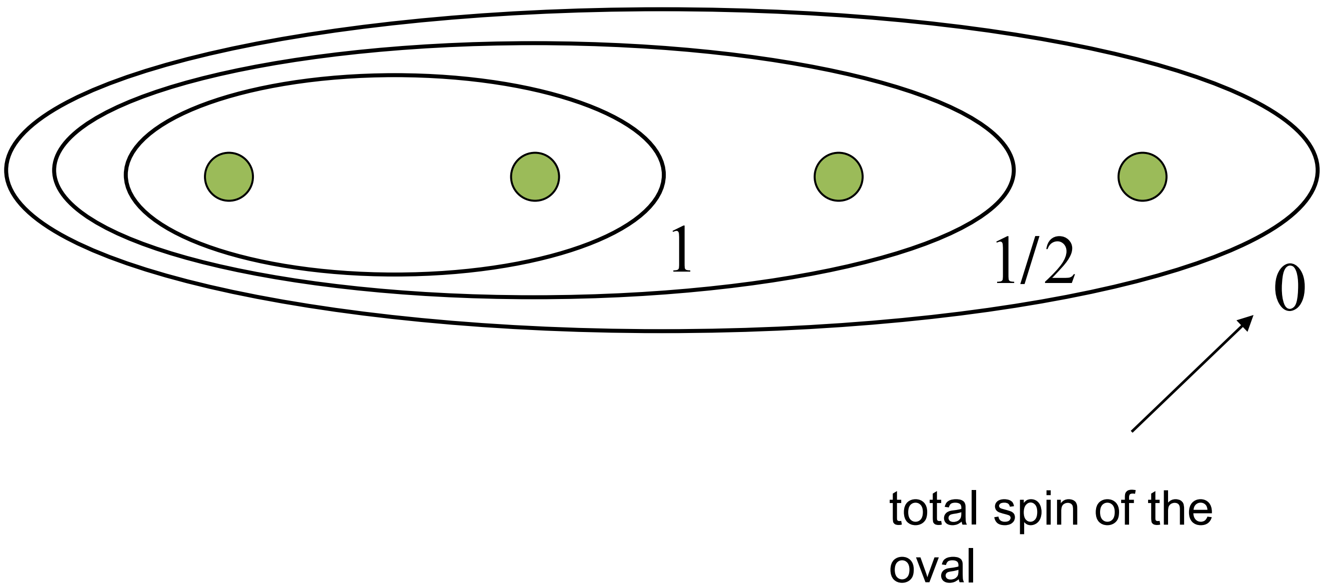
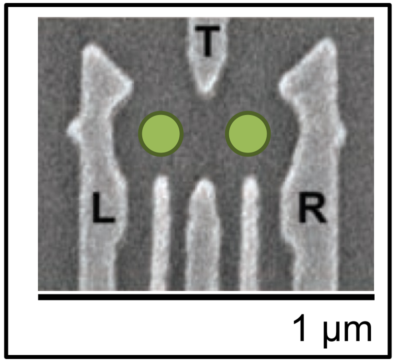


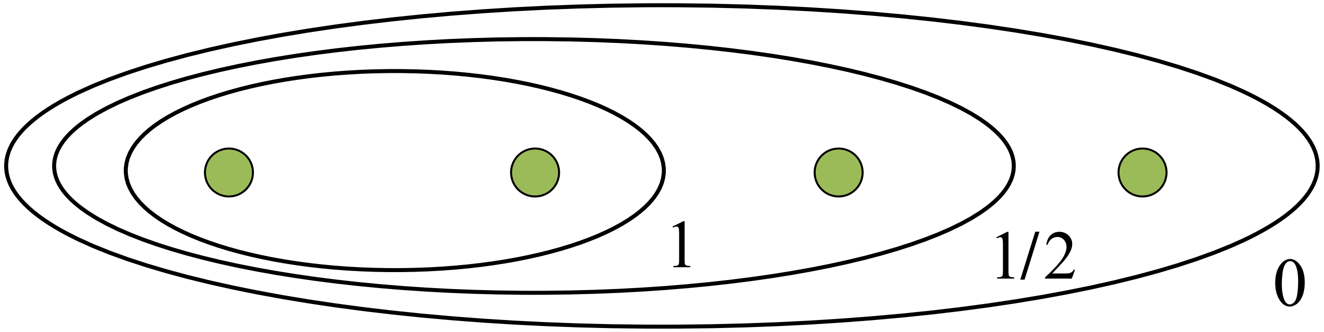


Exchange-Based QC

- Quantum gates through spin exchange



$$H_i = J\, \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

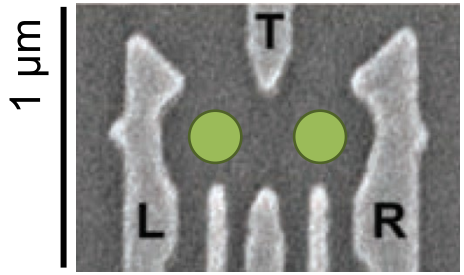


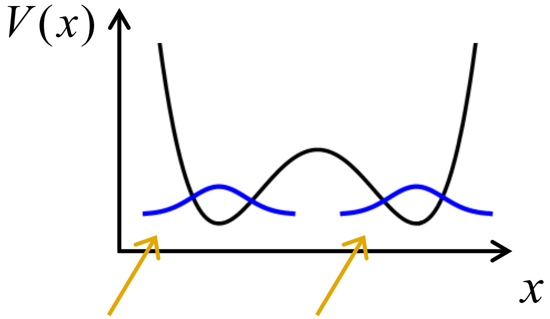
$$s_1 \otimes s_2 = |s_1 - s_2|,\ |s_1 - s_2 + 1|,\ \ldots,\ s_1 + s_2$$

Bacon, Kempe, Lidar & Whaley, *Phys. Rev. Lett.* (2000)

Controlling Exchange

Petta *et al.*, *Science* (2005)

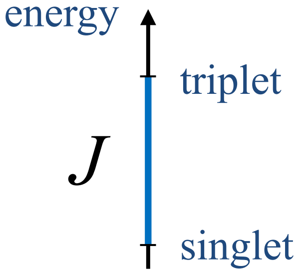




Electron wave functions in quantum dot potential $V(x)$

- Exchange Hamiltonian

$$H = J\, \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$



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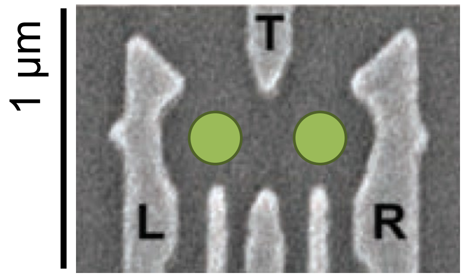

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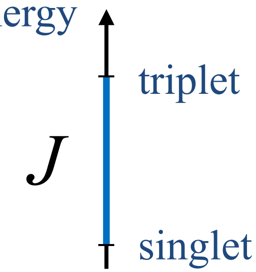

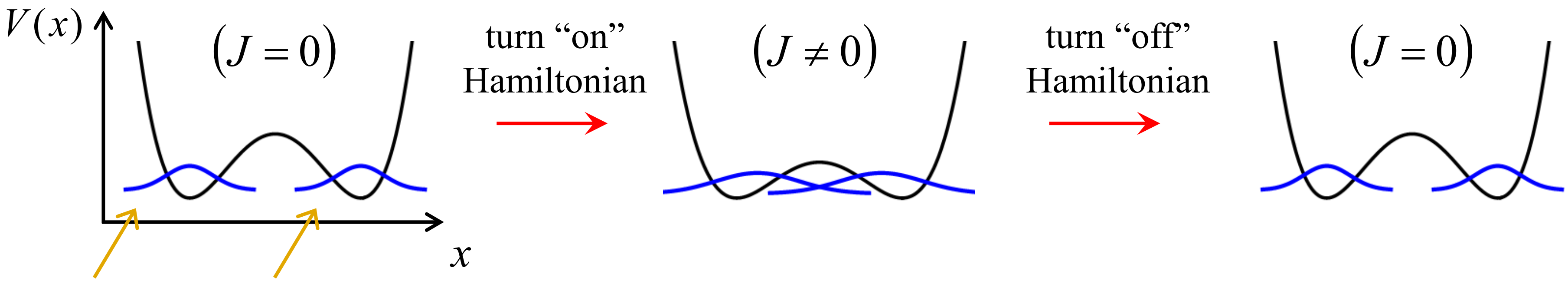

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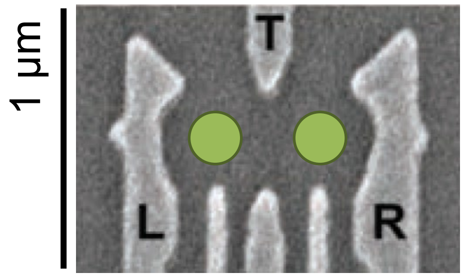

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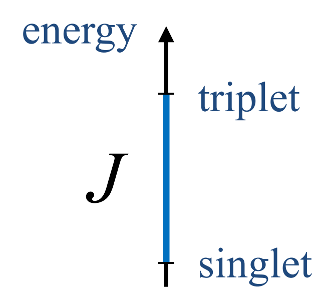
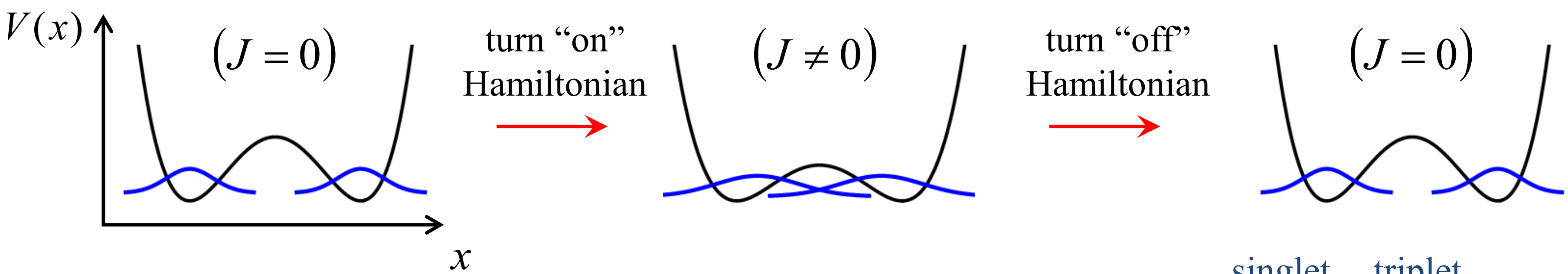
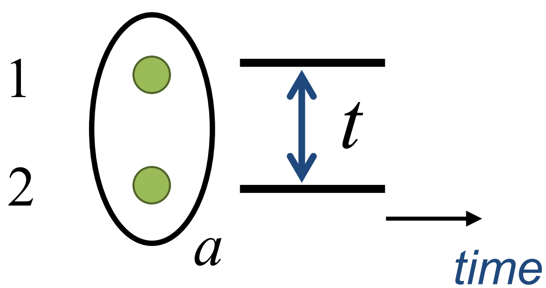






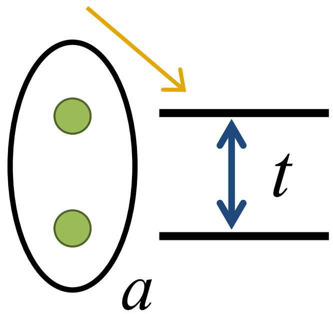
singlet triplet

$a =$ 0 1

$$\exp(-iHt) = \begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix} \qquad (J = 1)$$

Simple Exchange Pulses

exchange pulse of duration t

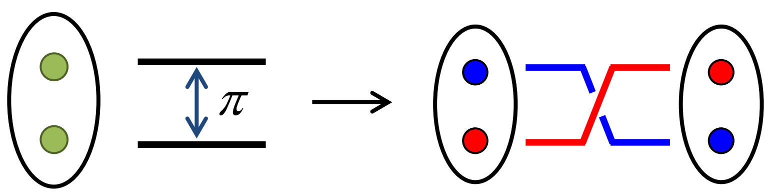


$$a = \begin{matrix} 0 & 1 \end{matrix}$$

$$\begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

- SWAP pulse

$t = \pi$

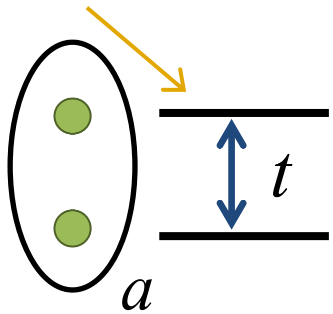


$$a = \begin{matrix} 0 & 1 \end{matrix}$$

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

Simple Exchange Pulses

exchange pulse of duration t



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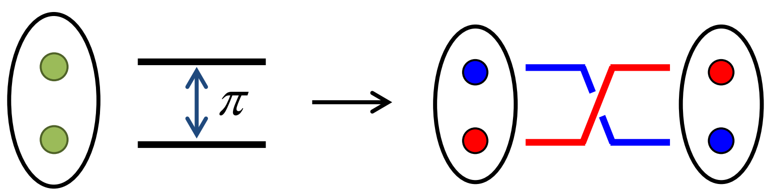


$$a = \begin{matrix} 0 & 1 & & 0 & 1 \end{matrix}$$

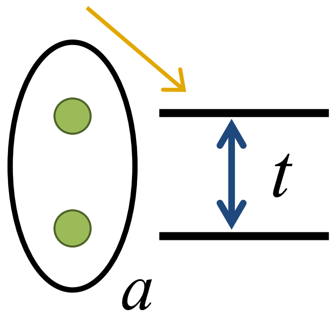
$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = -\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

singlet state $(a = 0)$ $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

triplet states $(a = 1)$ $= \begin{cases} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{cases}$

Simple Exchange Pulses

exchange pulse of duration t



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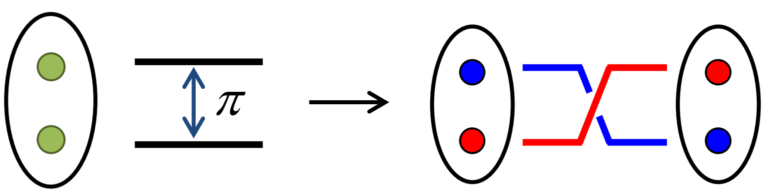
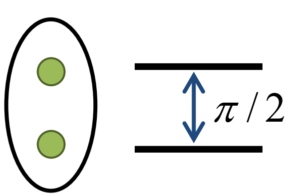


$$a = \begin{matrix} 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 \end{matrix}$$

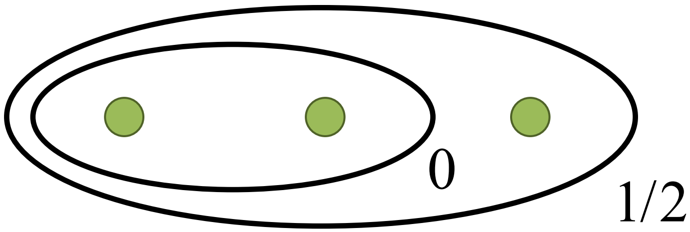
$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = -\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

- SWAP$^{1/2}$ pulse

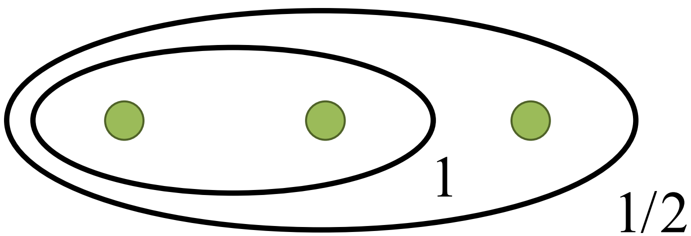
$t = \pi/2$



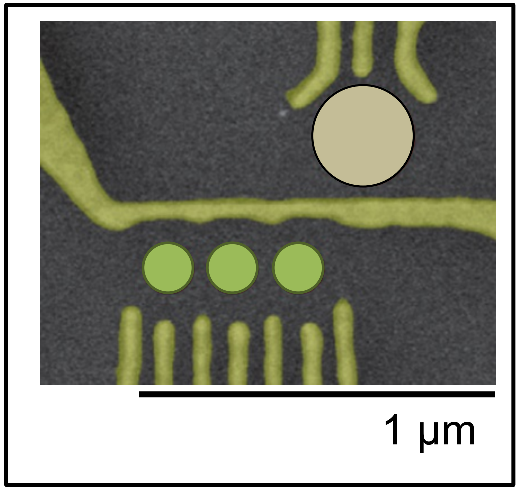
Three-Spin Qubit Encoding

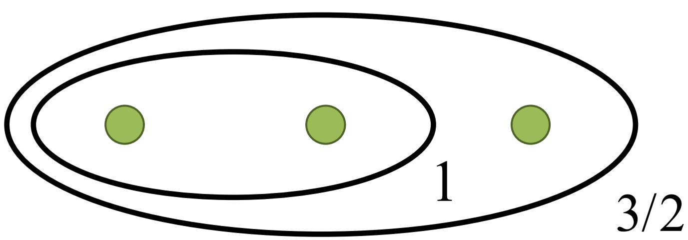


$= |0\rangle$



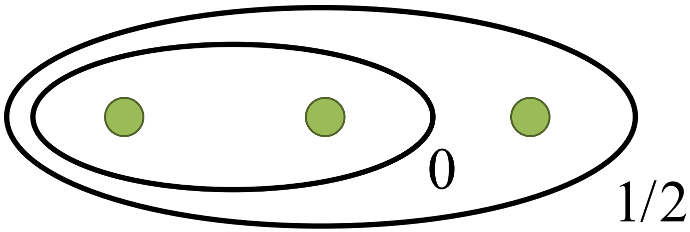
$= |1\rangle$



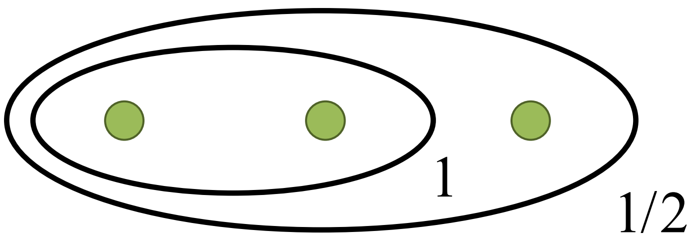


$= \left|\begin{matrix}\text{non}-\\ \text{computational}\end{matrix}\right\rangle$

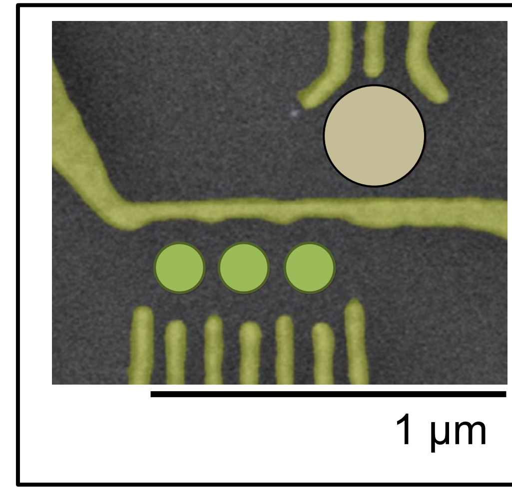
Three-Spin Qubit Encoding

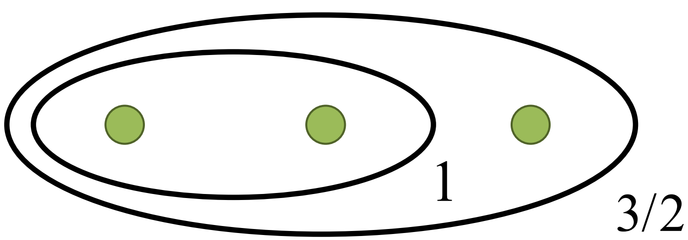


$= |0\rangle$



$= |1\rangle$





$= |\text{non} - \text{computational}\rangle$

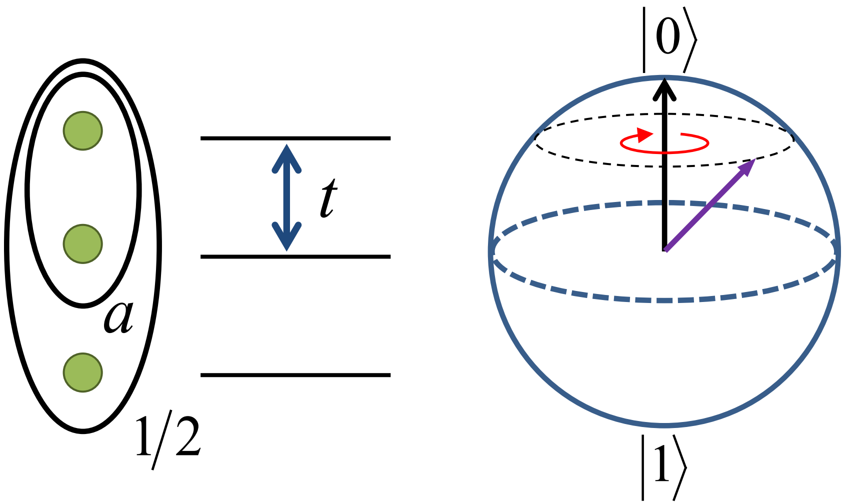
Transitions to this state are **leakage errors**.

DiVincenzo *et al.*, *Nature* (2000)

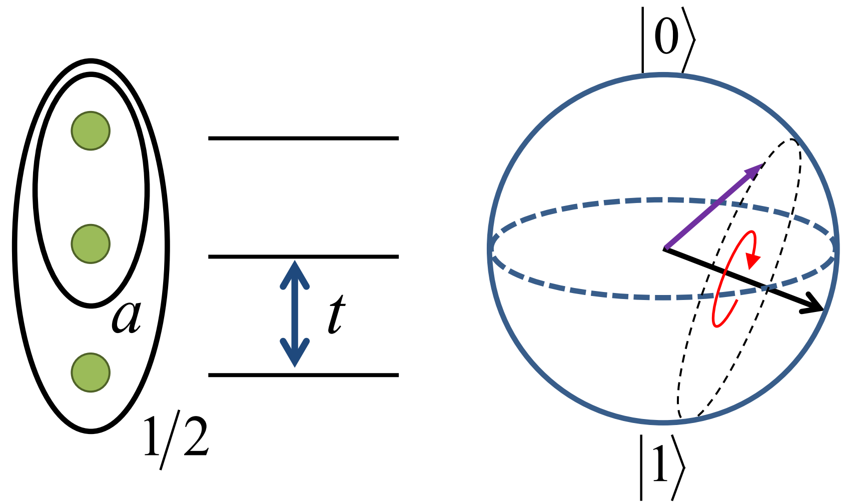
Medford *et al.*, *Nature Nanotechnology* (2013)

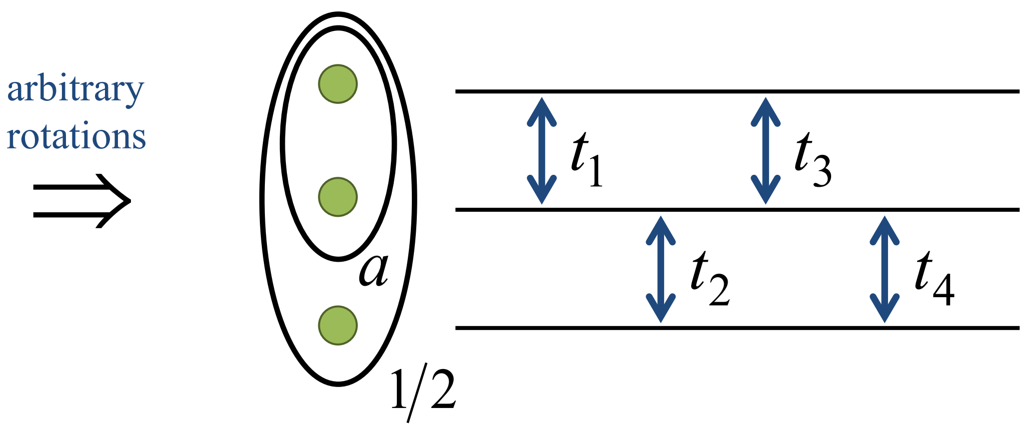
Single-Qubit Gates

- Rotation about *z*-axis :

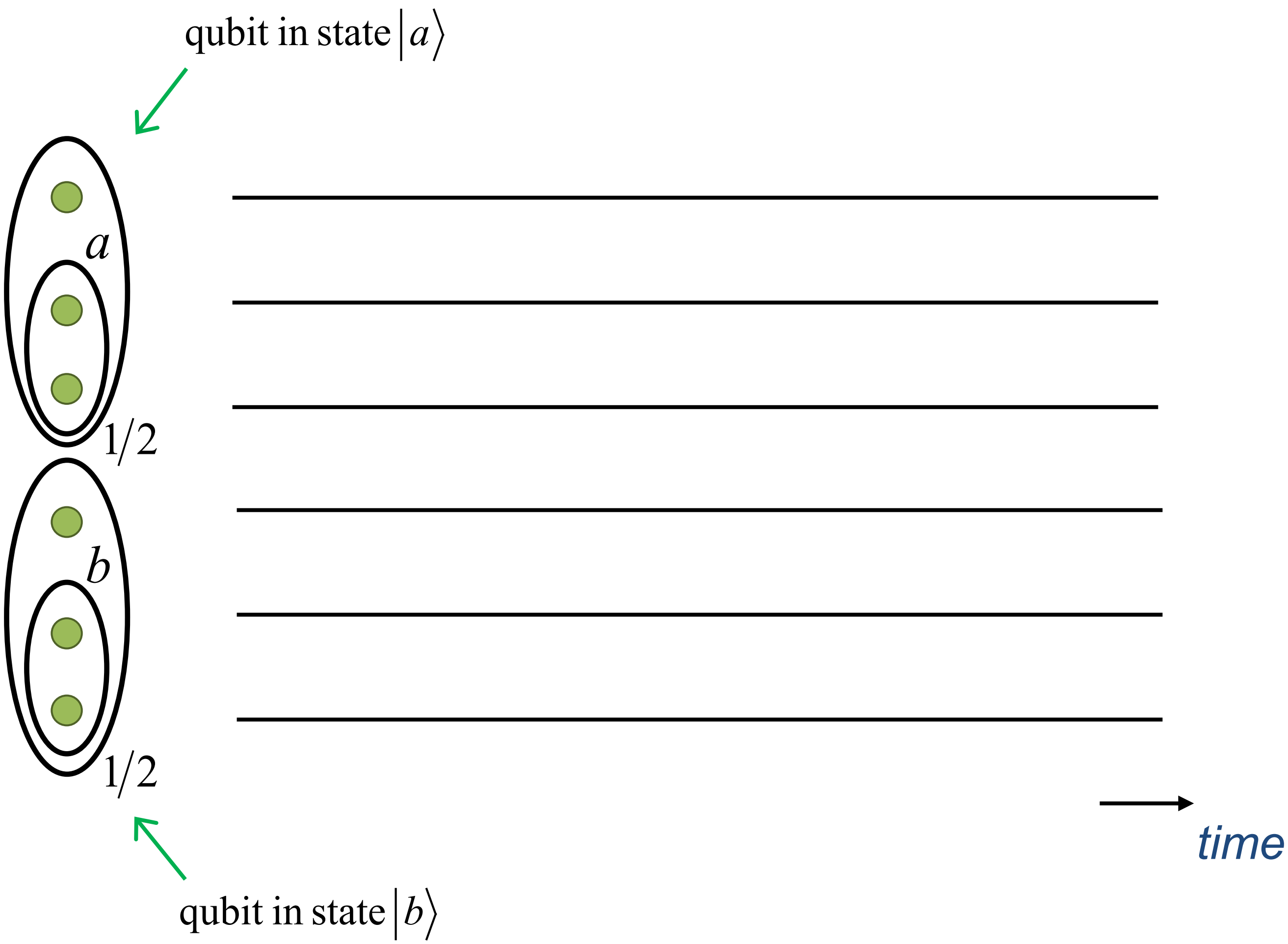


- Rotation about other axis:

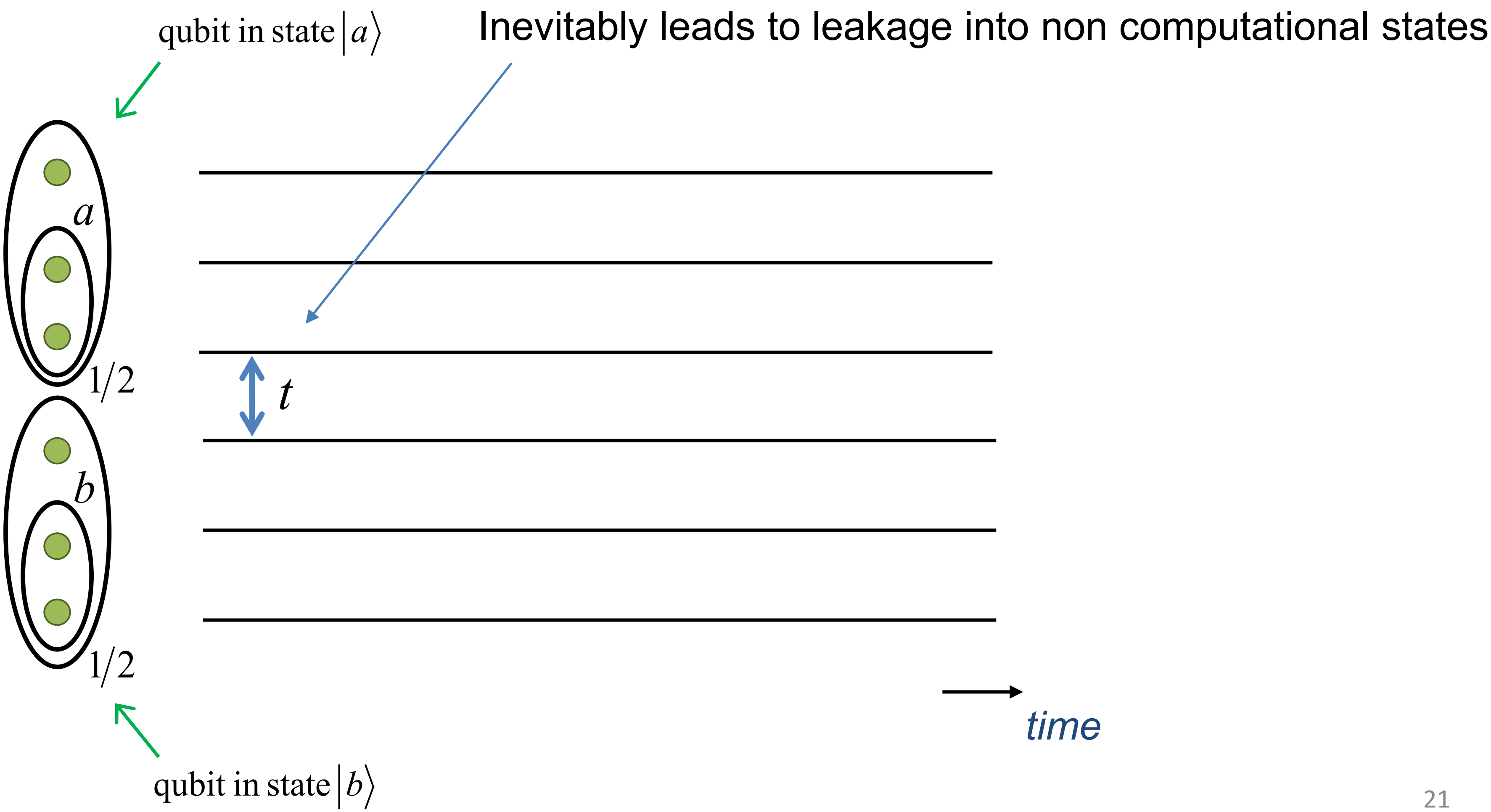




Two-Qubit Gates



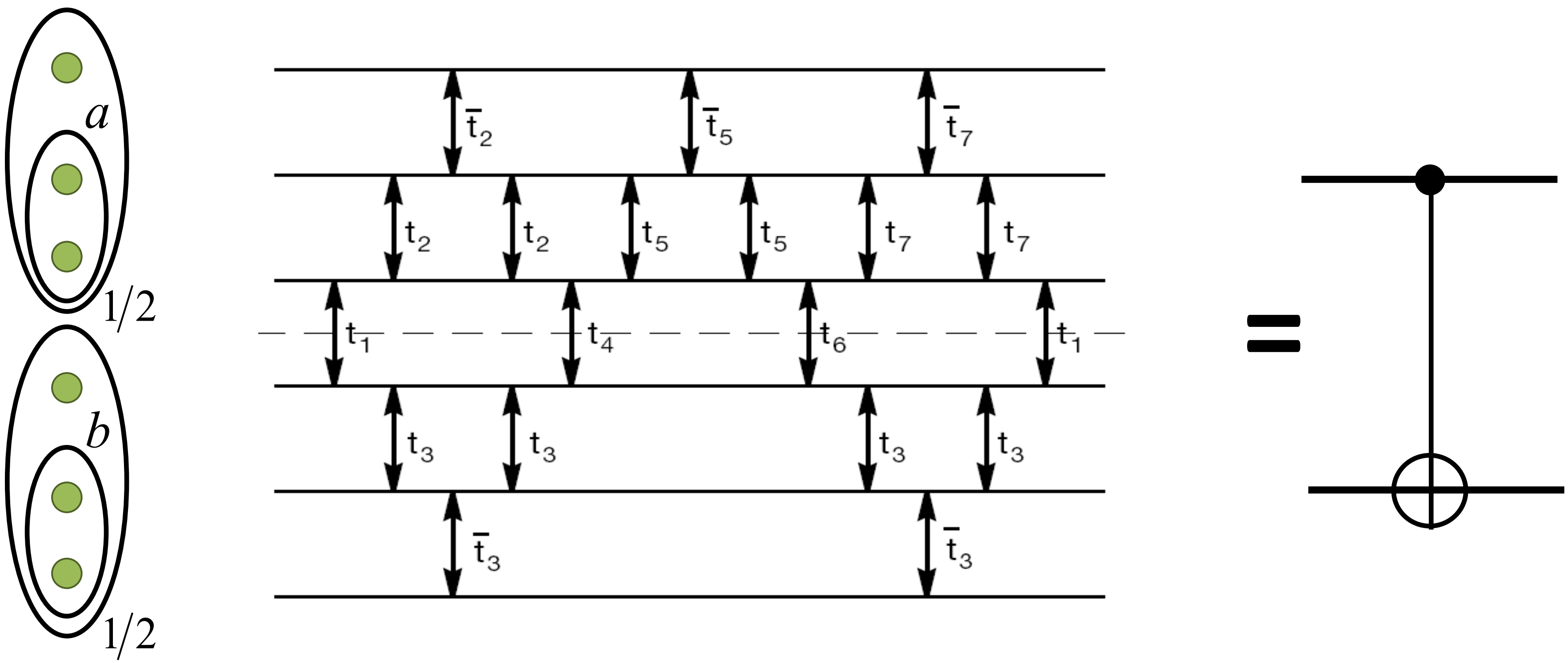
Two-Qubit Gates



Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically



$$t_1 = 2.581.. , \quad t_2 = 1.303.. , \quad t_3 = 1.753.. , \quad \ldots$$

Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

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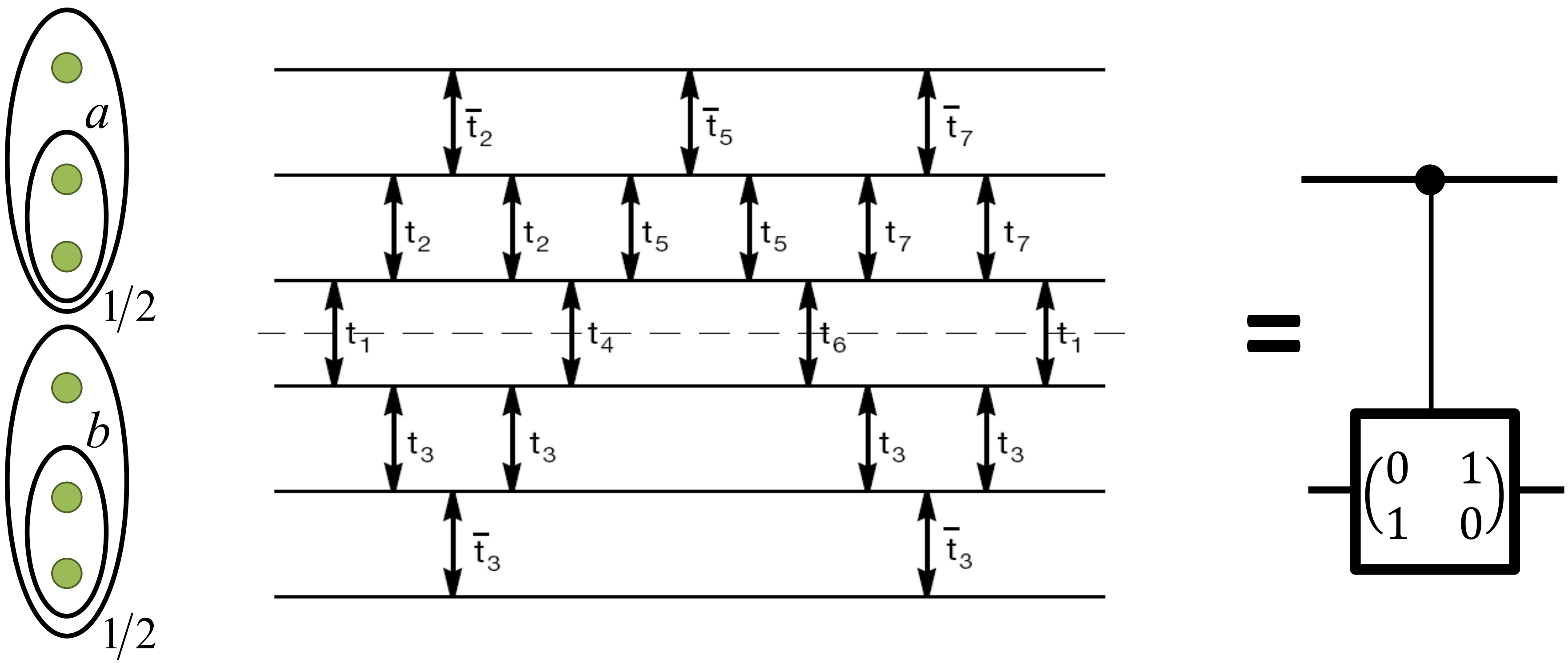

$$t_1 = 2.581.. \,,\quad t_2 = 1.303.. \,,\quad t_3 = 1.753.. \,,\quad \ldots$$

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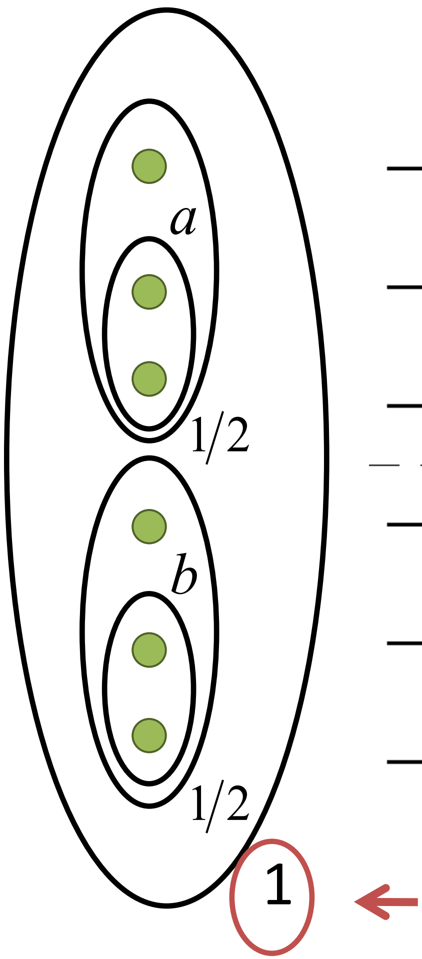
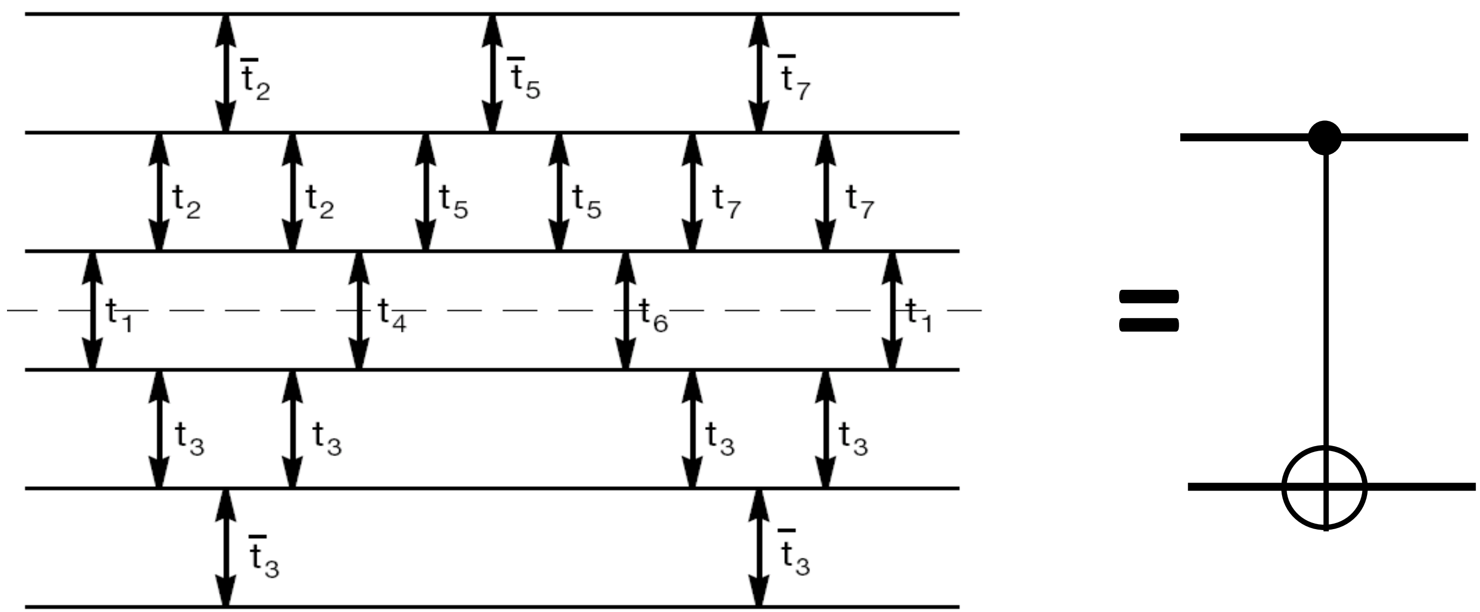


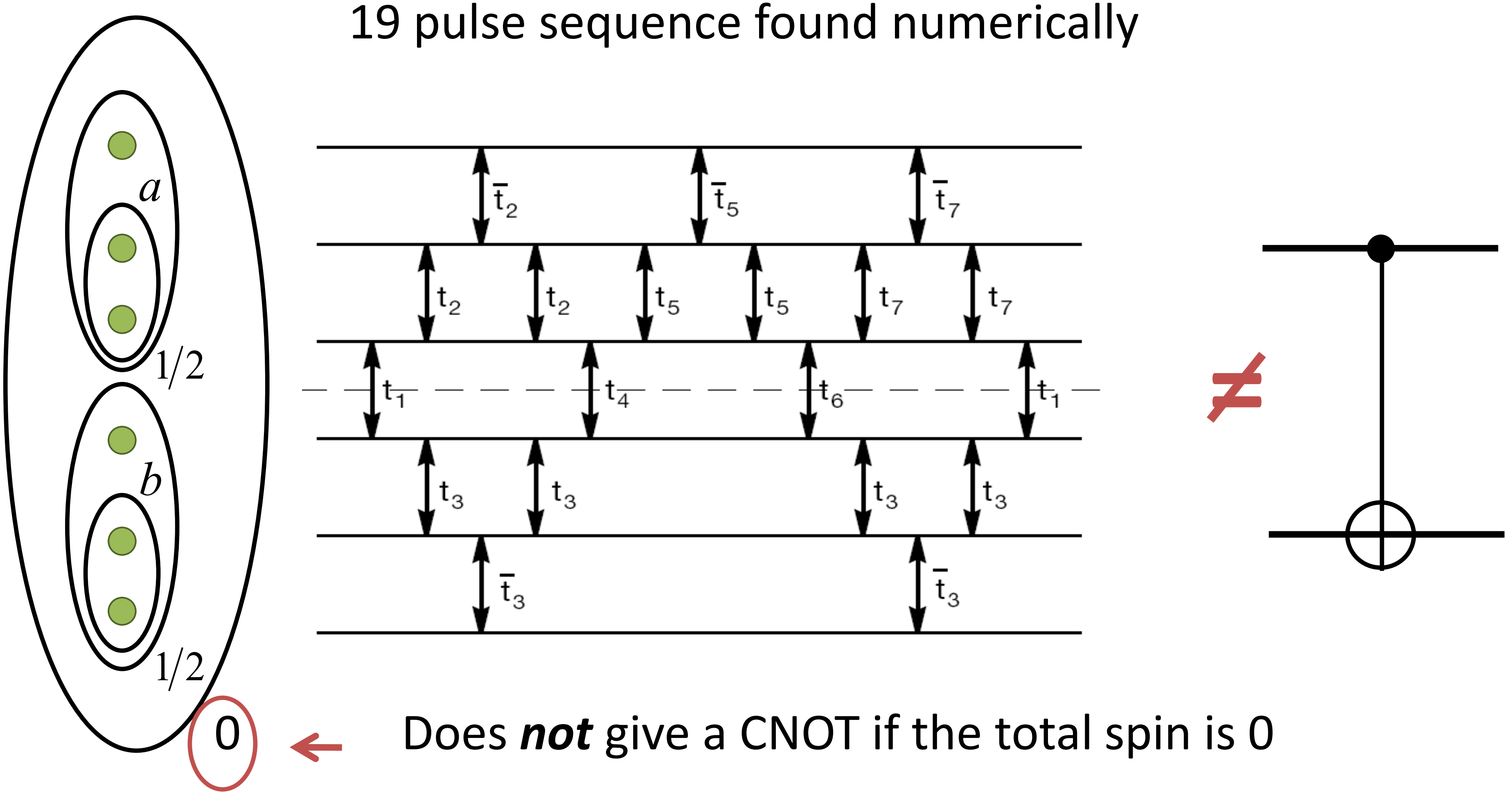


Gives a CNOT (up to single qubit operations) if the total spin is 1

Two-Qubit Gates

DiVincenzo, Bacon, Kempe, Burkard & Whaley, Nature (2000)

19 pulse sequence found numerically

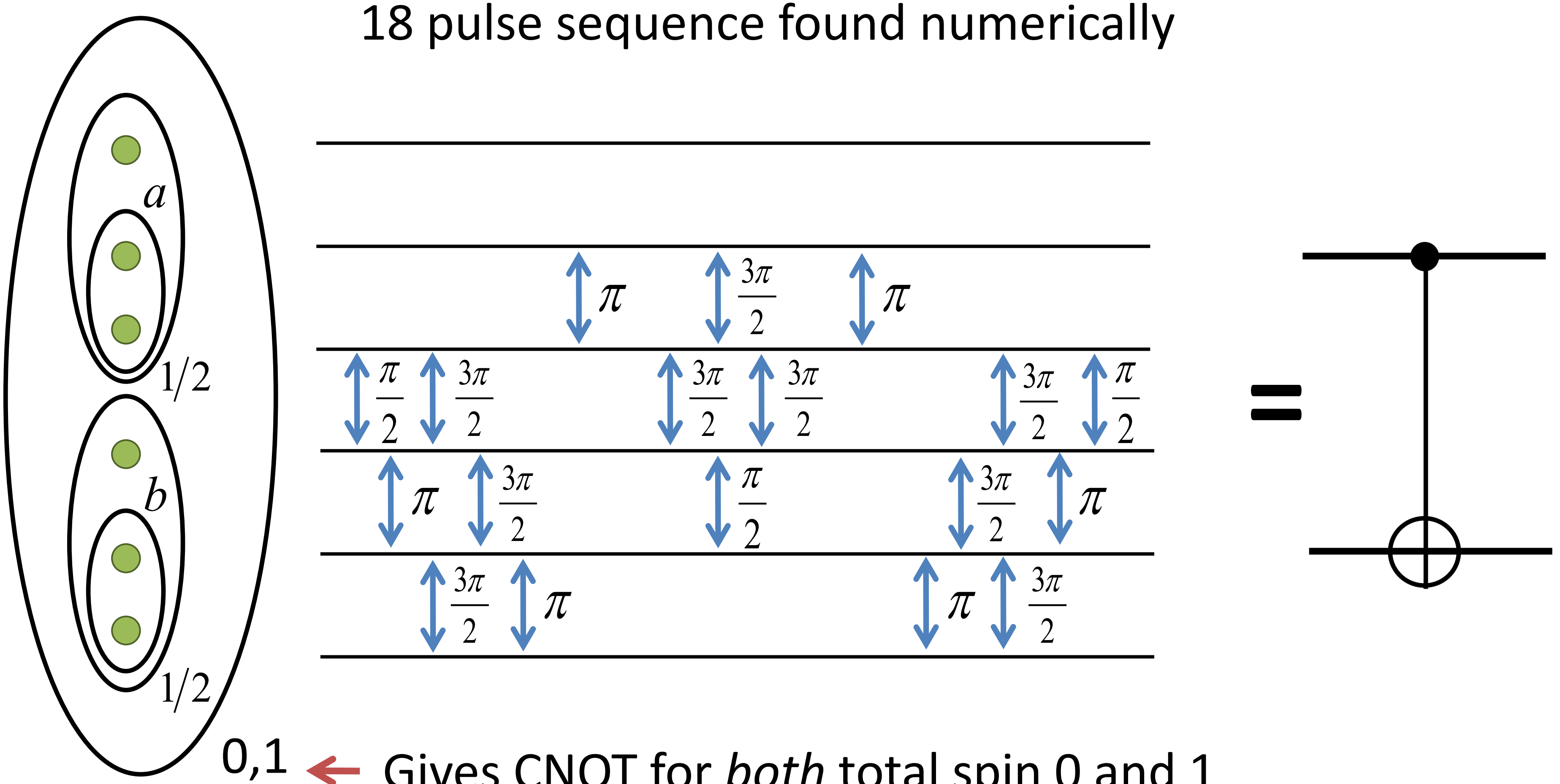


Does ***not*** give a CNOT if the total spin is 0

Fong-Wandzura Sequence

Fong & Wandzura, *Quantum Information and Computation* (2011)

18 pulse sequence found numerically

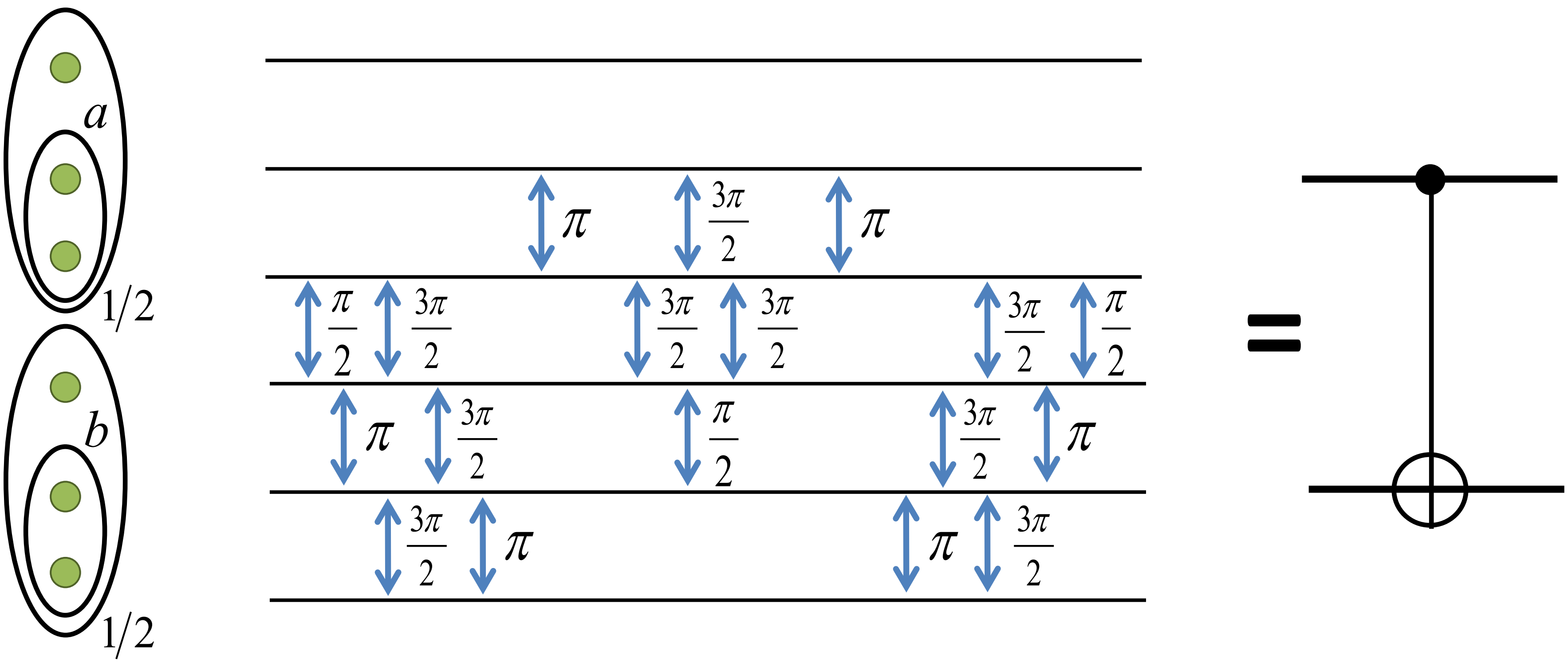


Gives CNOT for *both* total spin 0 and 1

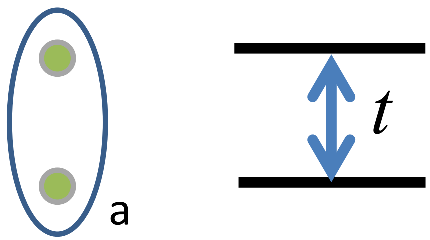
Fong-Wandzura Sequence

Fong & Wandzura, *Quantum Information and Computation* (2011)

18 pulse sequence found numerically



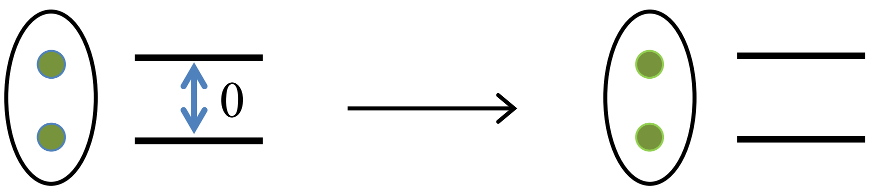
Two Simple Pulses



$$a = \quad 0 \qquad 1$$

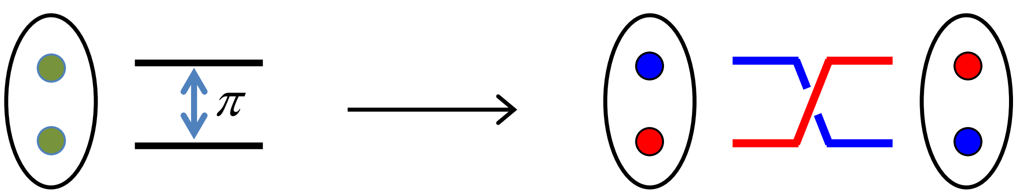
$$\begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

$t = 0$



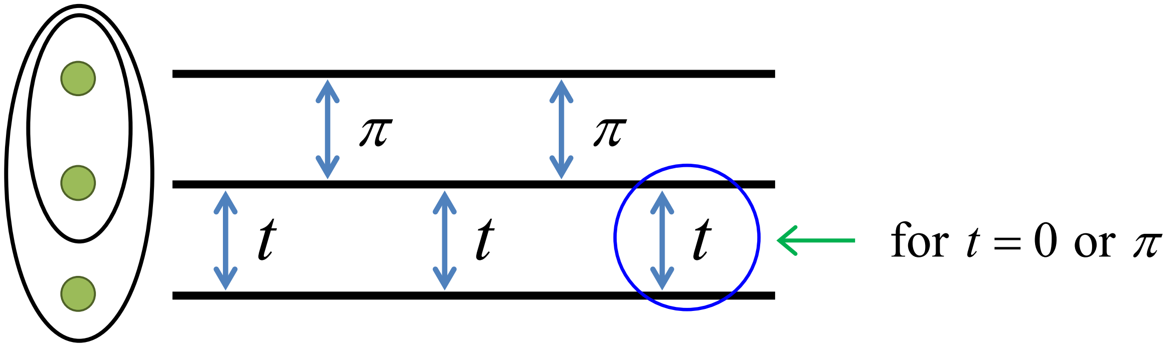
$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$t = \pi$



$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

Two Simple Sequences



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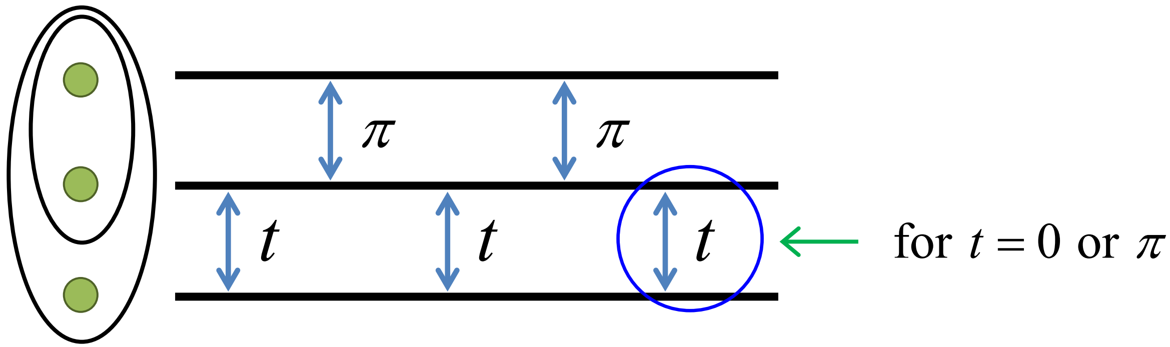
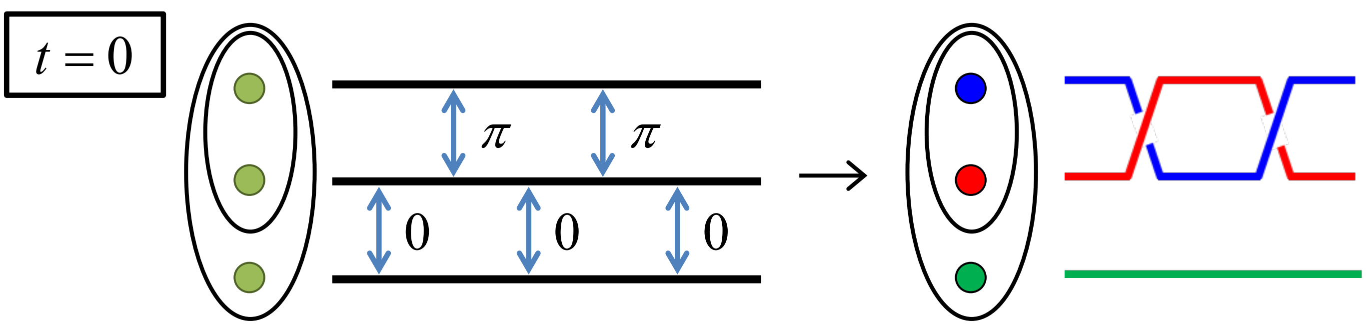
Two Simple Sequences





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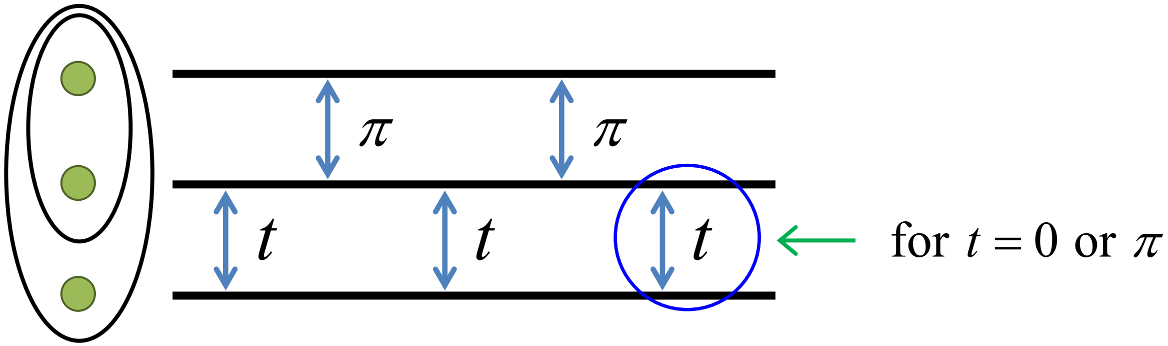
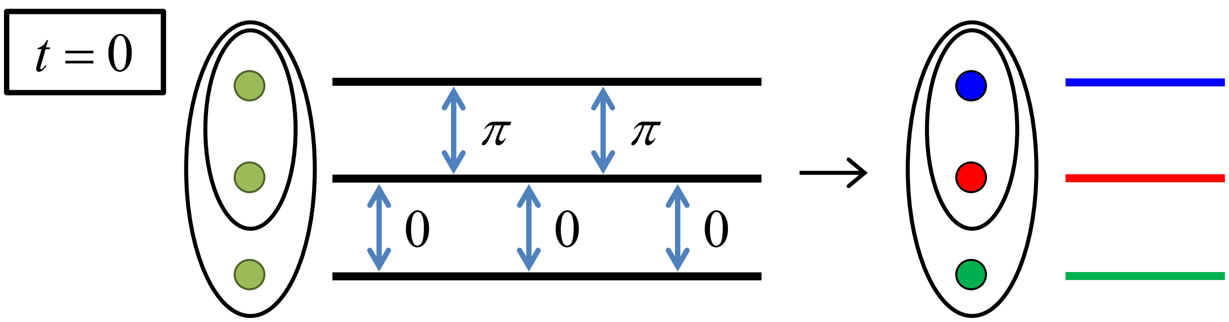
Two Simple Sequences





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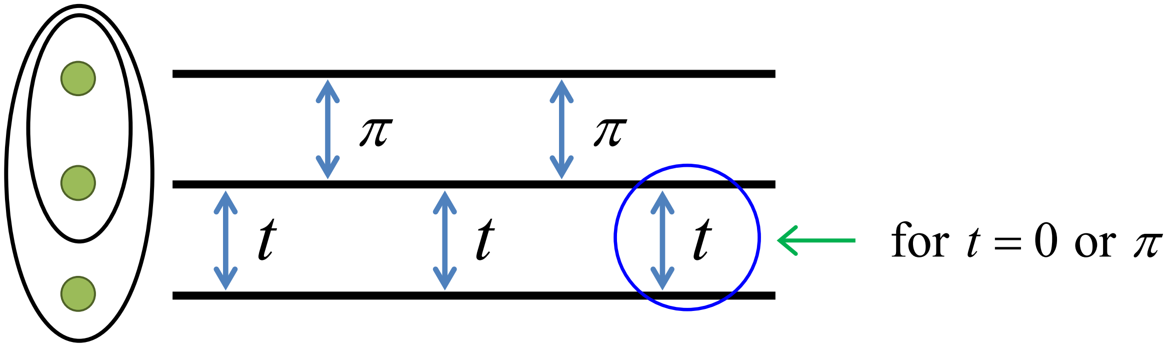
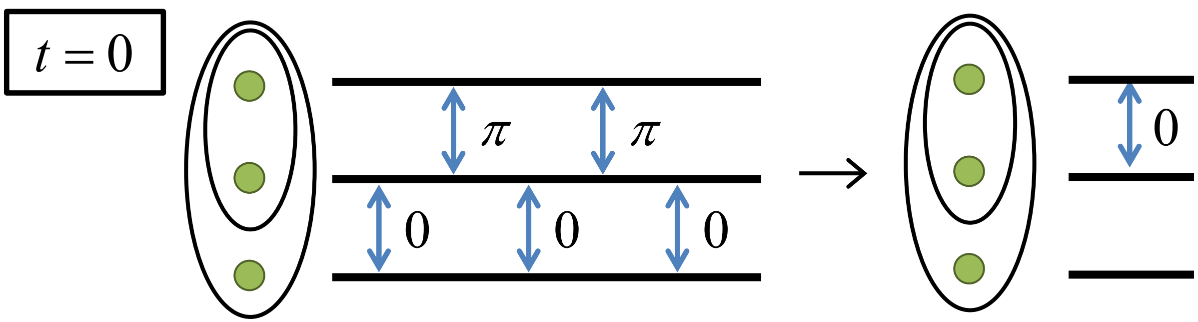
Two Simple Sequences





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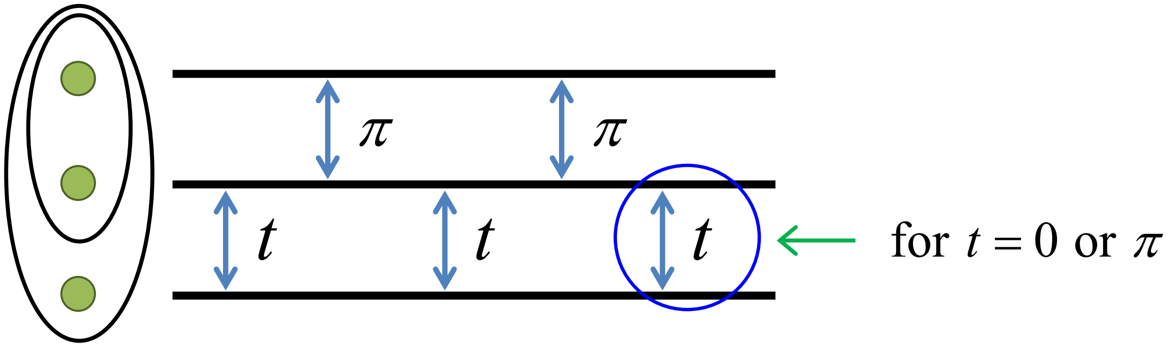
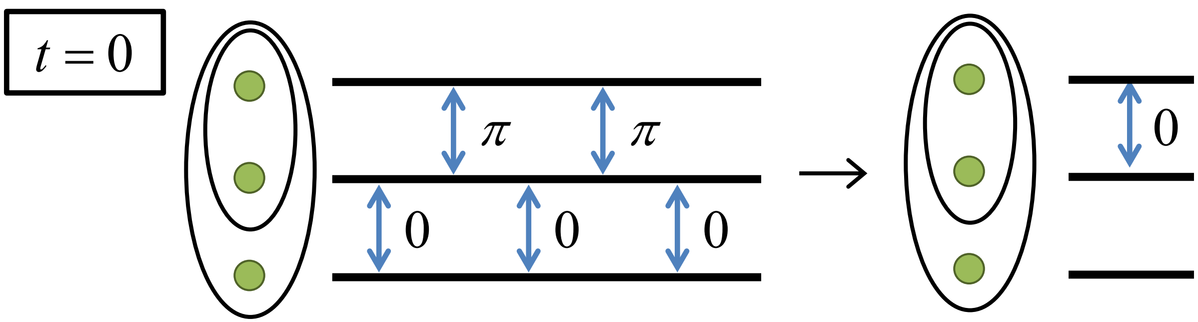
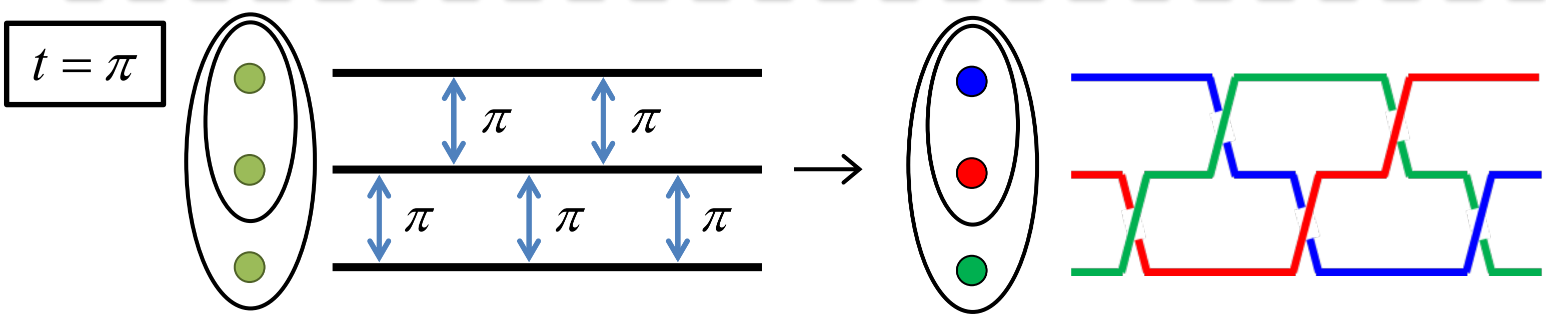
Two Simple Sequences







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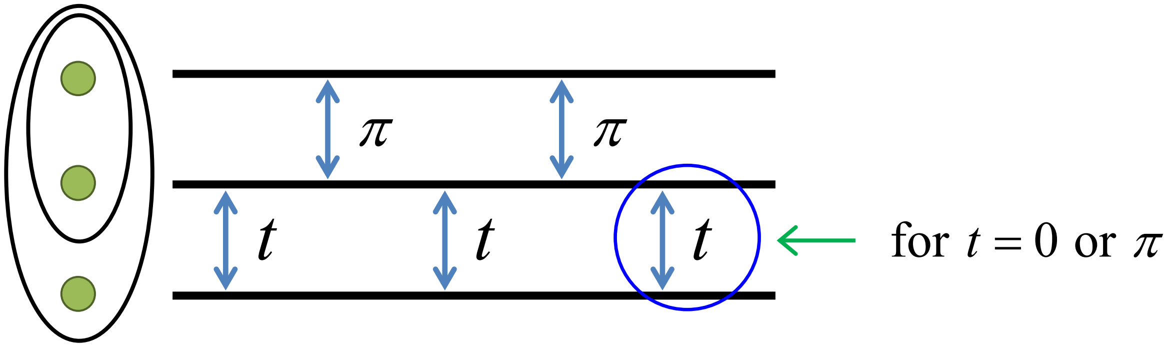
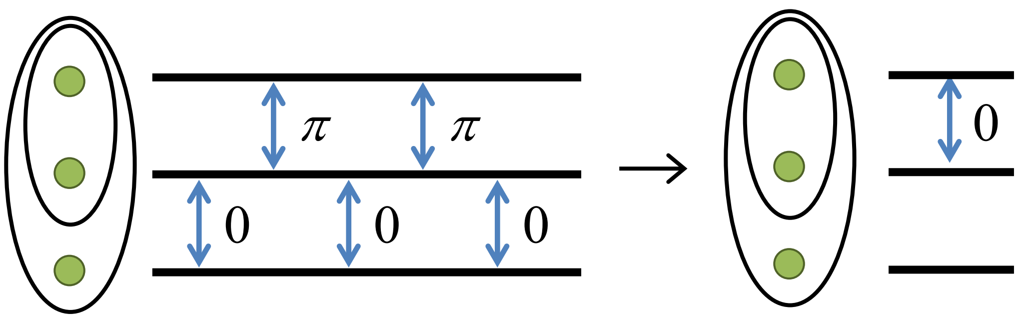
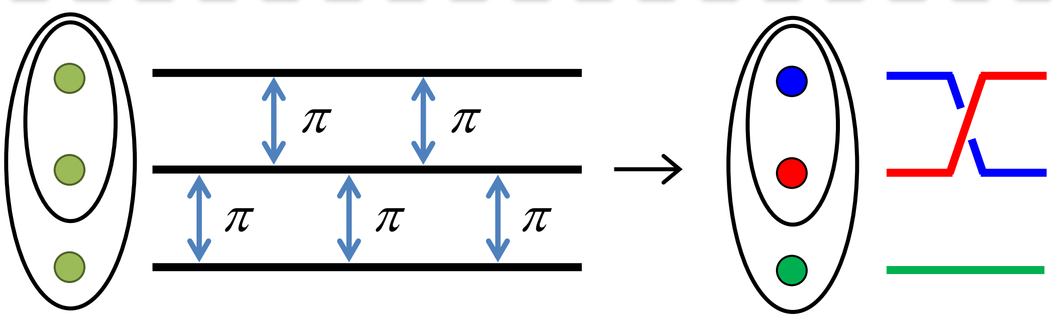
Two Simple Sequences



$t = 0$



$t = \pi$



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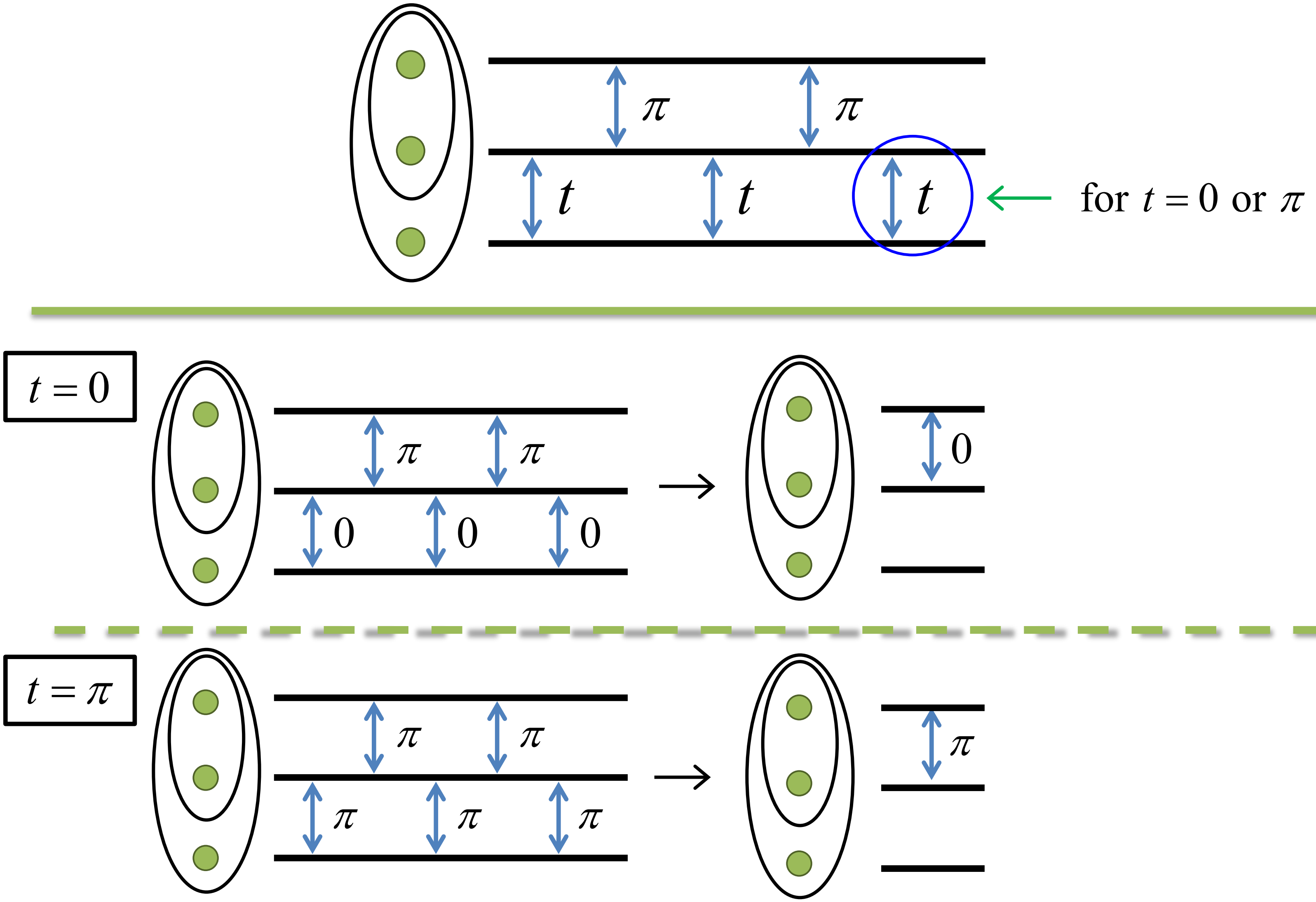
Two Simple Sequences



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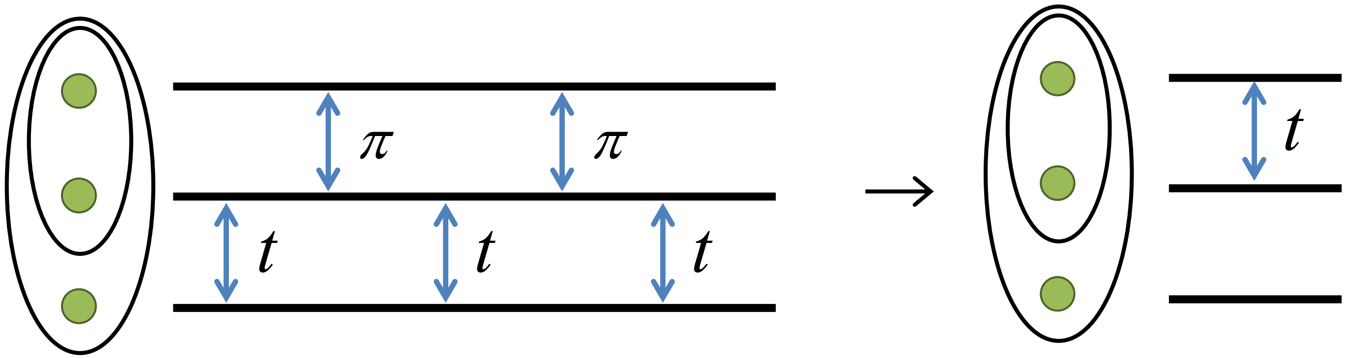
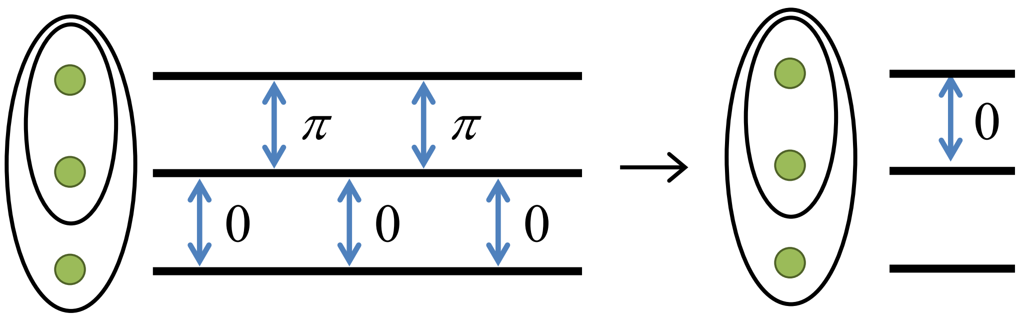
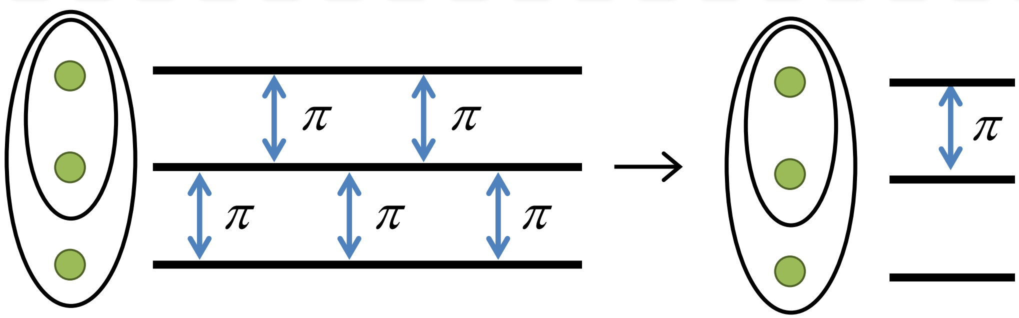
Two Simple Sequences



$t = 0$



$t = \pi$



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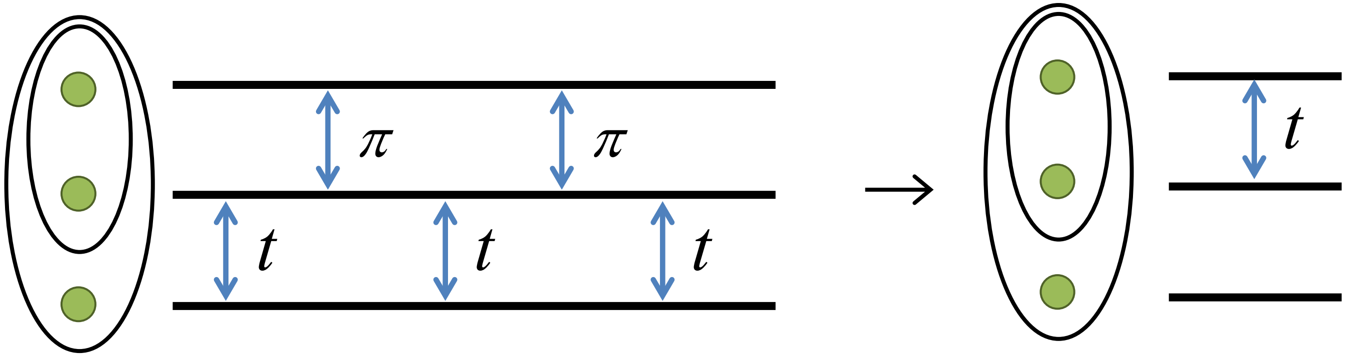
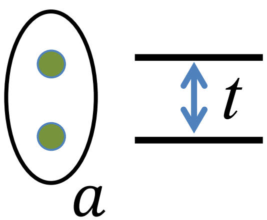
Sequence Identity





$$a = \quad 0 \quad 1$$

$$\begin{pmatrix} 1 & \\ & m \end{pmatrix}$$

Identity holds for

$$t = 0 \rightarrow m = +1$$
$$t = \pi \rightarrow m = -1$$

$m^2 = 1$

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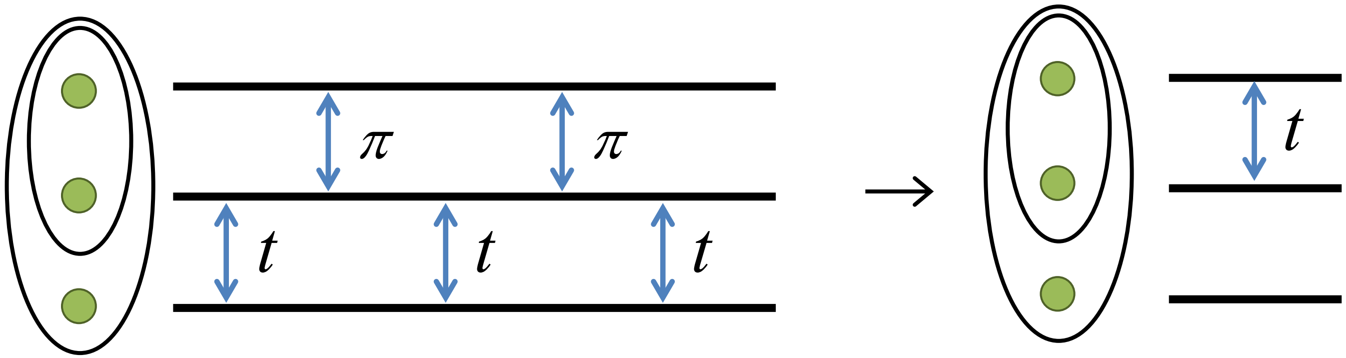
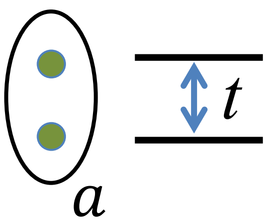
Sequence Identity





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Can show: Any pulse with m^2=1 will satisfy this identity

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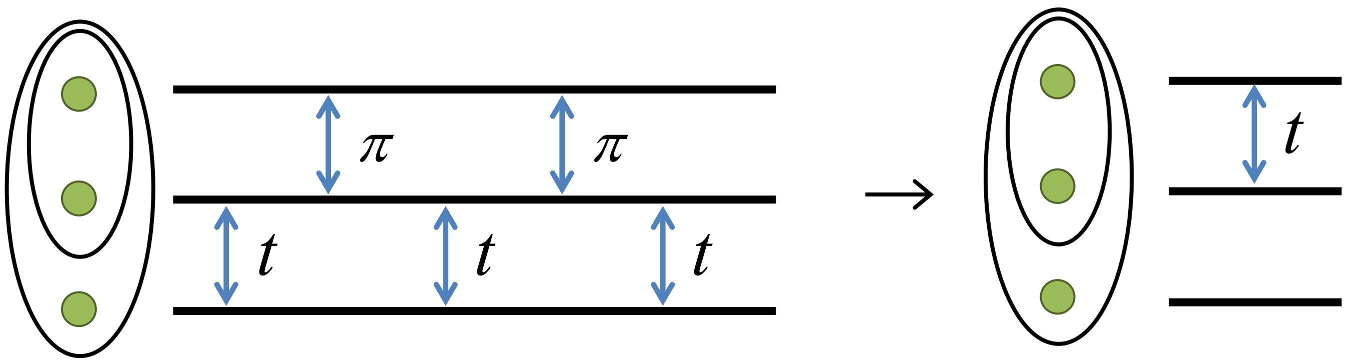
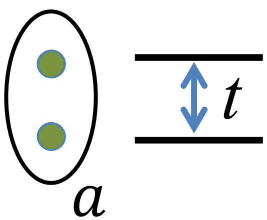
Sequence Identity





$a = \;\; 0 \quad 1$

$$\begin{pmatrix} 1 & \\ & m \end{pmatrix}$$

Identity holds for

$t = 0 \rightarrow m = +1$
$t = \pi \rightarrow m = -1$

$m^2 = 1$

Can show: Any pulse with m^2=1 will satisfy this identity

Does m have to be a number?
How about a matrix?

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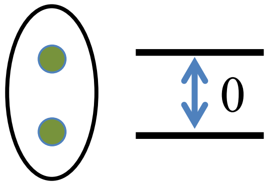
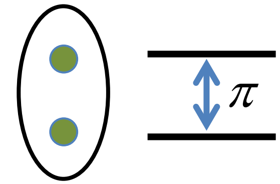
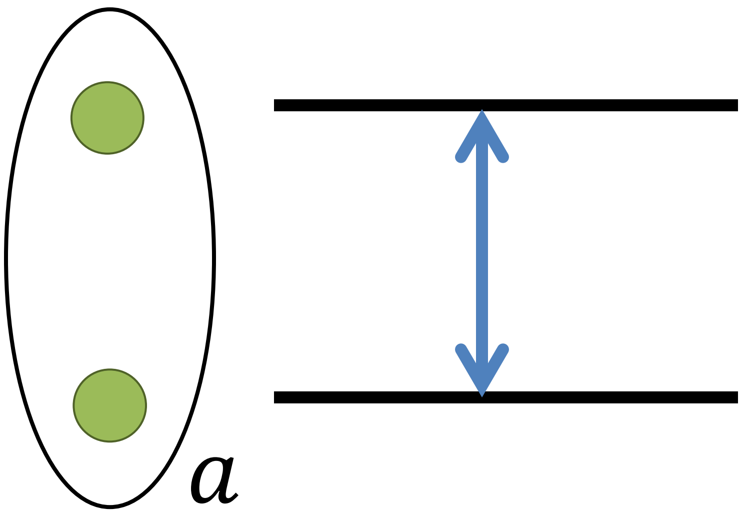
$m^2 = 1$ Pulses



$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$



$$a = \quad 0 \qquad 1$$

$$\begin{pmatrix} 1 & \\ & m \end{pmatrix}$$

$m^2 = 1 \longrightarrow m = +1,-1$

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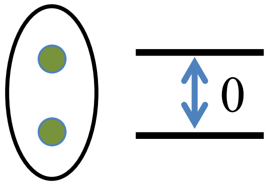
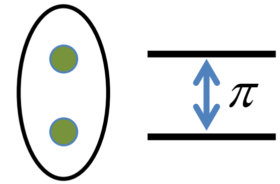
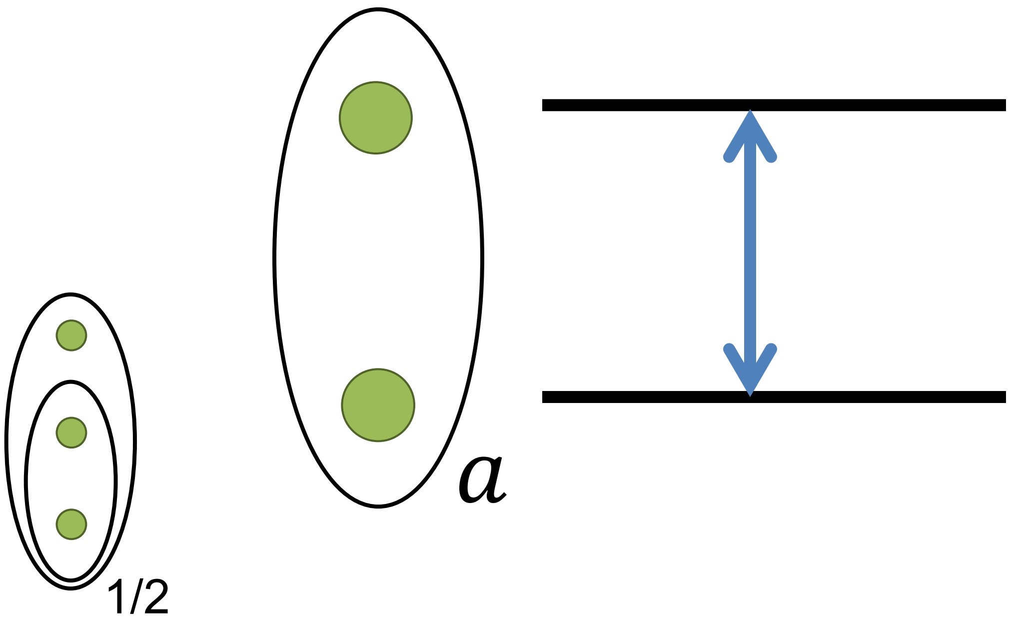
m^2 = 1 Pulses



$$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$



$$a \;=\; 0 \qquad 1$$

$$\begin{pmatrix} 1 & \\ & m \end{pmatrix}$$

$m^2 = 1 \longrightarrow m = +1,-1$

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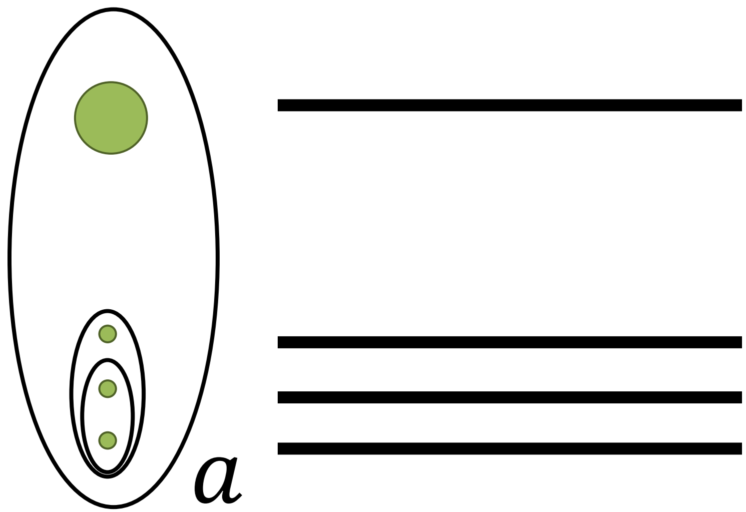
“Elevating” $m^2 = 1$ Pulses



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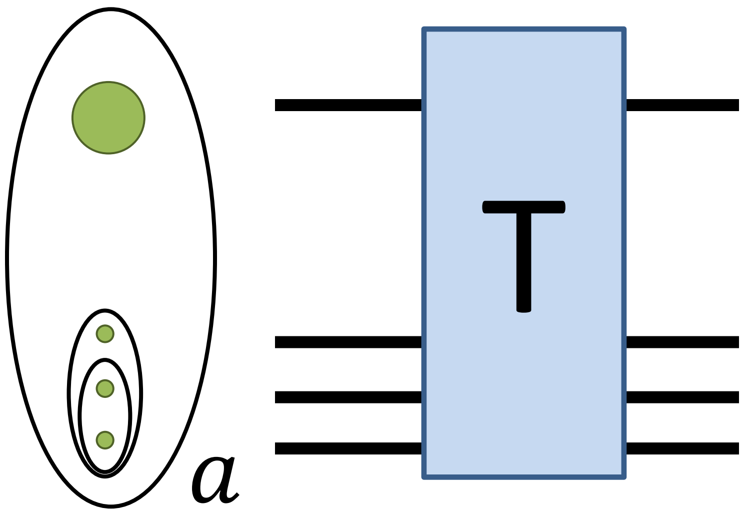
“Elevating” $m^2 = 1$ Pulses



$$a = \begin{matrix} 0 & 1 \end{matrix}$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

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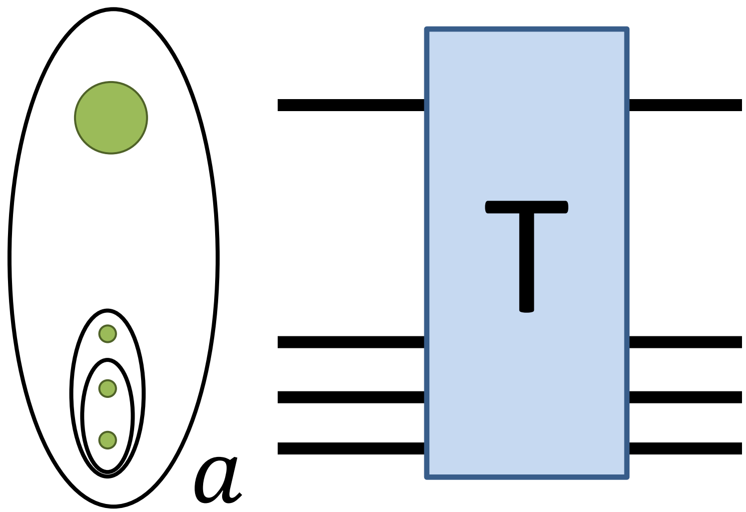
"Elevating" $m^2 = 1$ Pulses



$$a = \begin{matrix} 0 & 1 \end{matrix}$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

2×2 matrices, acting on "swapped in" qubit

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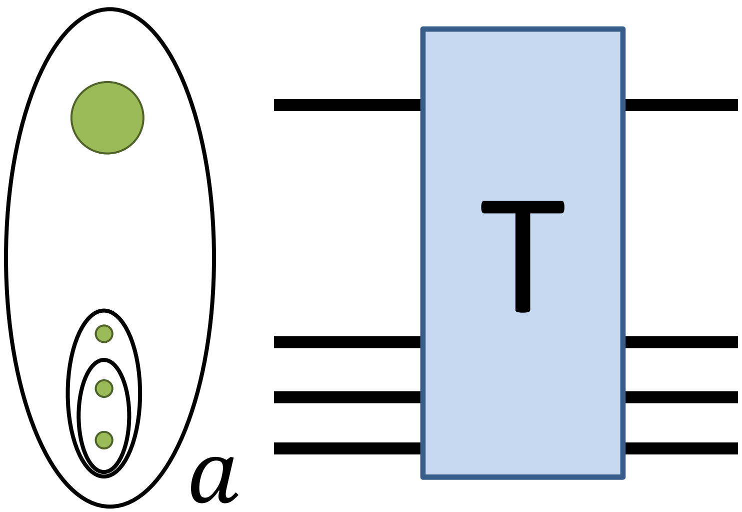
$M^2 = I$ "Pulse"



$$a = 0 \quad 1$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

$M^2 = I$ → $M = +I, -I$

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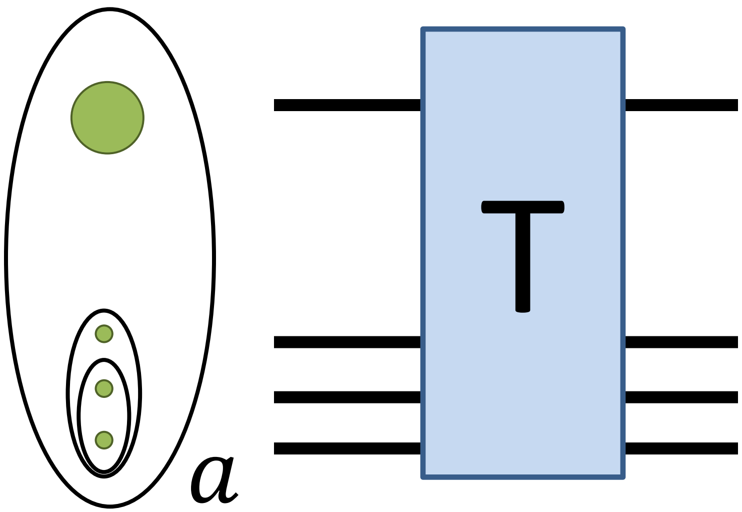
$M^2 = I$ "Pulse"



$$a = \quad 0 \quad 1$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

$M^2 = I$ → $M = +I, -I$ $\qquad M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

More solutions! → $M = \hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}$ $\qquad M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ldots$

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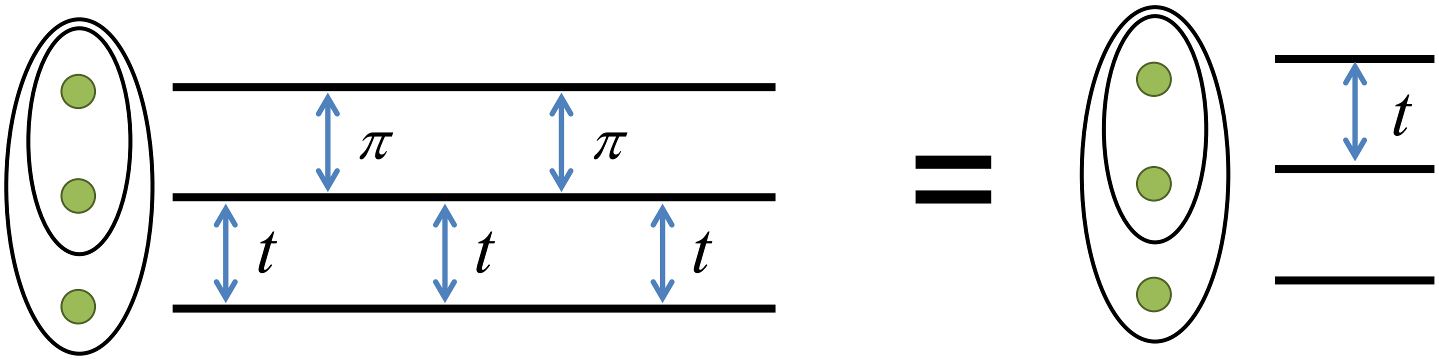
Sequence Elevation



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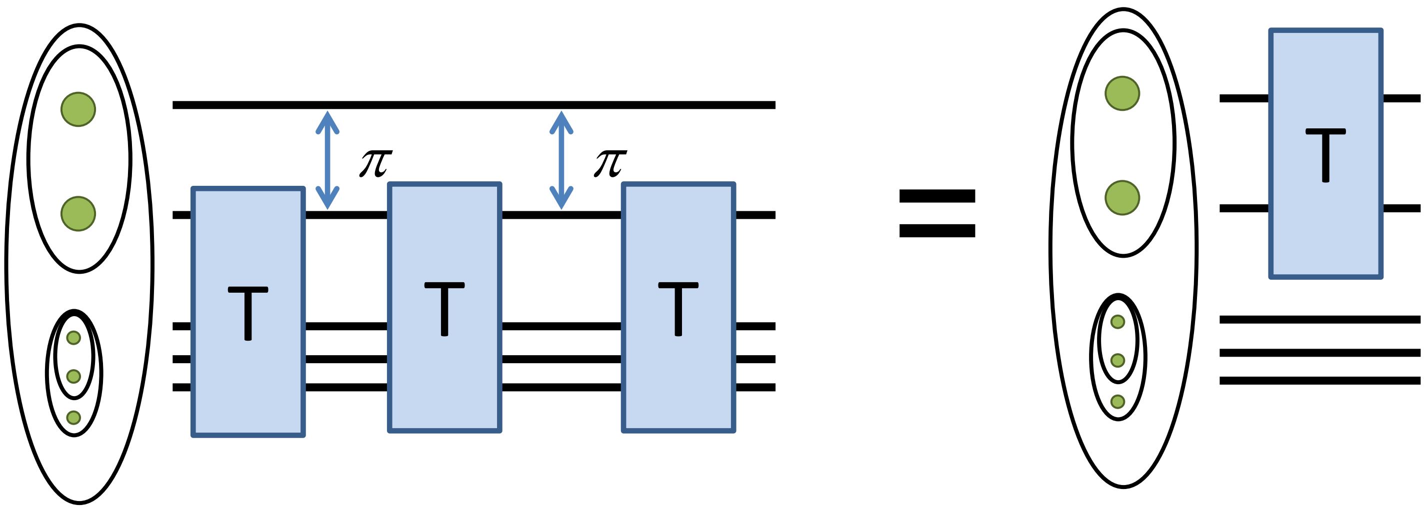
Sequence Elevation



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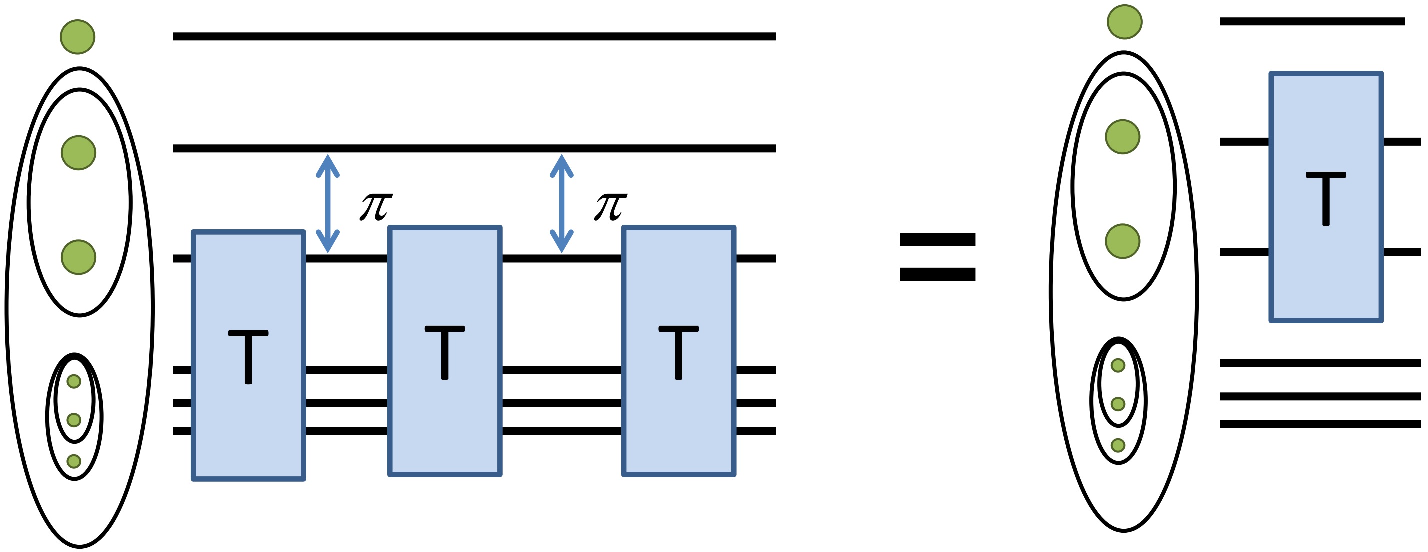
Sequence Elevation



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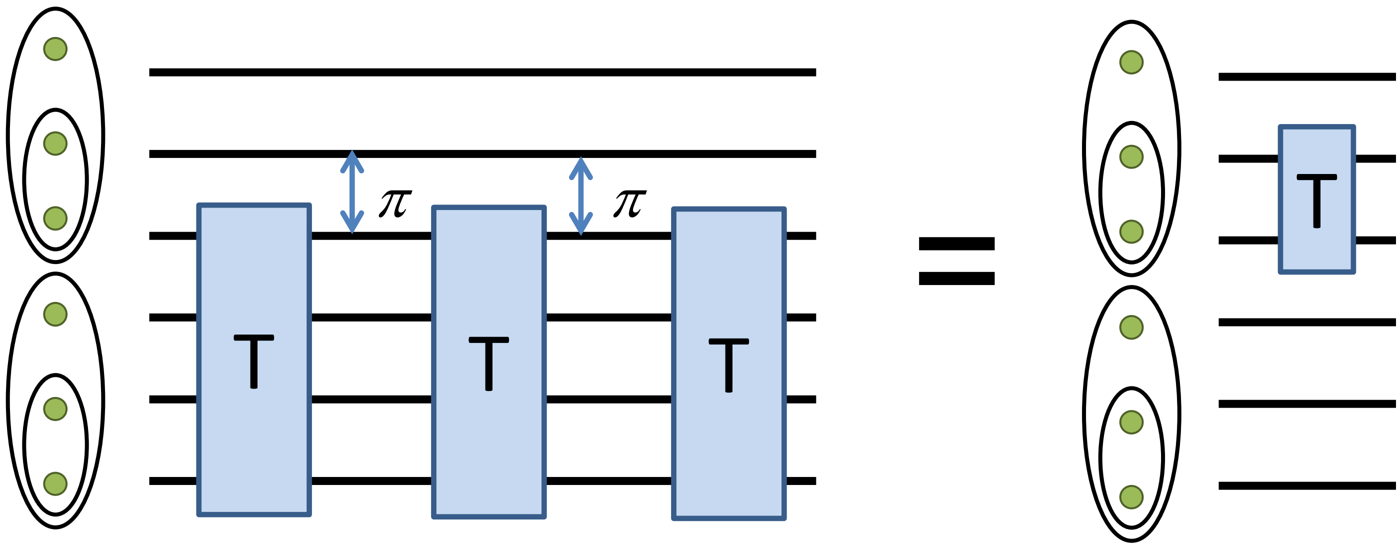
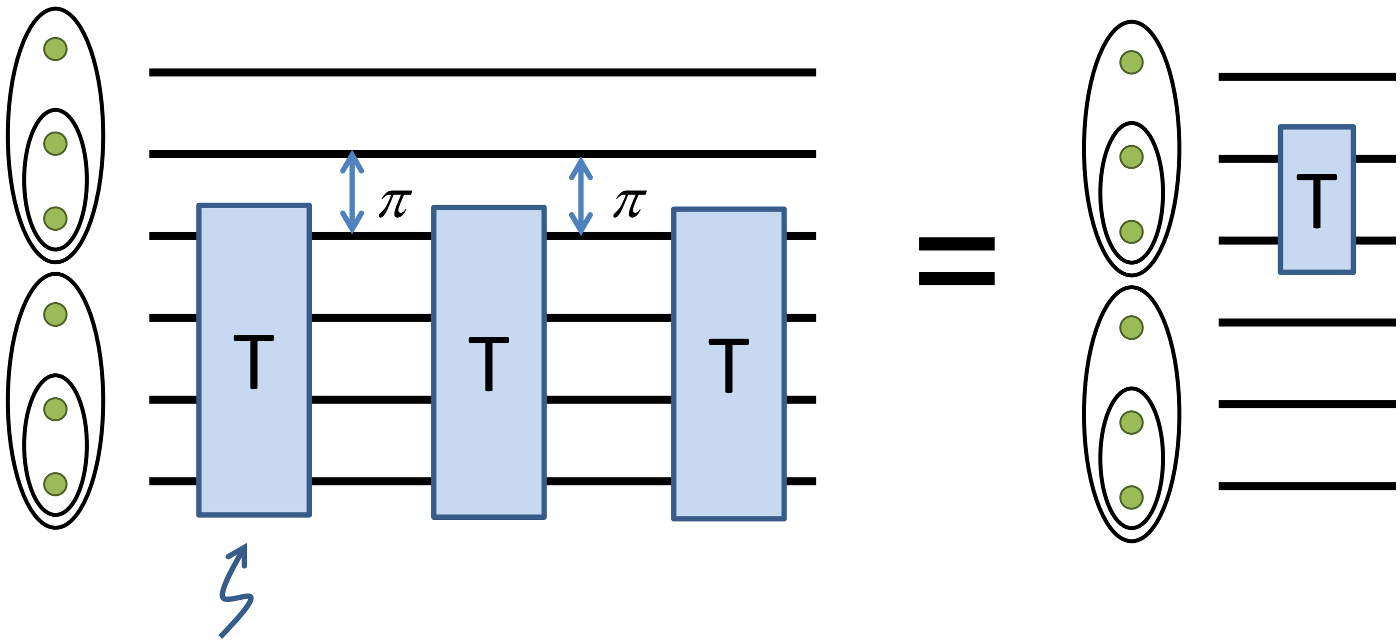
Sequence Elevation



Zeuch, NEB, *Phys. Rev. A* (2016)

Sequence Elevation



$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

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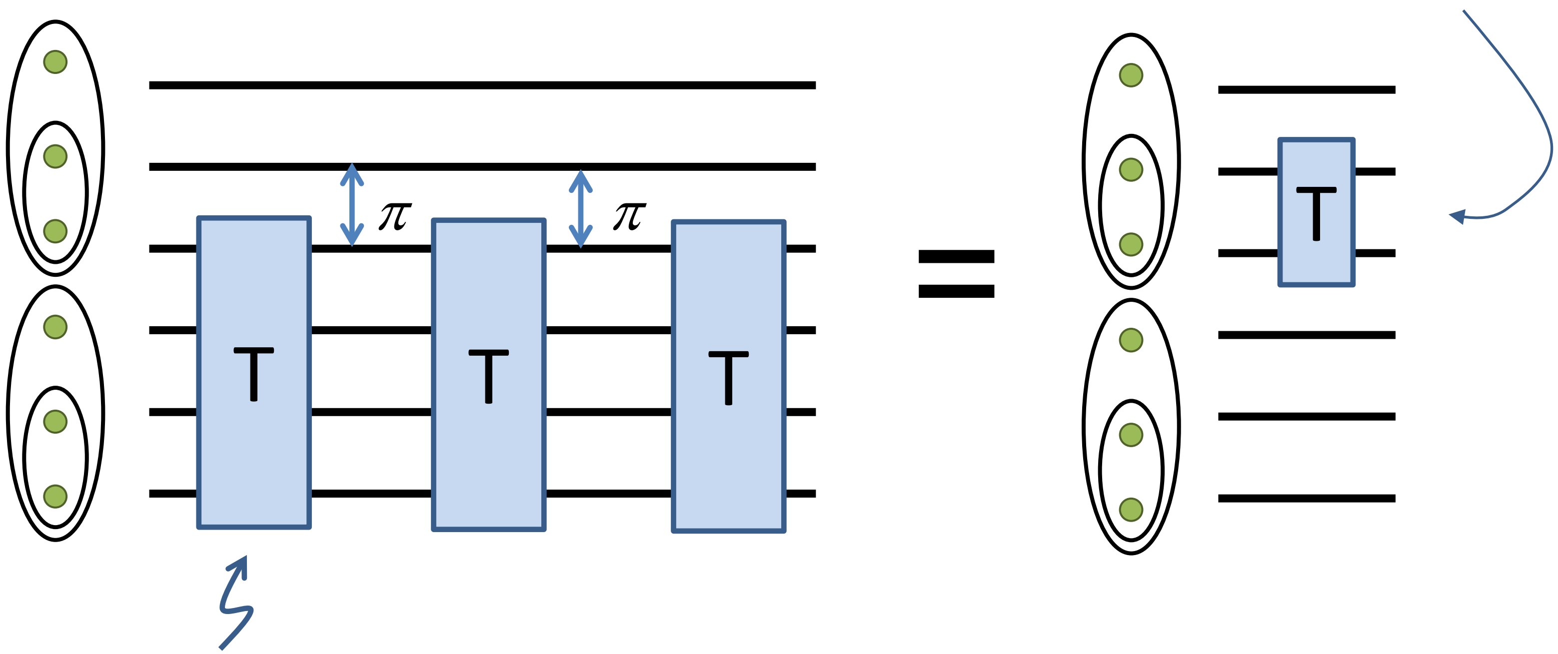
Sequence Elevation

Applies M to bottom qubit only if top qubit is in state $|1\rangle$



$\begin{pmatrix} I & \\ & M \end{pmatrix}$

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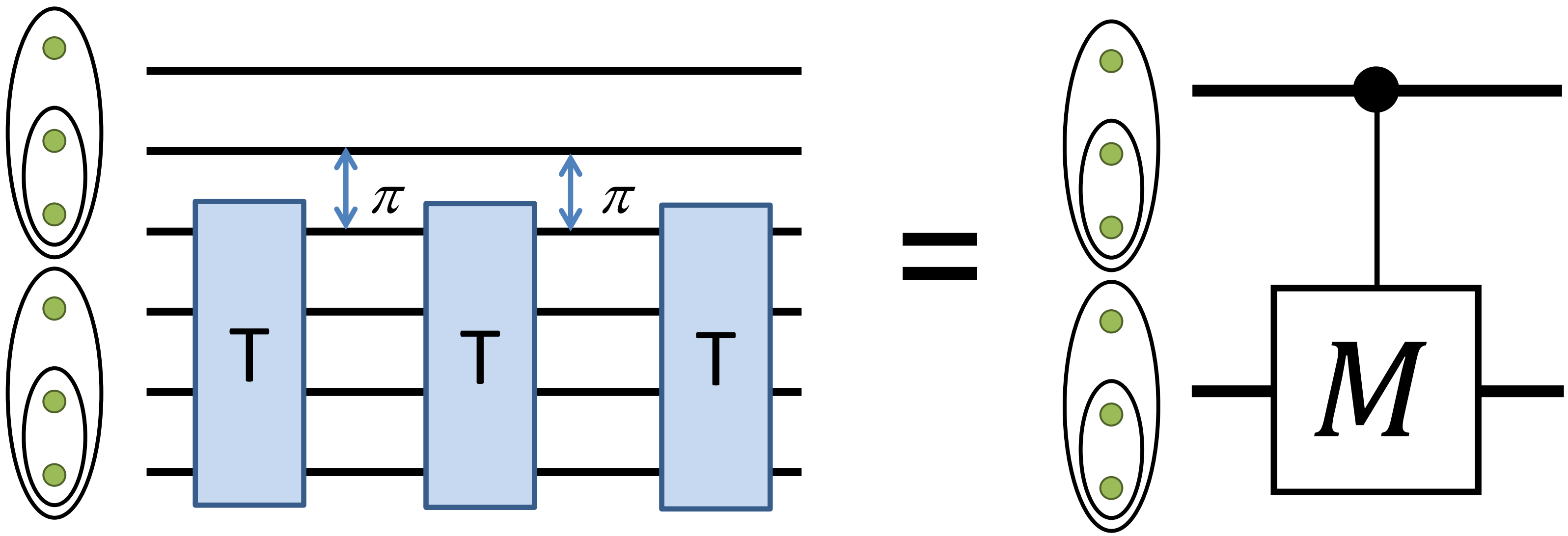
Sequence Elevation

Two-qubit controlled-M gate --- no leakage.



$$M = \hat{\boldsymbol{n}} \cdot \boldsymbol{\sigma}, \text{ e.g., } M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \cdots$$

controlled-NOT   controlled-Phase

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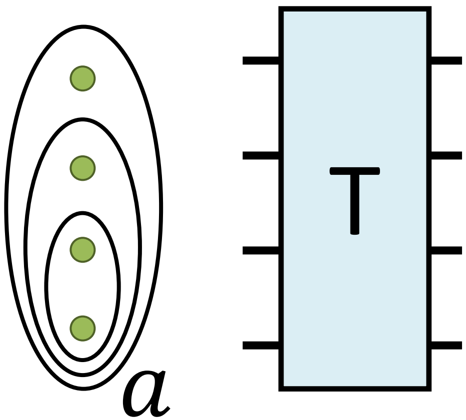
Constructing T

What we want



$$=$$

$$a = \quad 0 \qquad 1$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

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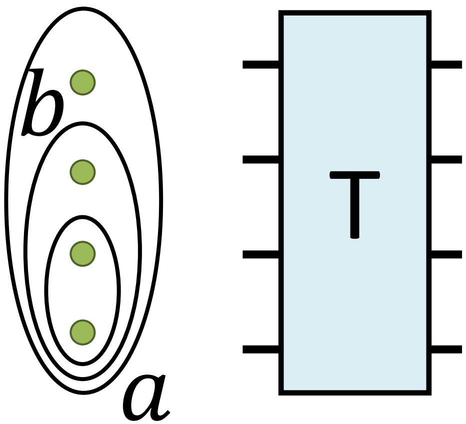
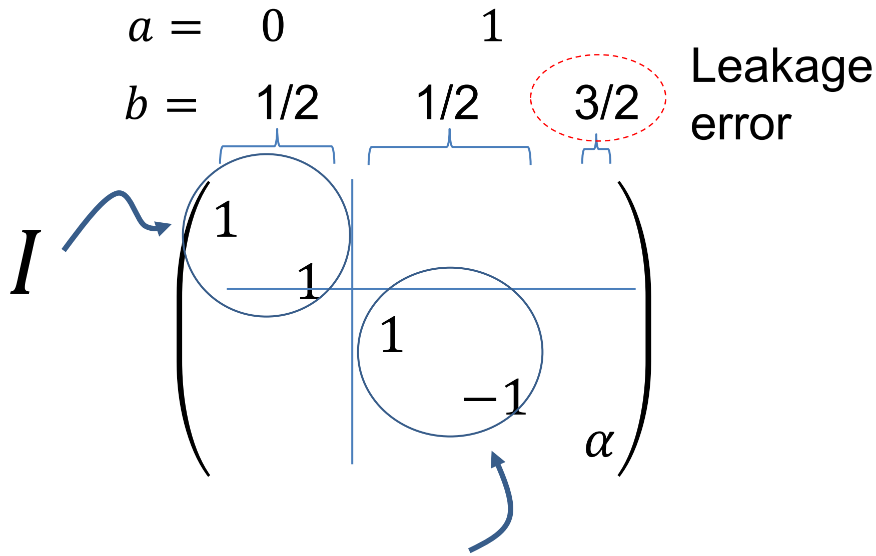
Constructing T

What we want



$=$



$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ for example

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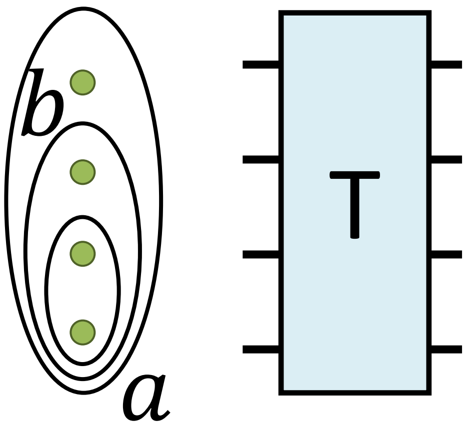
Constructing T

What we want



$=$

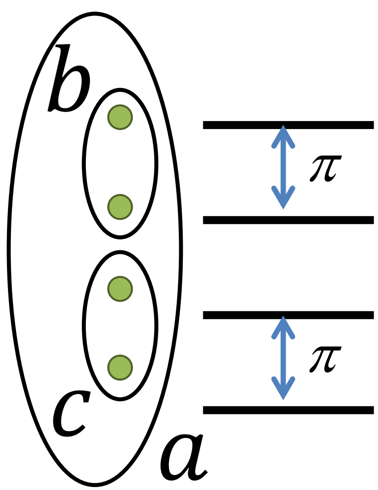
$a = \quad 0 \qquad\qquad 1$

$c = \quad 1/2 \qquad 1/2 \qquad 3/2$

Leakage error

$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & 1 & & \\ & & & -1 & \\ & & & & \alpha \end{array}\right)$$

This is close! (But wrong basis)



$a = \quad 0 \qquad\qquad 1$

$bc = 00 \;\; 11 \;\; 01 \;\; 10 \;\; 11$

$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)$$

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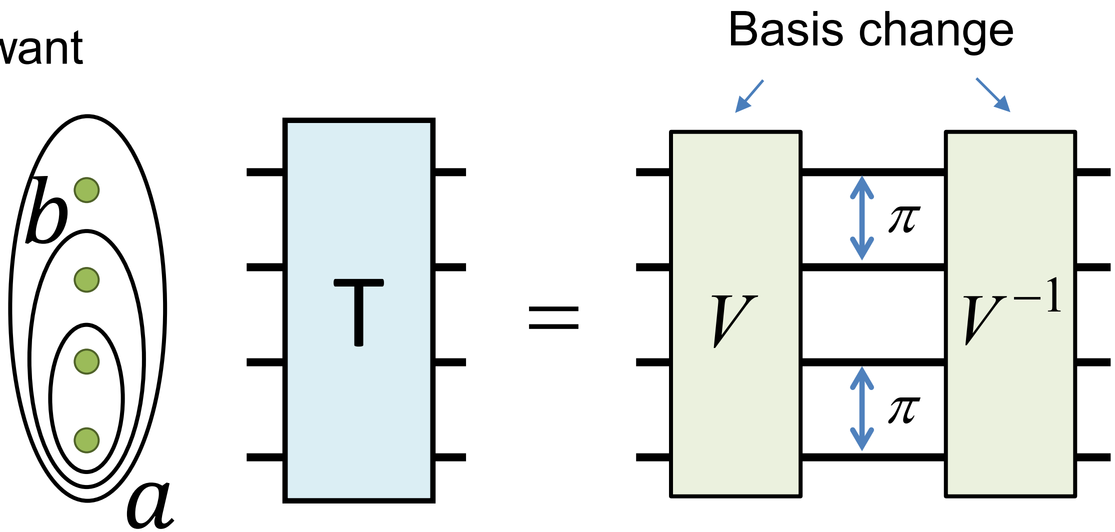
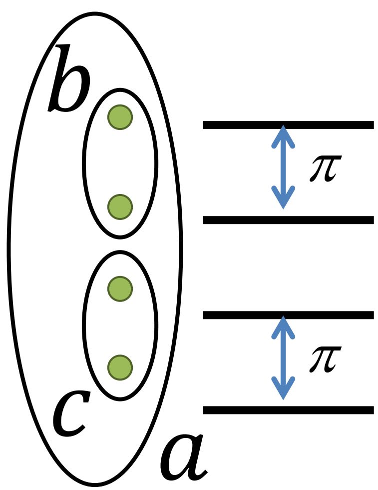
Constructing T

What we want



This is close! (But wrong basis)



$$a = \quad 0 \qquad\qquad 1$$

$$bc = 00\ \ 11\ \ 01\ \ 10\ \ 11$$

$$\left(\begin{array}{cc|ccc} 1 & & & & \\ & 1 & & & \\ \hline & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{array}\right)$$

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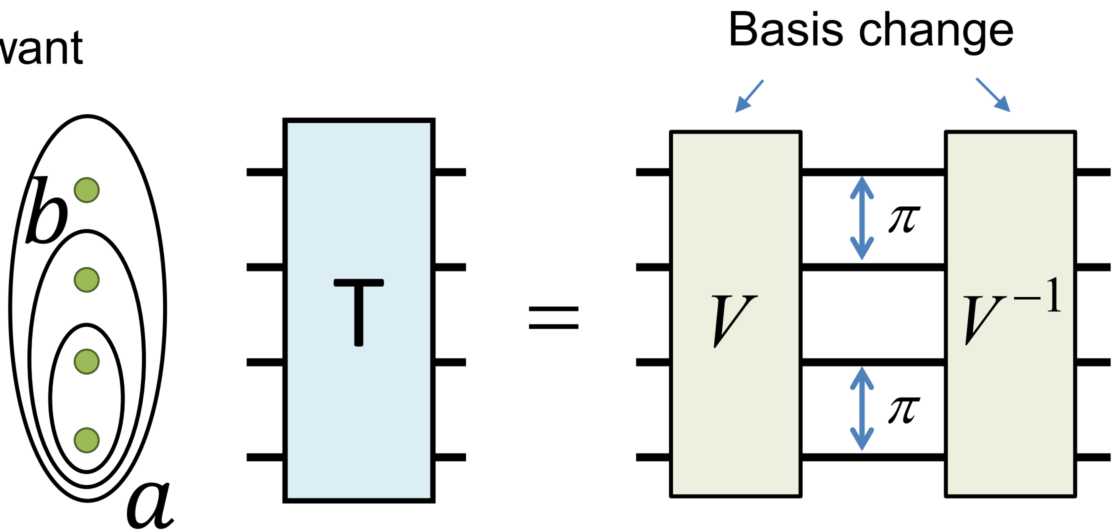
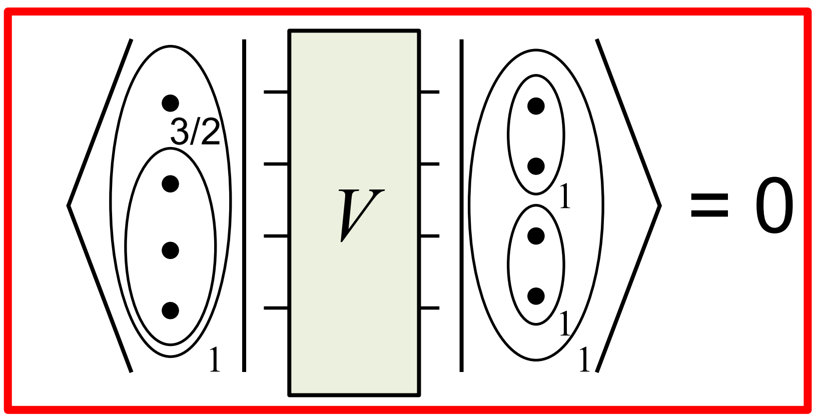
Constructing T

What we want



Constraint



Any *V* satisfying this constraint will do the job.

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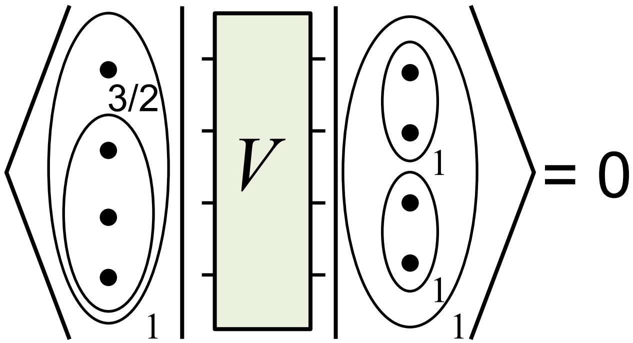
Satisfying the Constraint



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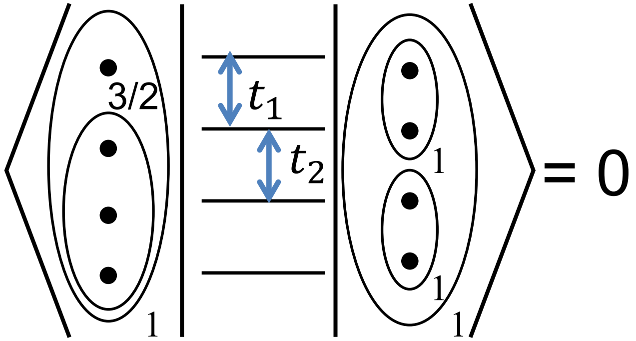
Satisfying the Constraint



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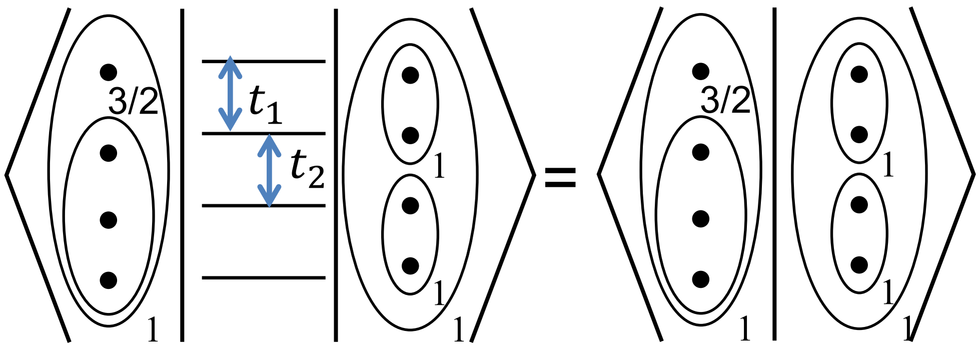
Satisfying the Constraint



$$\frac{1}{2}\left(-1+e^{it_1}+e^{it_2}+e^{i(t_1+t_2)}\right)$$

Form entirely fixed by $\boldsymbol{t_1 = 0, \pi}$ and $\boldsymbol{t_2 = 0, \pi}$ cases

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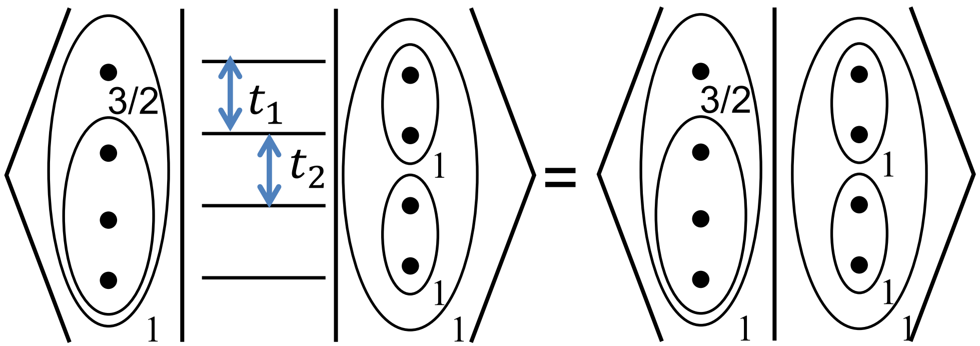
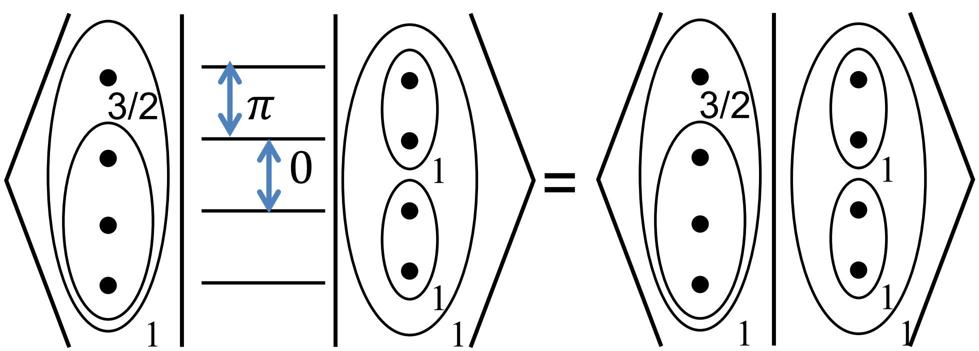
Satisfying the Constraint



$$\frac{1}{2}\left(-1+e^{it_1}+e^{it_2}+e^{i(t_1+t_2)}\right)$$

Form entirely fixed by $\boldsymbol{t_1 = 0, \pi}$ and $\boldsymbol{t_2 = 0, \pi}$ cases

For example, $\boldsymbol{t_1 = \pi}$ and $\boldsymbol{t_2 = 0}$



$$\frac{1}{2}\left(-1+e^{i\pi}+e^{i0}+e^{i(\pi+0)}\right)$$

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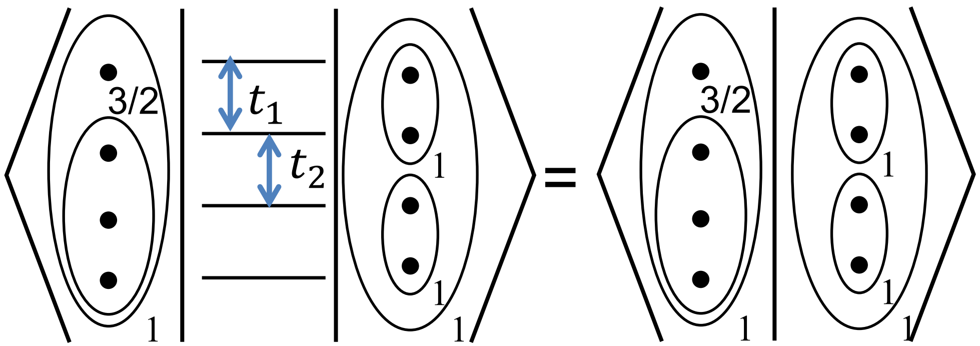
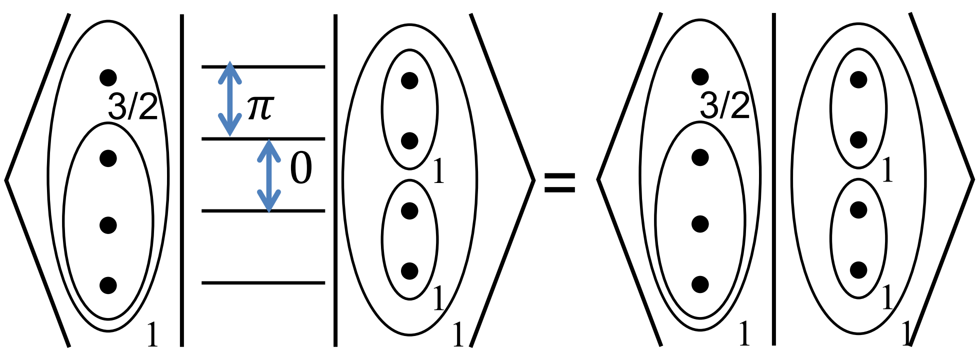
Satisfying the Constraint



$$\frac{1}{2}\left(-1+e^{it_1}+e^{it_2}+e^{i(t_1+t_2)}\right)$$

Form entirely fixed by $t_1 = 0, \pi$ and $t_2 = 0, \pi$ cases

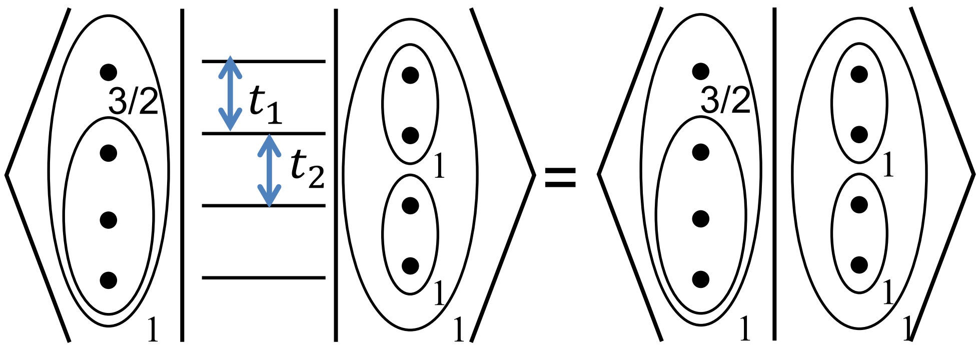
For example, $t_1 = \pi$ and $t_2 = 0$



$$\frac{1}{2}\left(-1+(-1)+1+(-1)\right)$$

Zeuch, NEB, *Phys. Rev. A* (2016)

Satisfying the Constraint



$$\frac{1}{2}\left(-1 + e^{it_1} + e^{it_2} + e^{i(t_1+t_2)}\right)$$

Form entirely fixed by $t_1 = 0, \pi$ and $t_2 = 0, \pi$ cases

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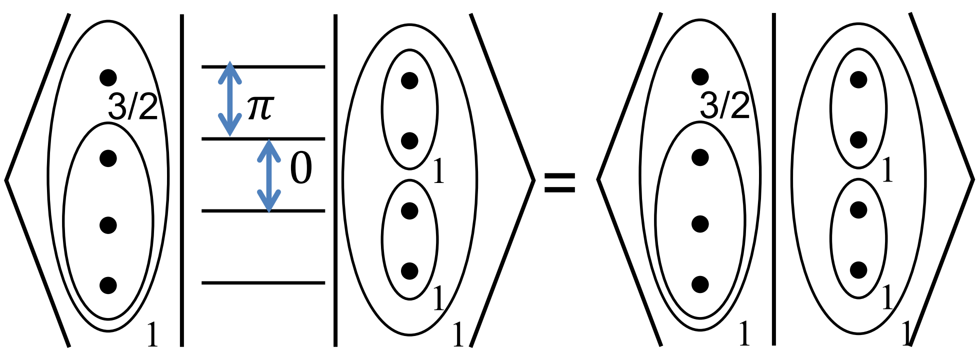
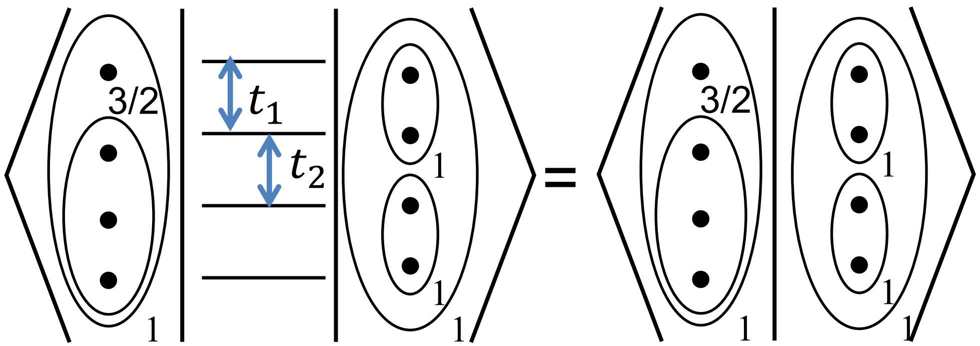


$$(-1)$$

Zeuch, NEB, *Phys. Rev. A* (2016)

Satisfying the Constraint



$$\frac{1}{2}\left(-1+e^{it_1}+e^{it_2}+e^{i(t_1+t_2)}\right)$$

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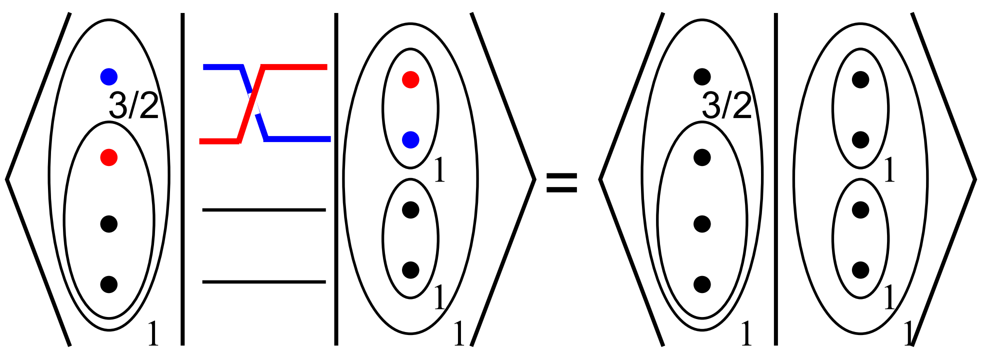
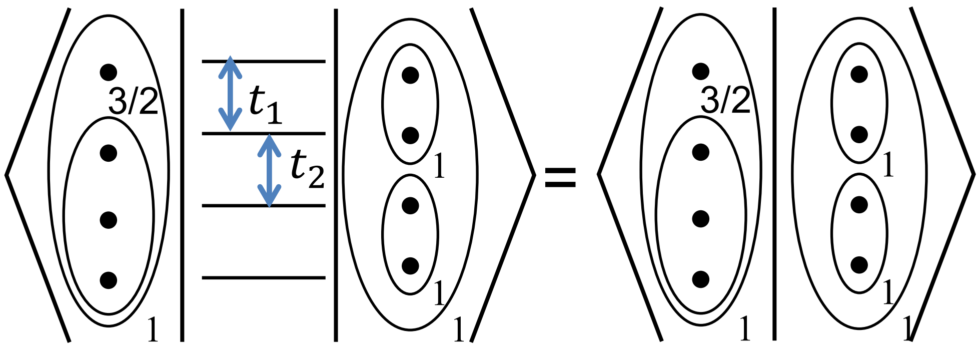


(-1)

Zeuch, NEB, *Phys. Rev. A* (2016)

Satisfying the Constraint

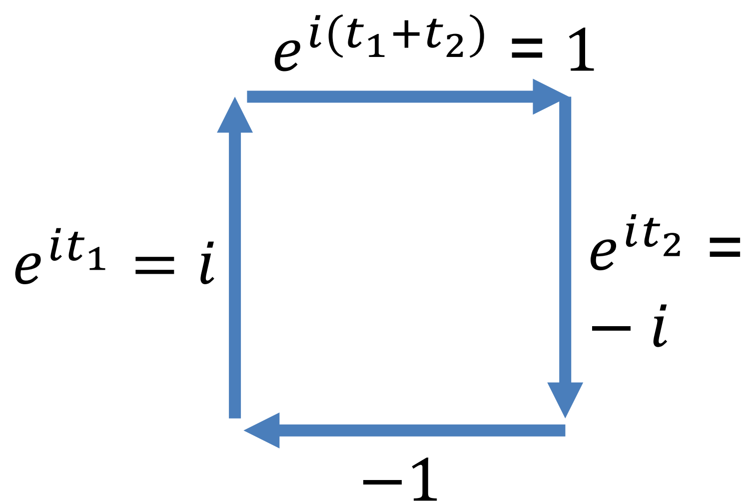


$\frac{1}{2}\left(-1+e^{it_1}+e^{it_2}+e^{i(t_1+t_2)}\right)$

Need destructive interference

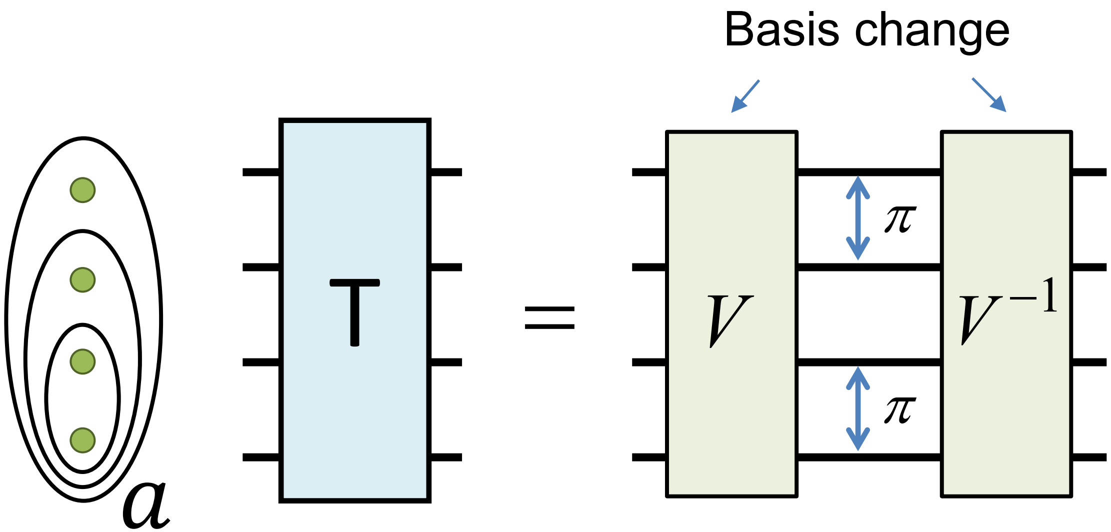
$$t_1 = \frac{\pi}{2}$$

$$t_2 = \frac{3\pi}{2}$$

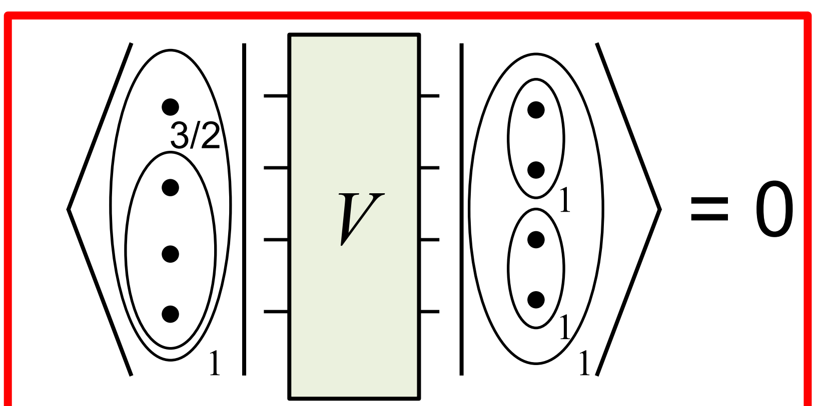


Zeuch, NEB, *Phys. Rev. A* (2016)

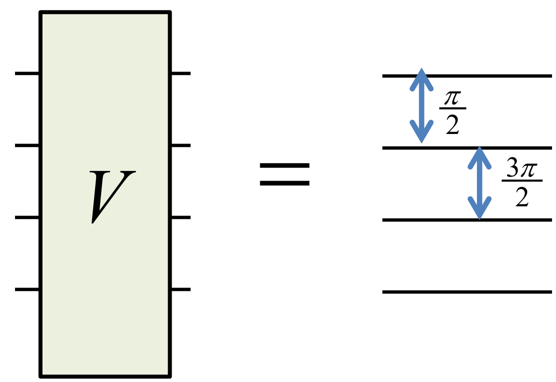
Constructing T



Constraint

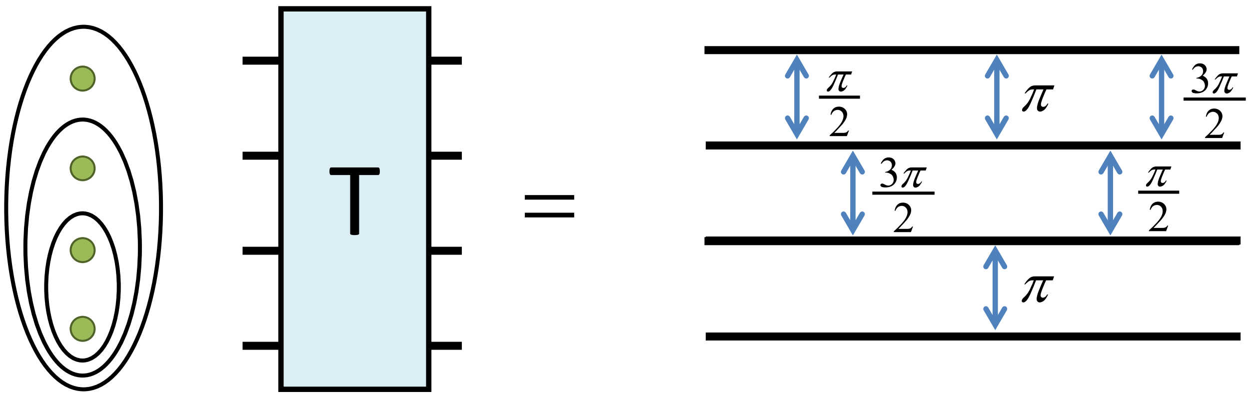


Optimal Solution

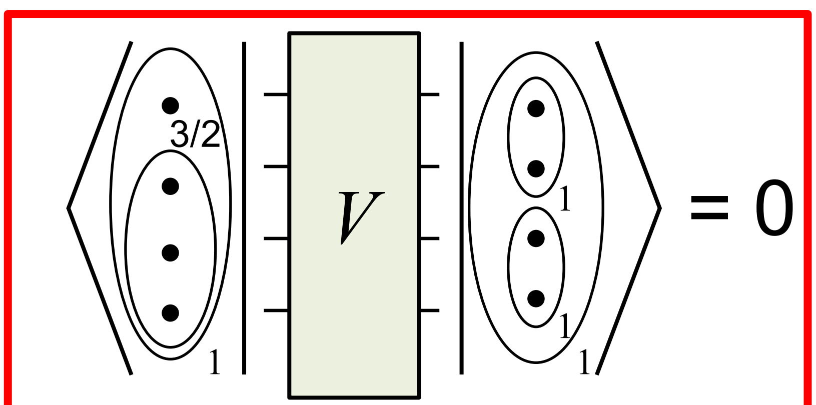


Zeuch, NEB, *Phys. Rev. A* (2016)

Constructing T



Constraint



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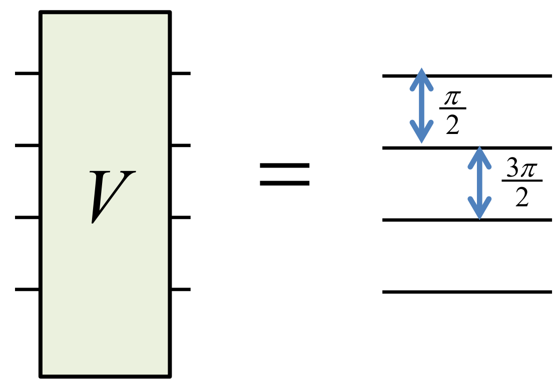
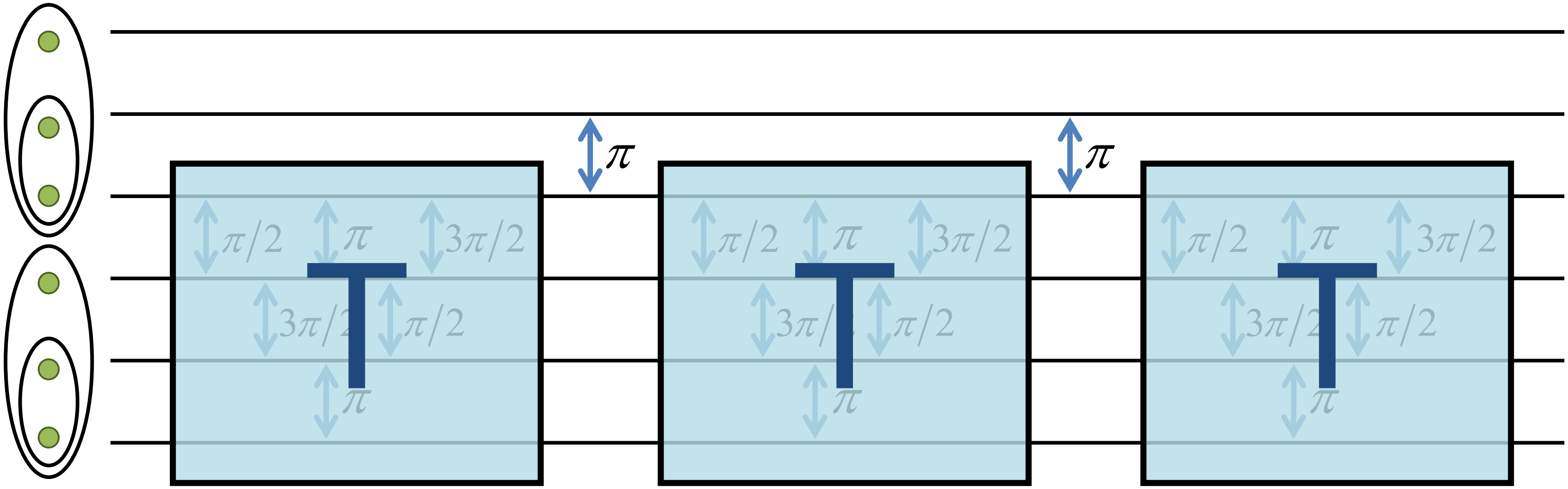


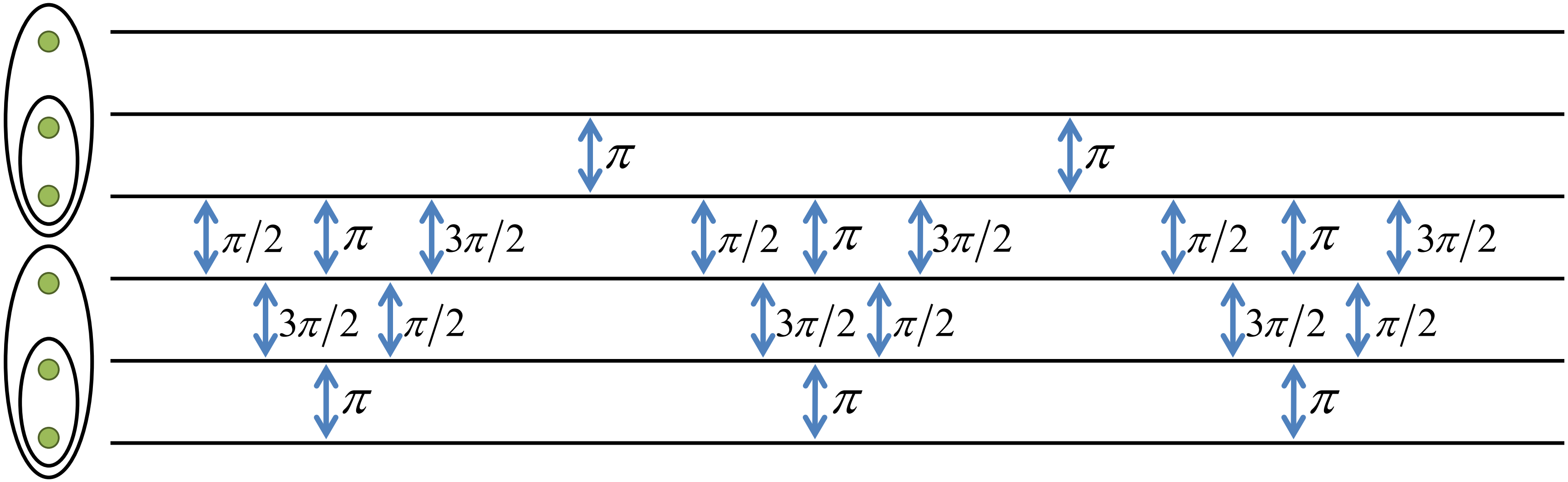
Zeuch, NEB, *Phys. Rev. A* (2016)

Full Sequence



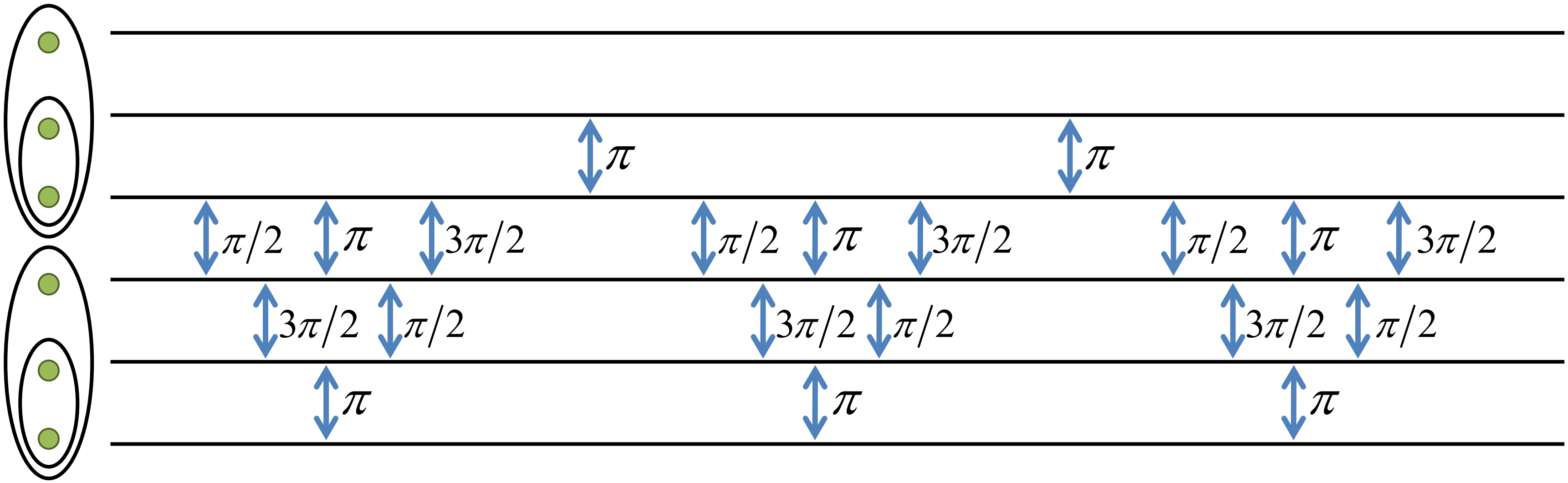
Zeuch, NEB, *Phys. Rev. A* (2016)

Full Sequence

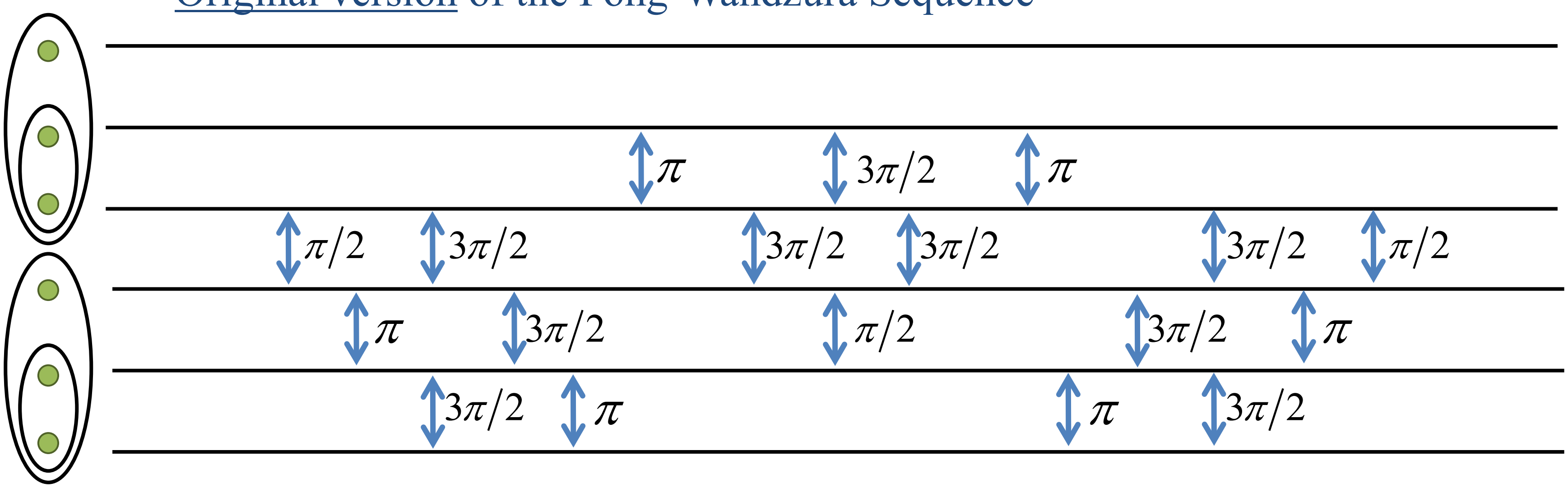


Zeuch, NEB, *Phys. Rev. A* (2016)

Full Sequence



- Original version of the Fong-Wandzura Sequence



RAPID COMMUNI

PHYSICAL REVIEW A **93**, 010303(R) (2016)

Simple derivation of the Fong-Wandzura pulse sequence

Daniel Zeuch and N. E. Bonesteel

Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32310, USA

(Received 4 September 2015; published 25 January 2016)

We give an analytic construction of a class of two-qubit gate pulse sequences that act on five of the six spin-$\frac{1}{2}$ particles used to encode a pair of exchange-only three-spin qubits. Within this class, the problem of gate construction reduces to that of finding a smaller sequence that acts on four spins and is subject to a simple constraint. The optimal sequence satisfying this constraint yields a two-qubit gate sequence equivalent to that found numerically by Fong and Wandzura. Our construction is sufficiently simple that it can be carried out entirely with pen, paper, and knowledge of a few basic facts about quantum spin.

DOI: 10.1103/PhysRevA.93.010303

RAPID COMMUNI

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DOI: 10.1103/PhysRevA.93.010303

PHYSICAL REVIEW B **102**, 075311 (2020)

Efficient two-qubit pulse sequences beyond CNOT

D. Zeuch [1] and N. E. Bonesteel [2]
[1] *Peter Grünberg Institute, Theoretical Nanoelectronics, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[2] *Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32310, USA*

(Received 30 January 2020; revised 6 August 2020; accepted 6 August 2020; published 26 August 2020)

We design efficient controlled-rotation gates with arbitrary angle acting on three-spin encoded qubits for exchange-only quantum computation. Two pulse sequence constructions are given. The first is motivated by an analytic derivation of the efficient Fong-Wandzura sequence for an exact CNOT gate. This derivation, briefly reviewed here, is based on elevating short sequences of SWAP pulses to an entangling two-qubit gate. To go beyond CNOT, we apply a similar elevation to a modified short sequence consisting of SWAPs and one pulse of arbitrary duration. This results in two-qubit sequences that carry out controlled-rotation gates of arbitrary angle. The second construction streamlines a class of arbitrary CPHASE gates established earlier. Both constructions are based on building two-qubit sequences out of subsequences with special properties that render each step of the construction analytically tractable.

DOI: 10.1103/PhysRevB.102.075311

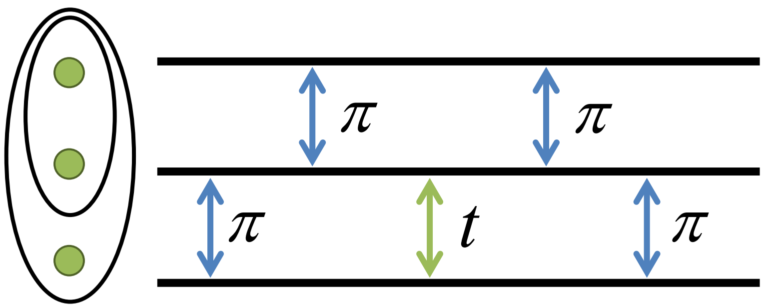
New Sequences?

Strategy: "Elevate" simple three-spin pulse sequence to a two-qubit gate.

Zeuch, NEB, *Phys. Rev. B* (2020)

Another Simple Sequence

Strategy: "Elevate" simple three-spin pulse sequence to a two-qubit gate.



Zeuch, NEB, *Phys. Rev. B* (2020)

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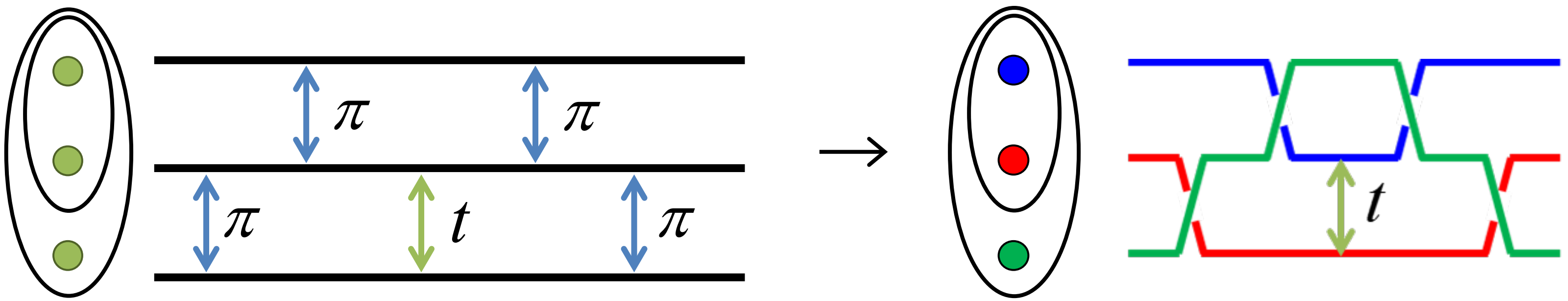
Strategy: “Elevate” simple three-spin pulse sequence to a two-qubit gate.



Zeuch, NEB, *Phys. Rev. B* (2020)

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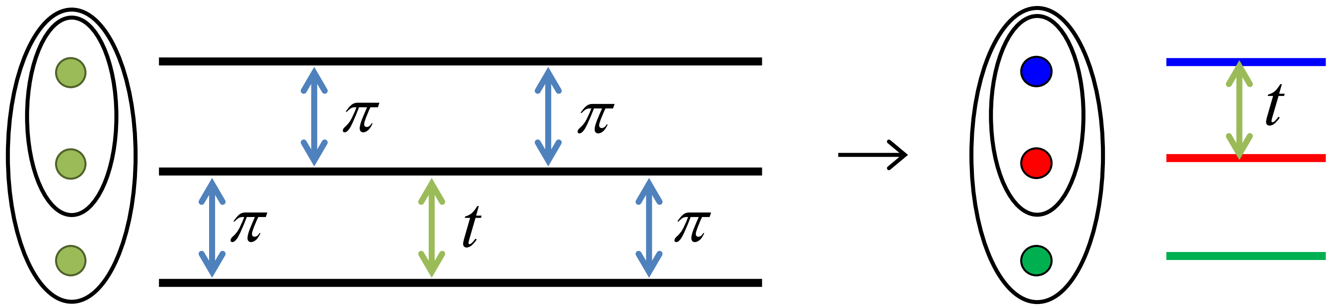
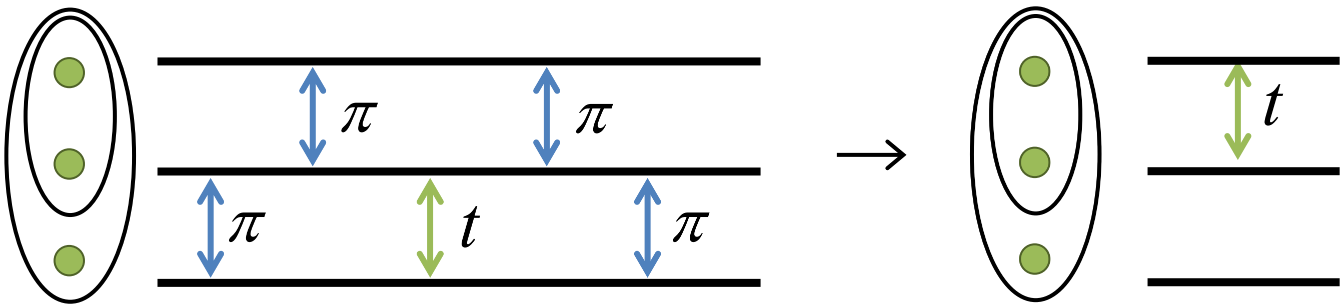


Zeuch, NEB, *Phys. Rev. B* (2020)

Another Simple Sequence

Strategy: “Elevate” simple three-spin pulse sequence to a two-qubit gate.



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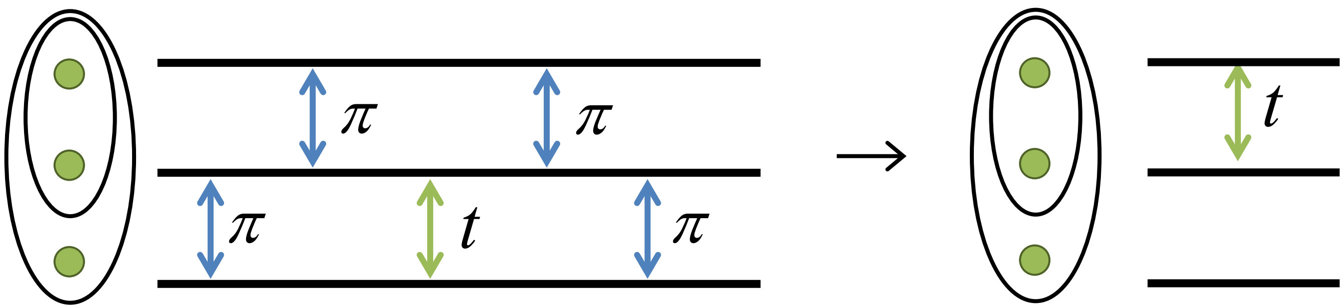
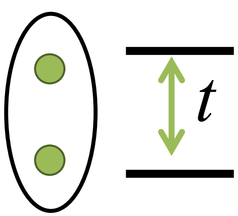
Sequence Identity

Strategy: "Elevate" simple three-spin pulse sequence to a two-qubit gate.





$$\begin{pmatrix} 1 & \\ & e^{-it} \end{pmatrix}$$

Identity holds for *any* *t*, not just *t=0,*π

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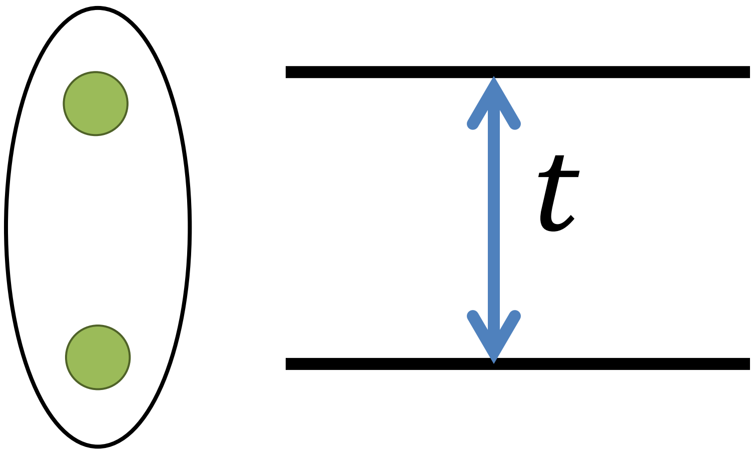
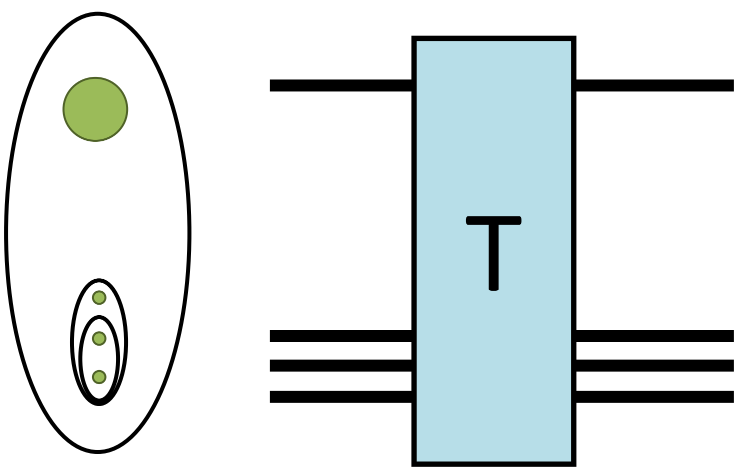
“Elevating” t Pulse



$$\begin{pmatrix} 1 & \\ & e^{it} \end{pmatrix}$$

M can be any 2x2 unitary.
No longer require $\boldsymbol{M^2 = I}$



$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

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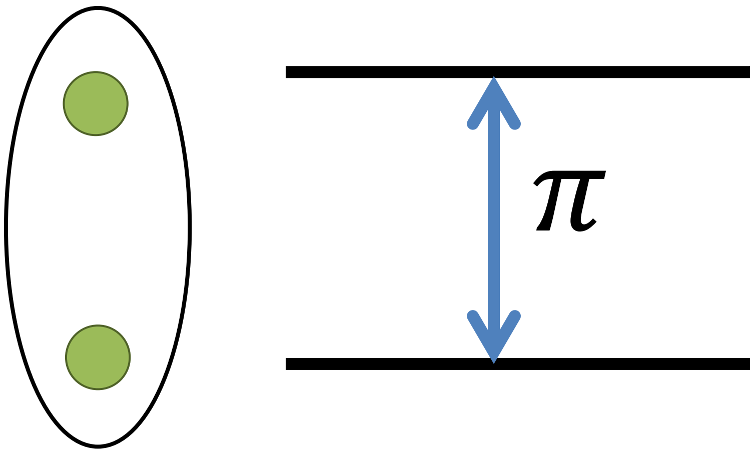
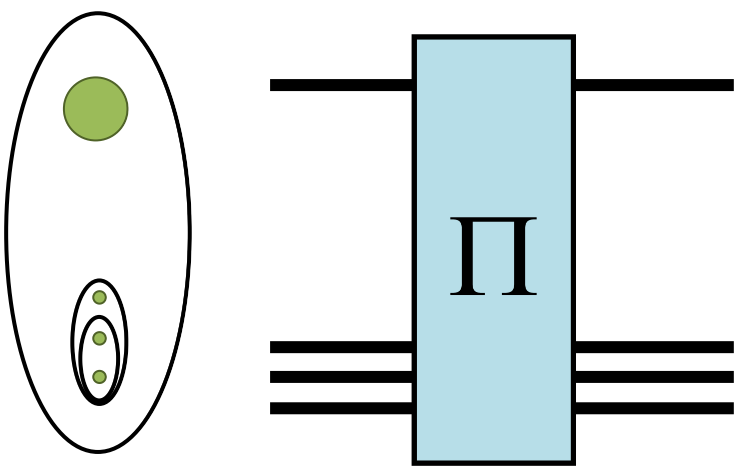
“Elevating” $SWAP$ Pulse



$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$



$$\begin{pmatrix} I & \\ & -I \end{pmatrix}$$

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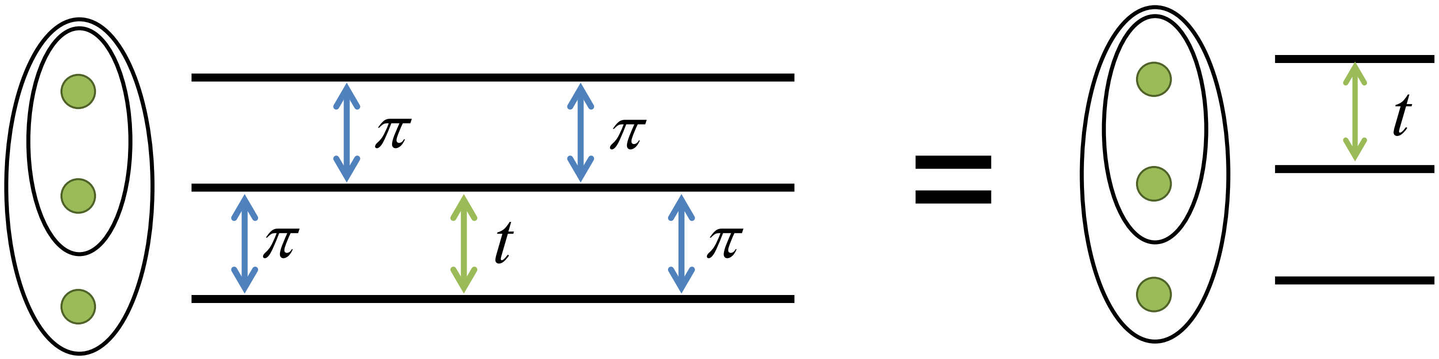
Sequence Elevation



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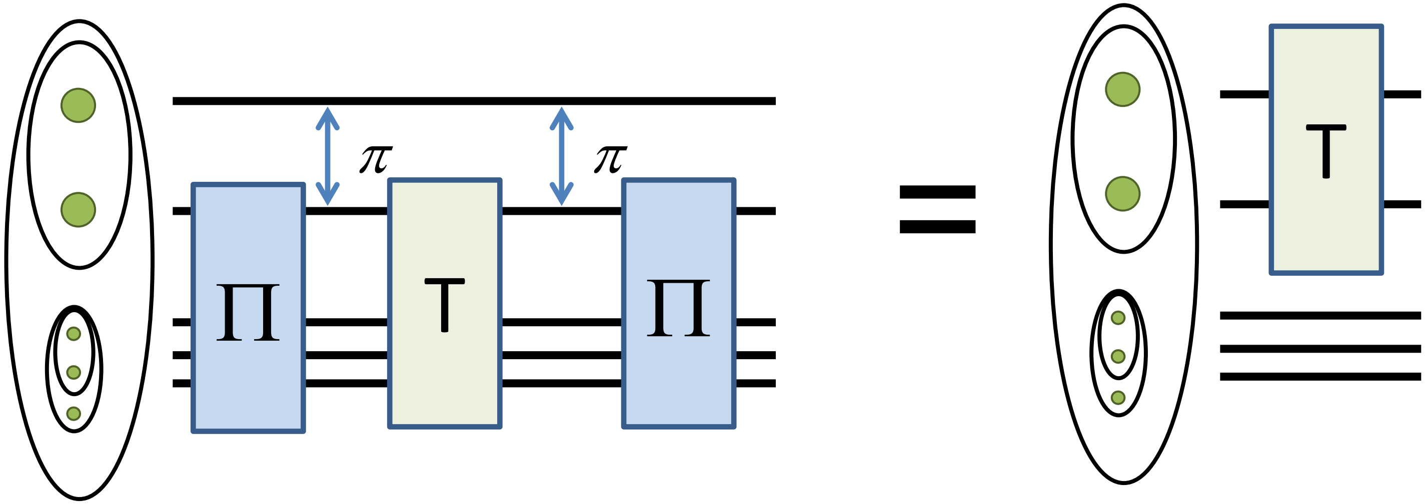
Sequence Elevation



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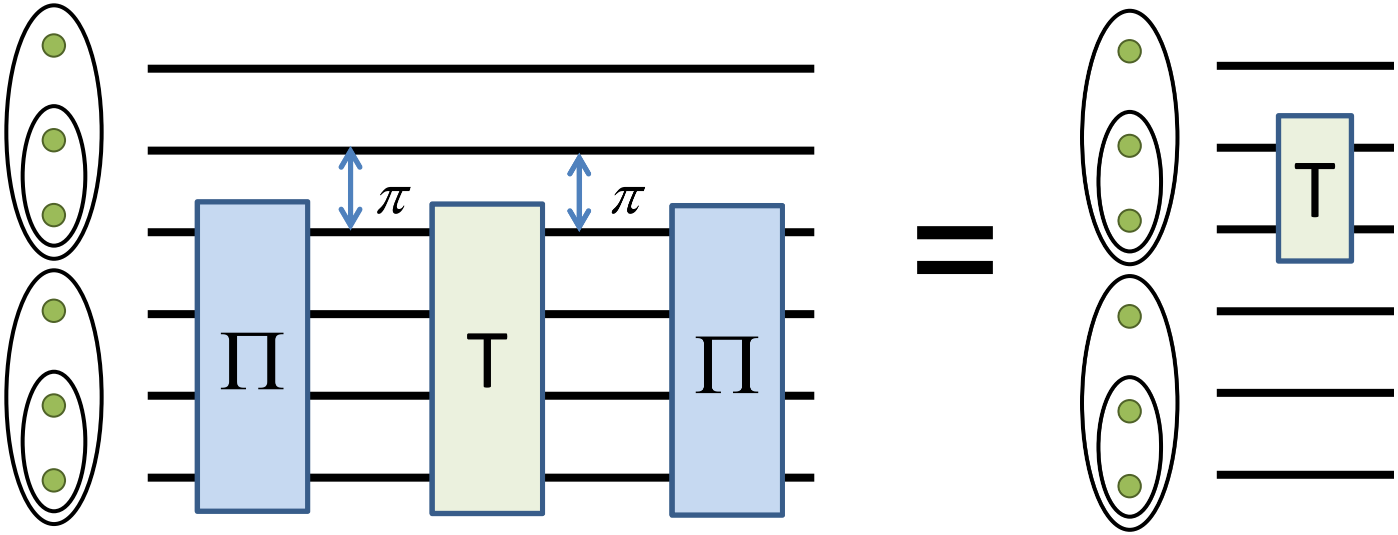
Sequence Elevation



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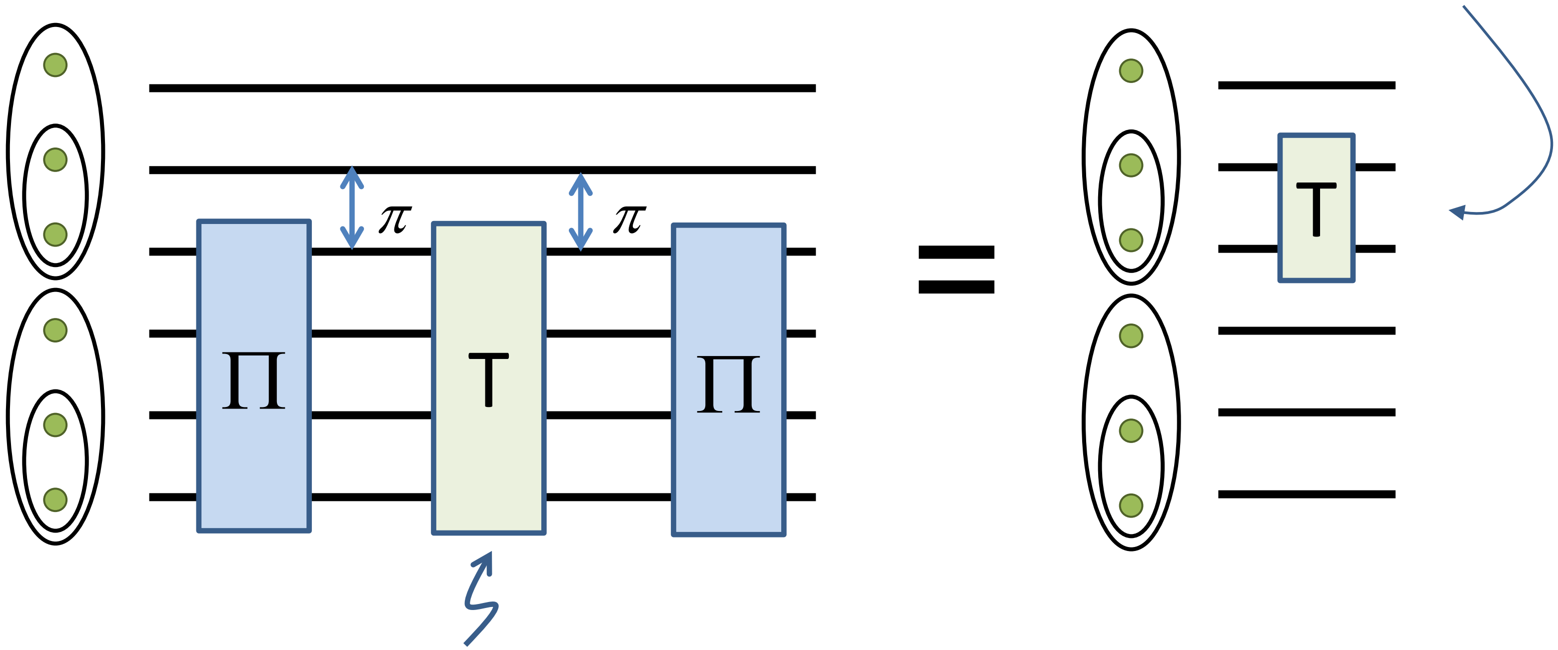
Sequence Elevation

Applies M to bottom qubit only if top qubit is in state $|1\rangle$



$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

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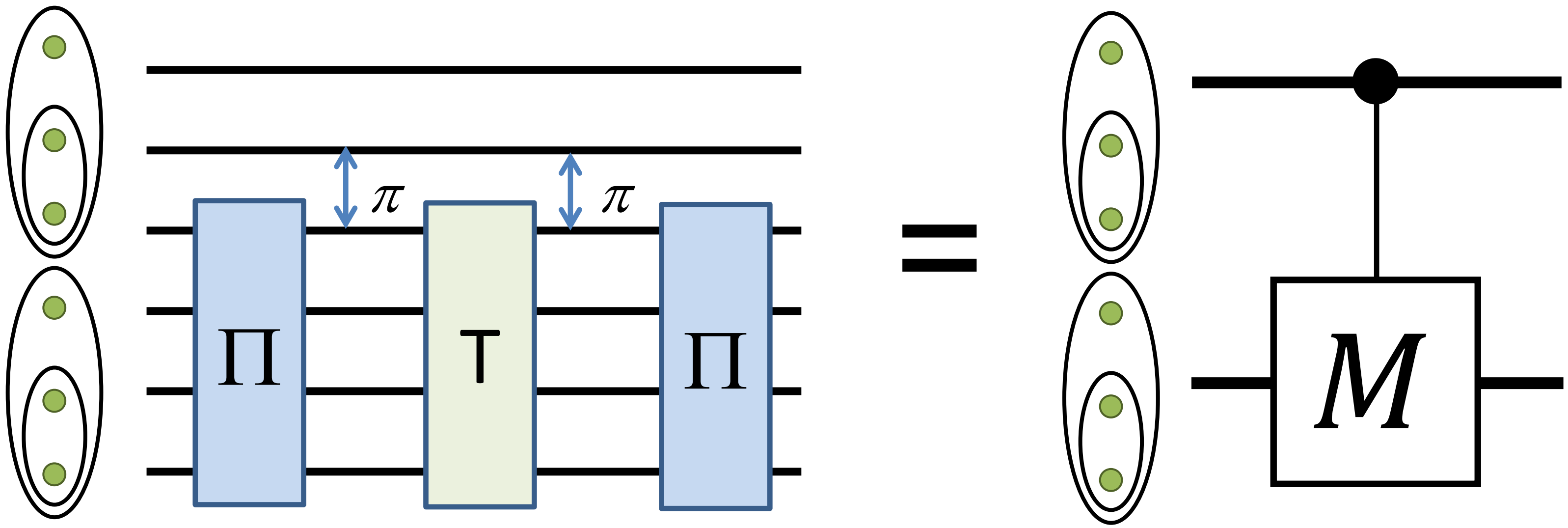
Sequence Elevation



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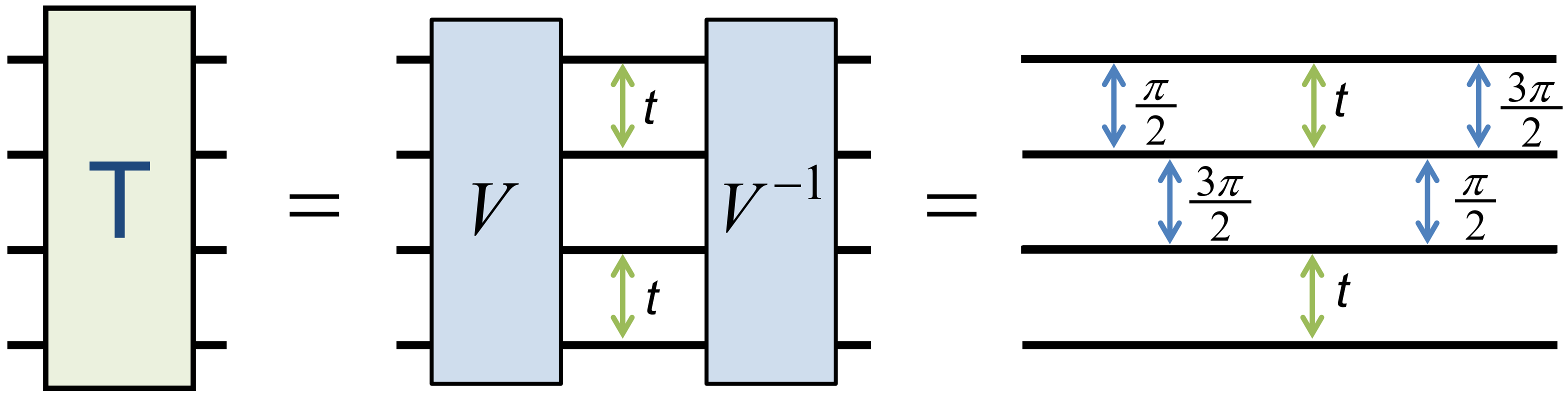
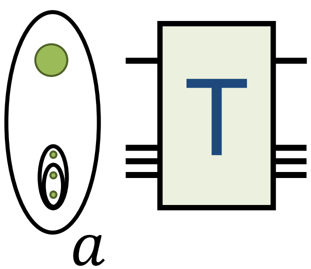
Constructing T





$$a = 0 \qquad 1$$

$$\begin{pmatrix} I & \\ & M \end{pmatrix}$$

$$M(t) = e^{i\,\xi(t)} e^{i\phi(t)\hat{\boldsymbol{n}}(t)\cdot\boldsymbol{\sigma}/2}$$

$$\boxed{\phi(t) = 2\arccos\big((5\cos(t/2) + 3\cos(3t/2))/8\big)}$$

$$\xi(t) = -t/2$$

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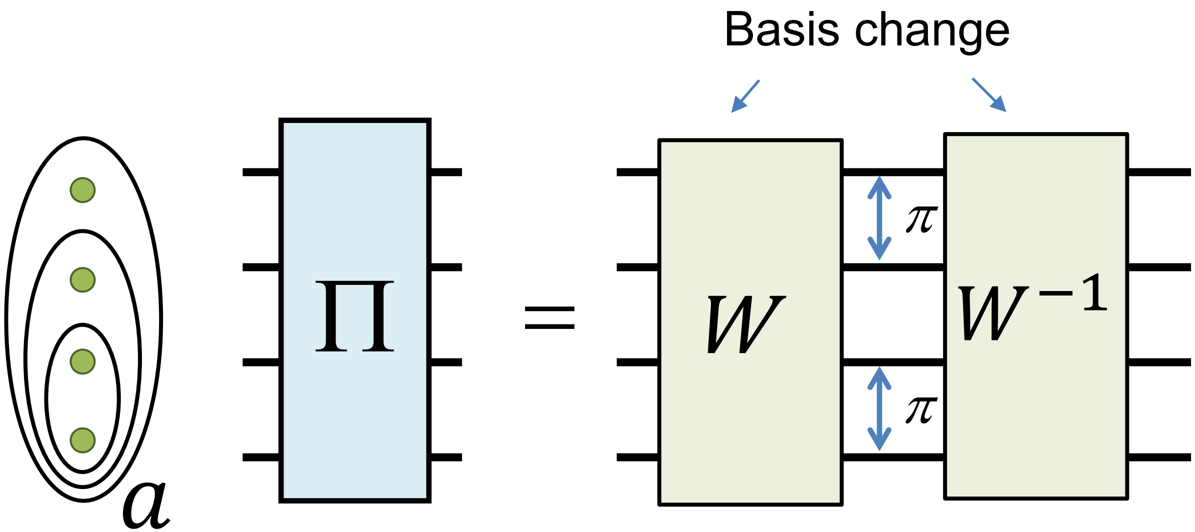
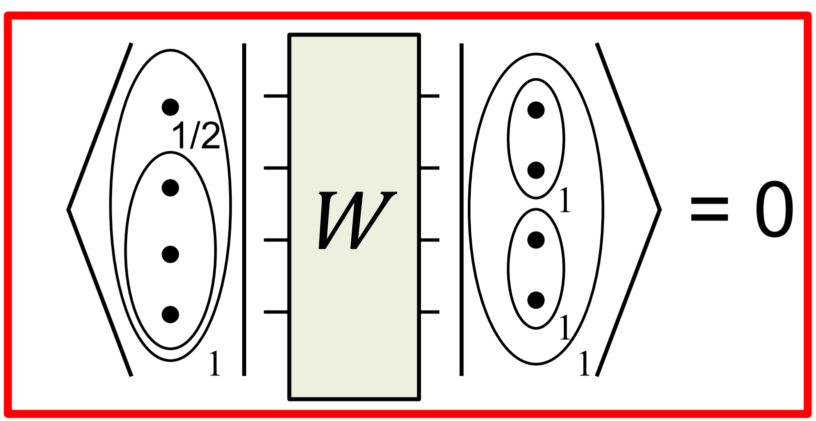
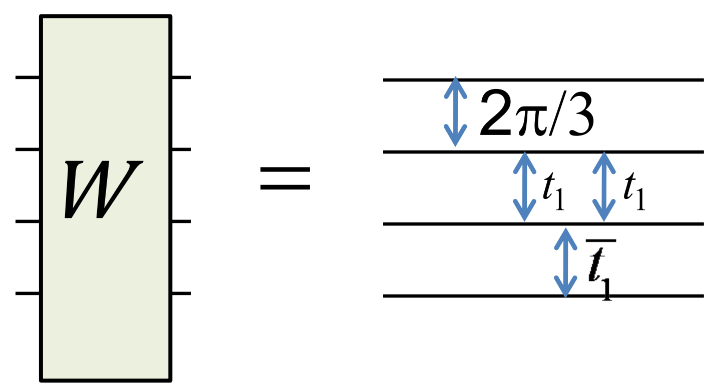
Constructing Π



Constraint



Solution



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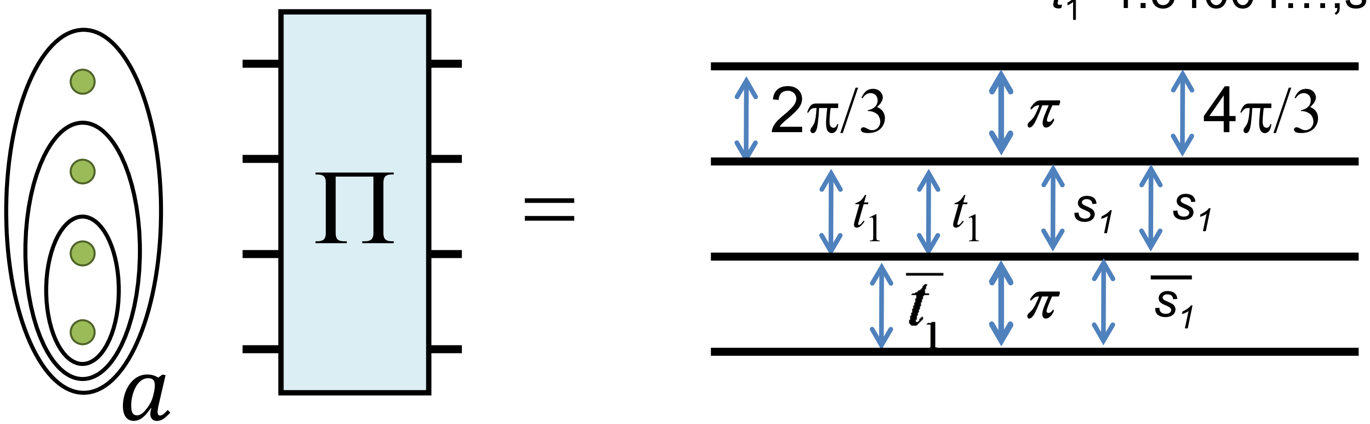
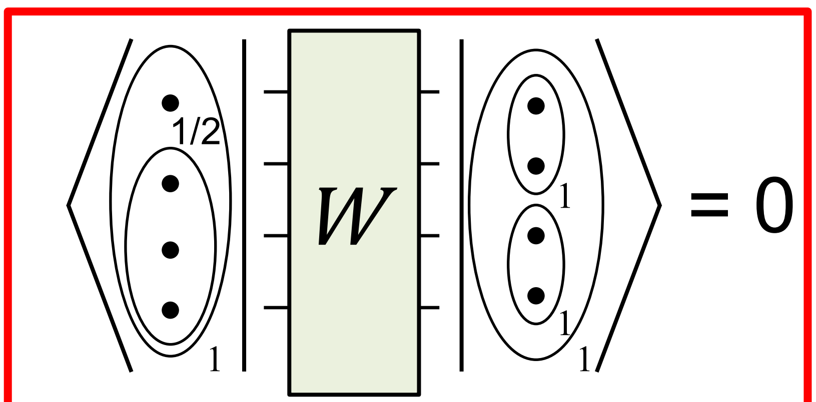
Constructing Π

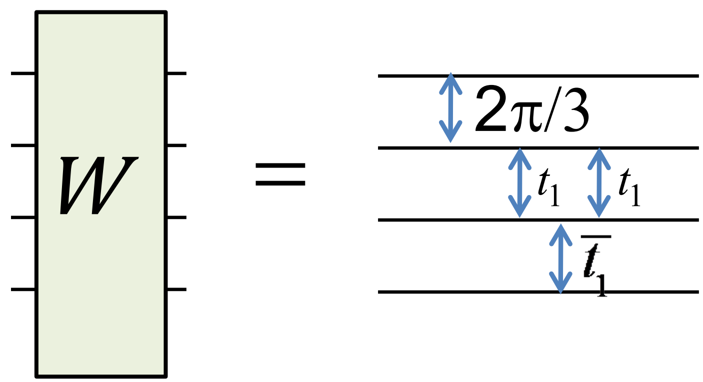
t_1=1.34004…,s_1= $2\pi - t_{1,\ldots}$



Constraint



Solution



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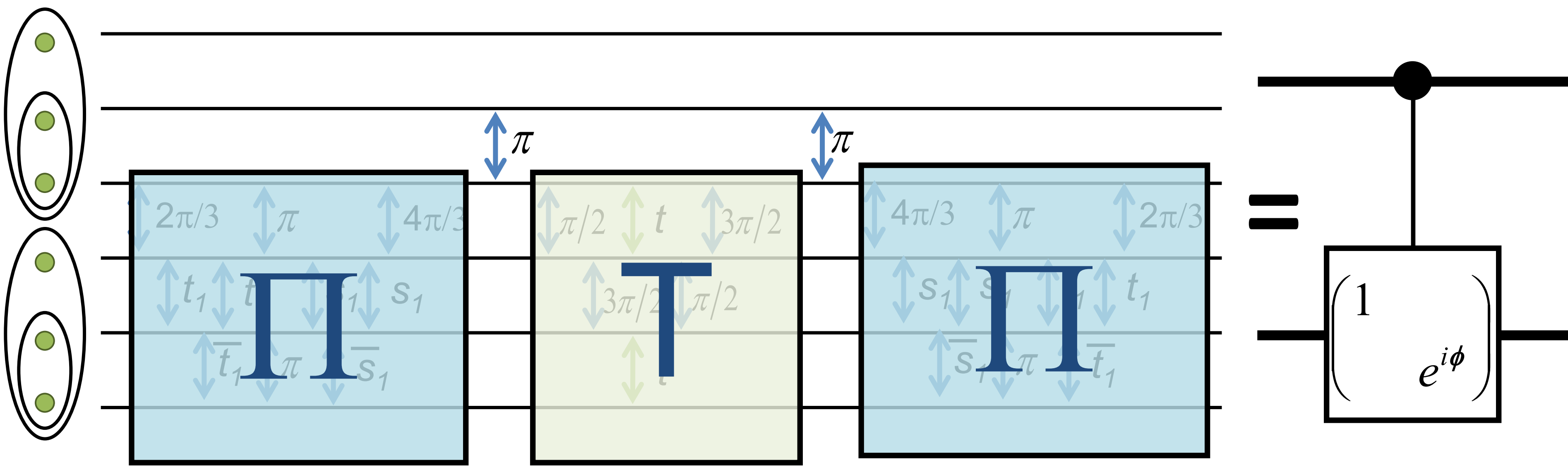
Full Sequence



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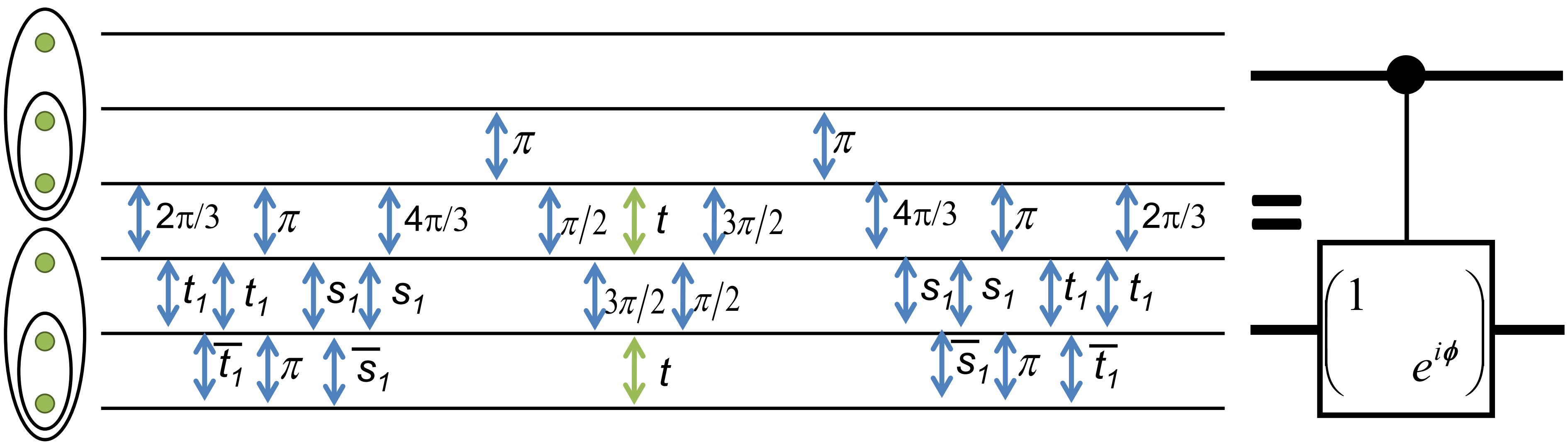
Full Sequence



t_1=1.34004…,s_1= 2π–$t_{1,\ldots}$

$$\phi(t) = 2\arccos\big((5\cos(t/2) + 3\cos(3t/2))/8\big)$$

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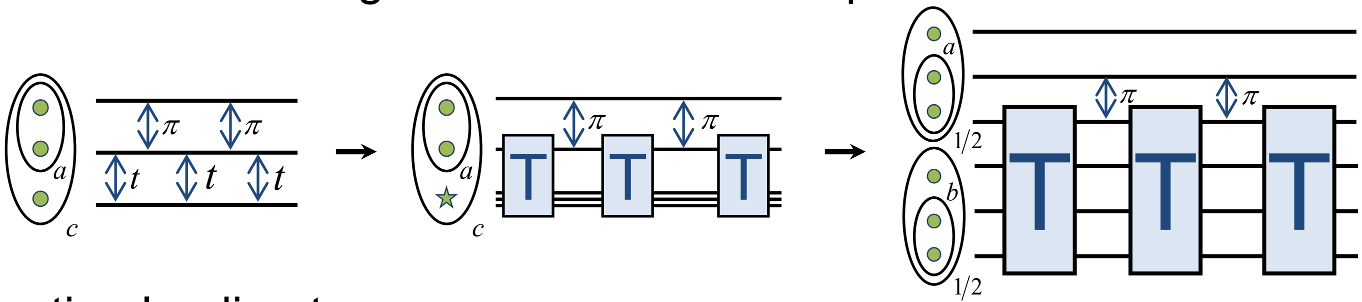
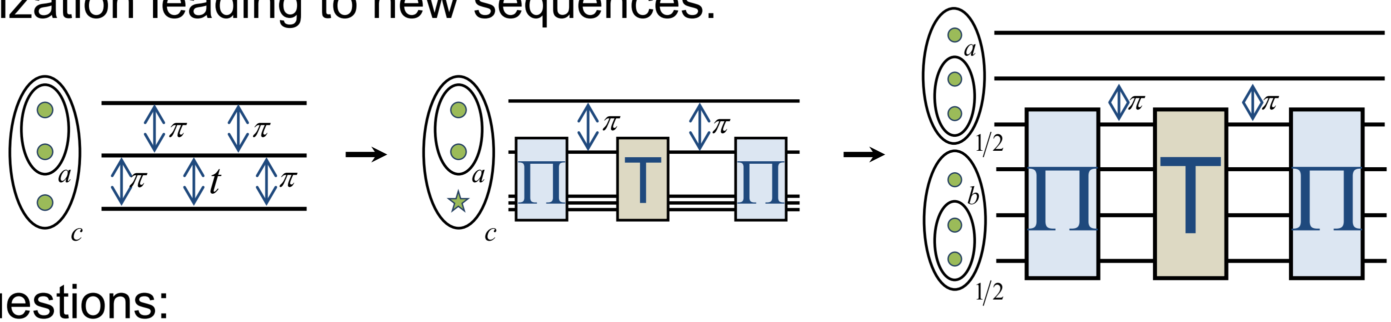
Summary

Analytic Derivation of Fong-Wandzura CNOT sequence:



Generalization leading to new sequences:



Open questions:

1) Can we prove Fong-Wandzura sequence is truly optimal?
2) More efficient general gate constructions?
3) Can these tools be used to construct more “robust” sequences?

Zeuch, NEB, Phys. Rev. A **98**, 010303 (2016)
Zeuch, NEB, Phys. Rev. B **102,** 075311 (2020)