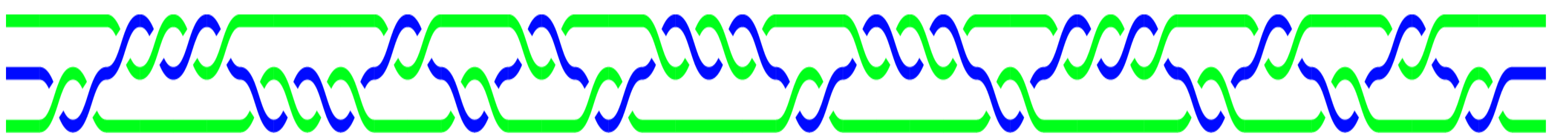
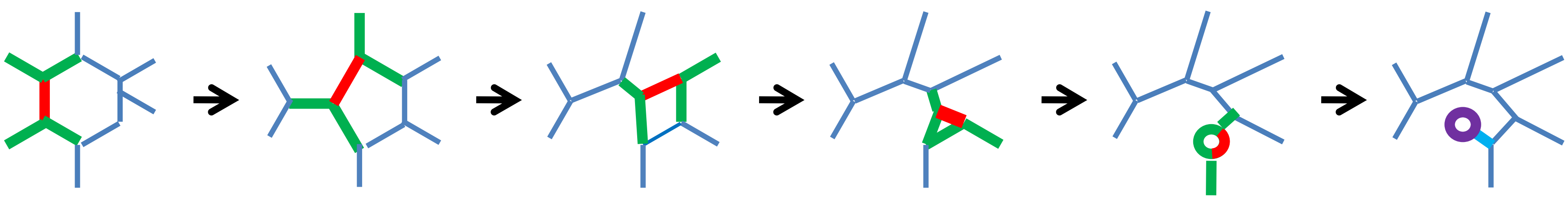
Simulating Anyons on a Quantum Computer

Nick Bonesteel

Florida State University

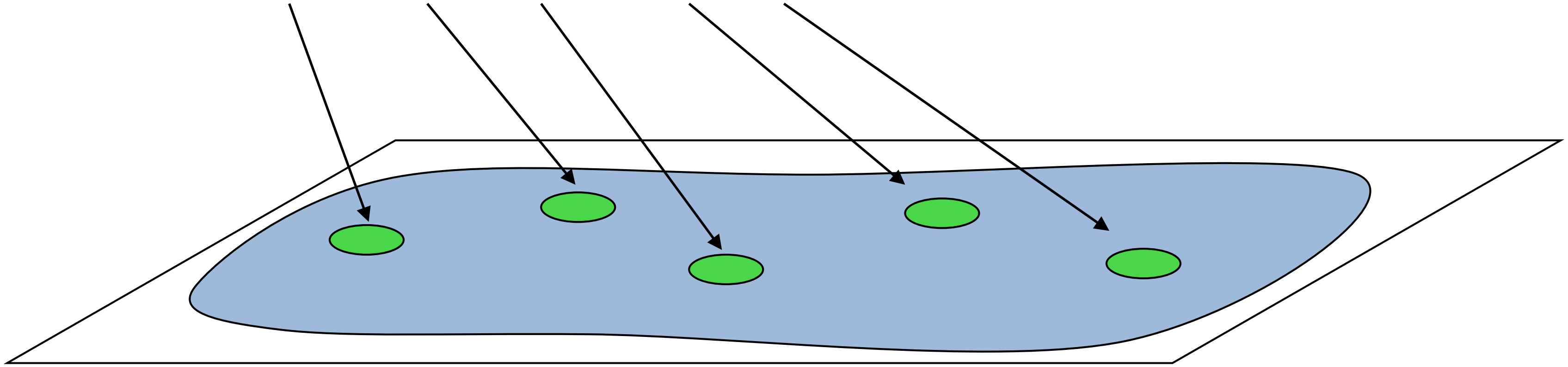




CINT Annual Meeting, 9/24/18

"Non-Abelian" FQH States (Moore & Read '91)

Fractionalized quasiparticles = **Non-Abelian anyons**



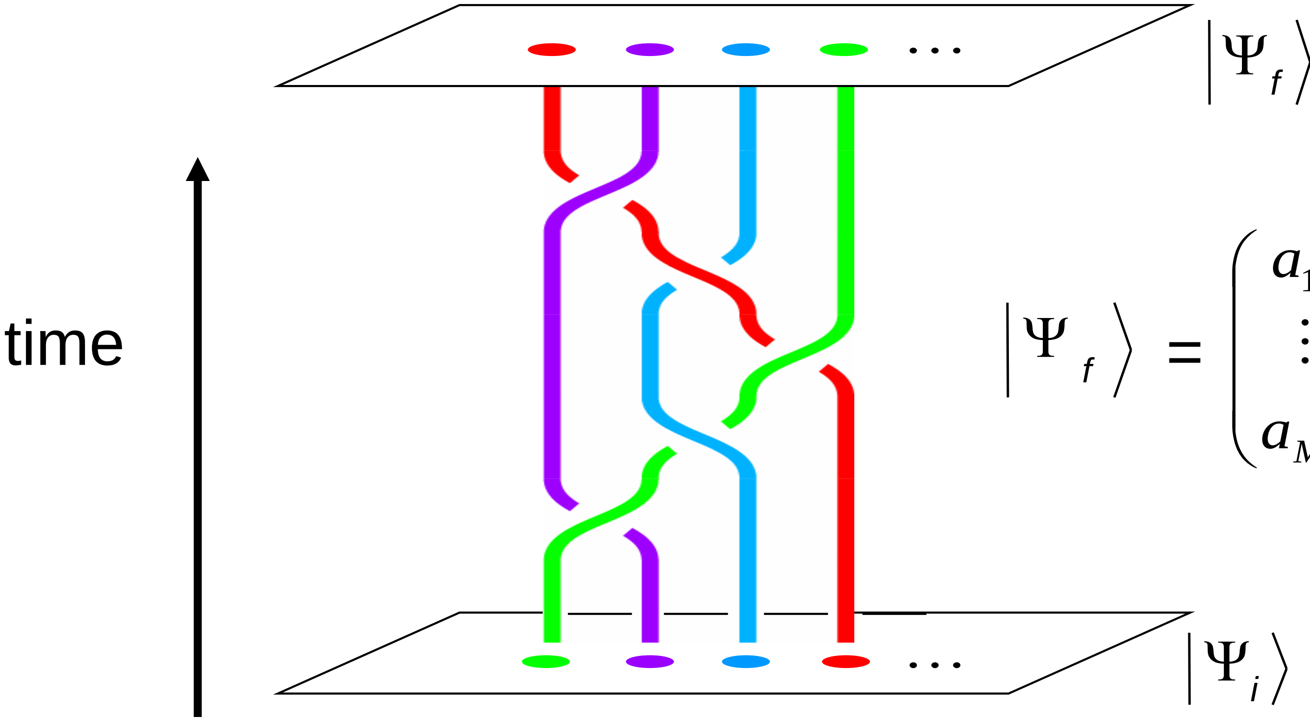
Essential features:

A degenerate Hilbert space whose dimensionality is **exponentially large in the number of quasiparticles**.

States in this space **can only be distinguished by global measurements** provided quasiparticles are far apart.

→ ***A perfect place to hide quantum information!***

Topological Quantum Computation



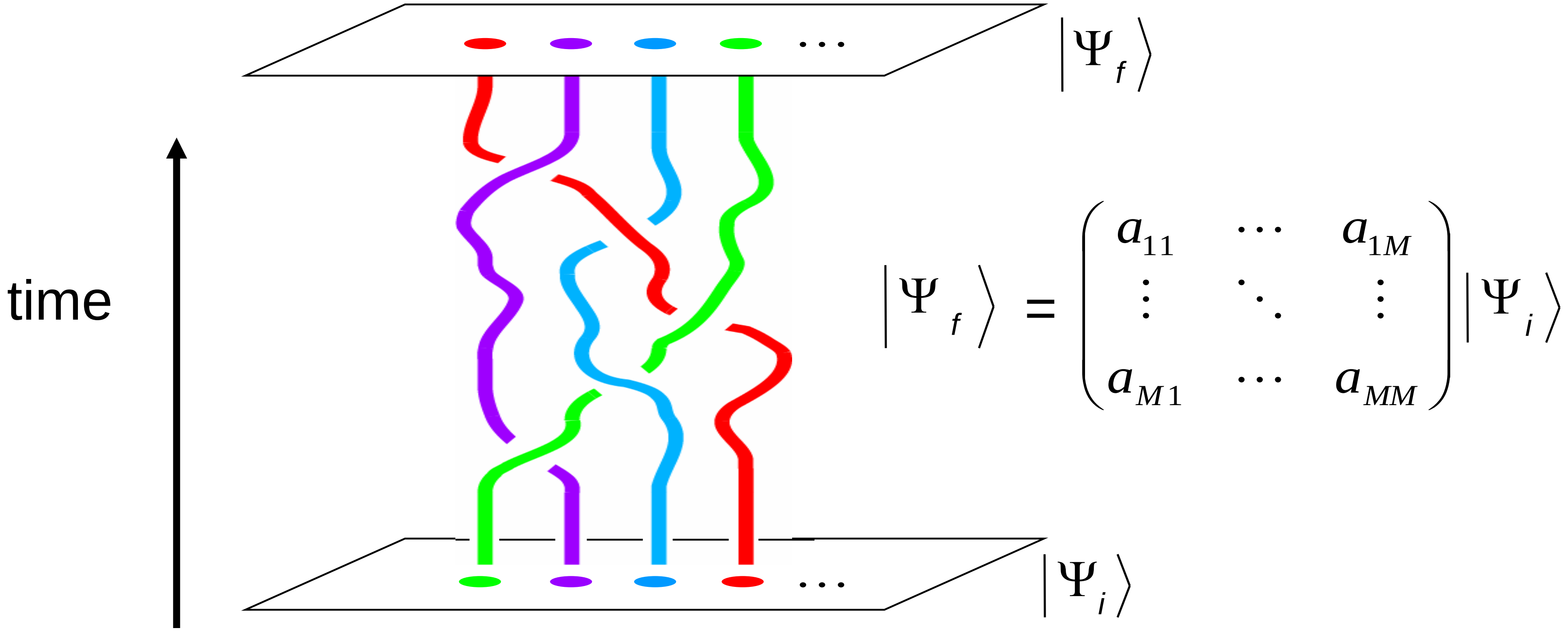
$$|\Psi_f\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{pmatrix} |\Psi_i\rangle$$

Matrix depends only on the topology of the braid swept out by anyon world lines!

Robust quantum computation?

Kitaev, ‘97 ; Freedman, Larsen, and Wang, ‘01

Topological Quantum Computation



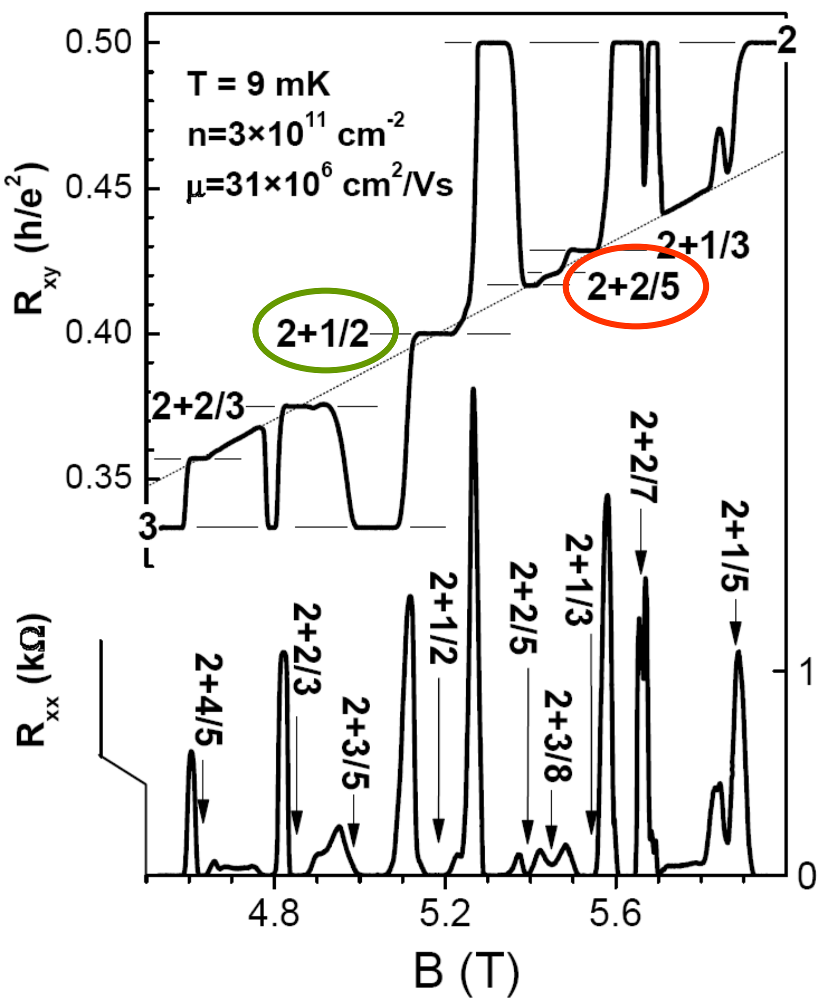
Matrix depends only on the topology of the braid swept out by anyon world lines!

Robust quantum computation?

Kitaev, '97 ; Freedman, Larsen, and Wang, '01

Possible Non-Abelian FQH States

J.S. Xia et al., PRL (2004).



ν = **5/2**: Probable Moore-Read Pfaffian state.

Charge *e/4* quasiparticles are **Ising Anyons**
Moore & Read '91

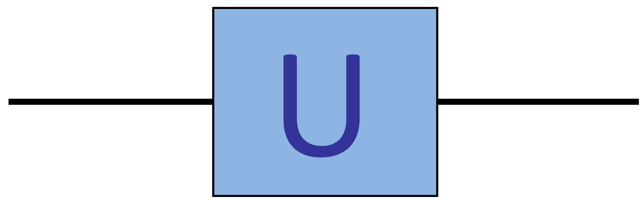
Not Universal for Quantum Computation

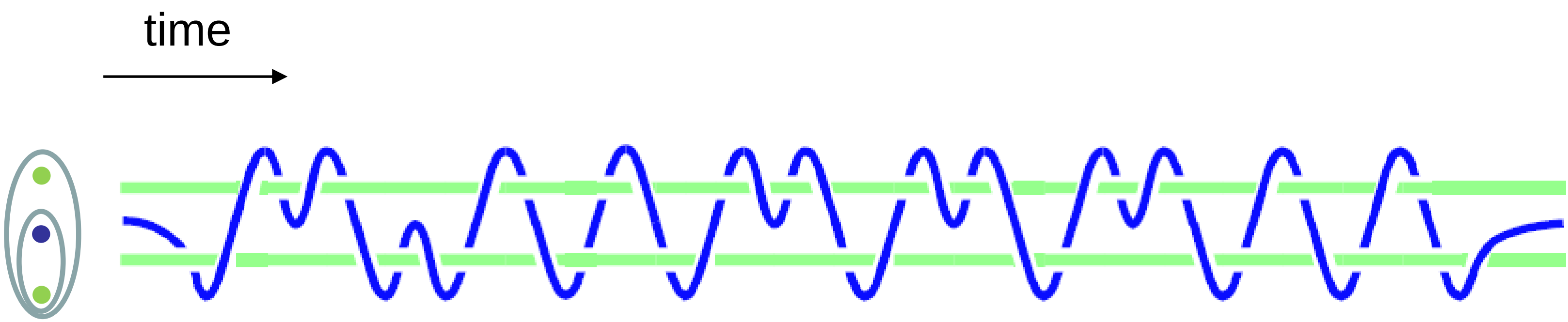
ν = **12/5**: Possible Read-Rezayi "Parafermion" state. Read & Rezayi, '99

Charge *e/5* quasiparticles are **Fibonacci anyons.**
Slingerland & Bais '01

Universal for Quantum Computation!
Freedman, Larsen & Wang '02

Single Qubit Operations

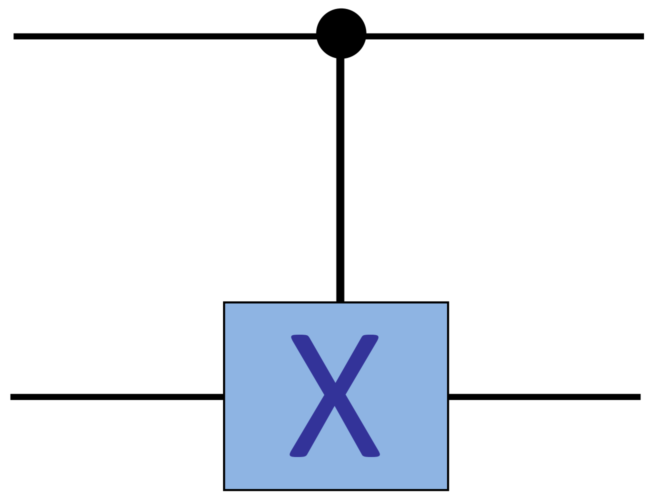


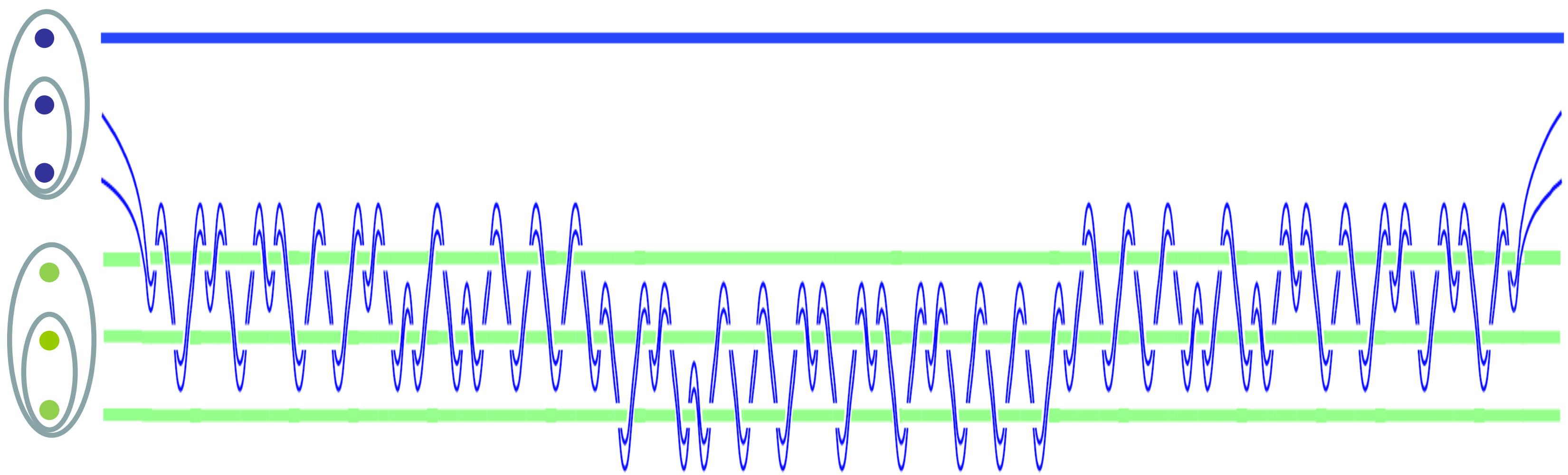


NEB, L. Hormozi, G. Zikos, & S. Simon, PRL 2005
L. Hormozi, NEB, & S. Simon, PRL 2009

Two Qubit Gates (CNOT)

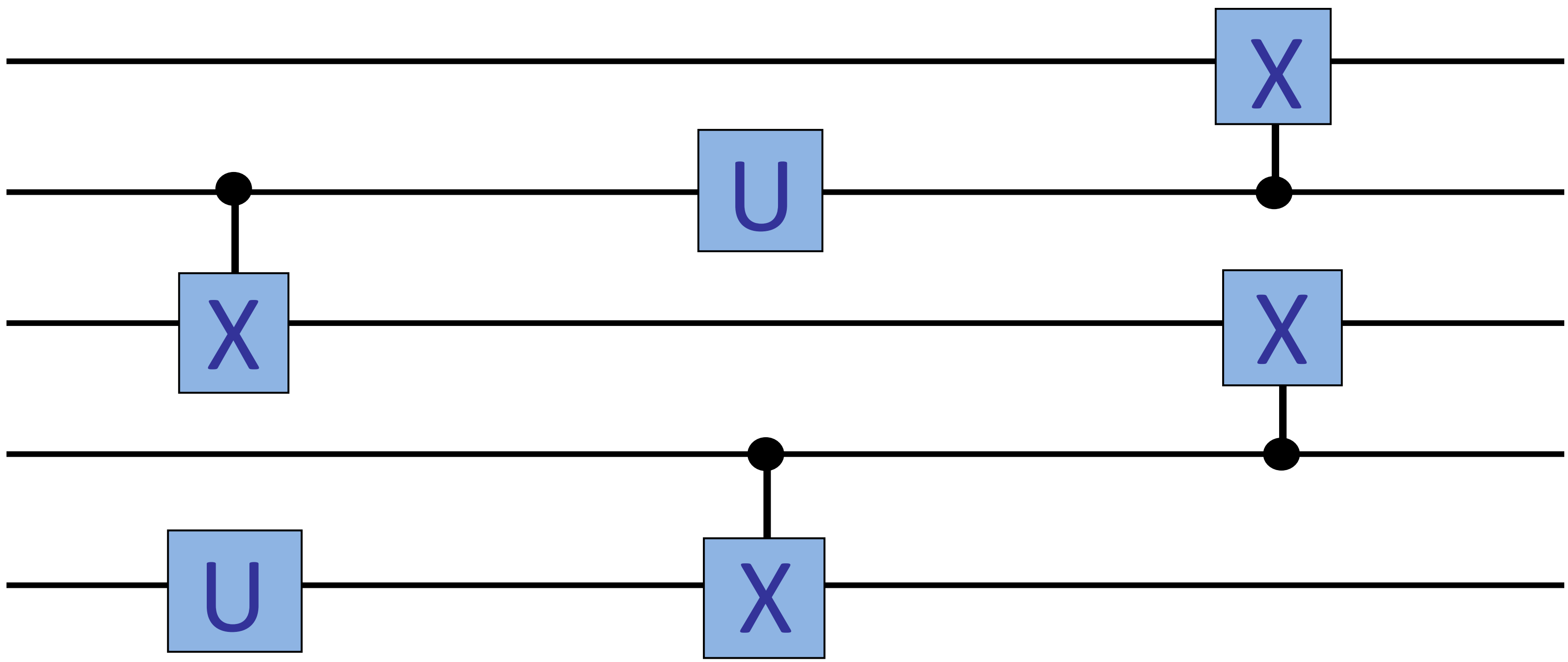
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



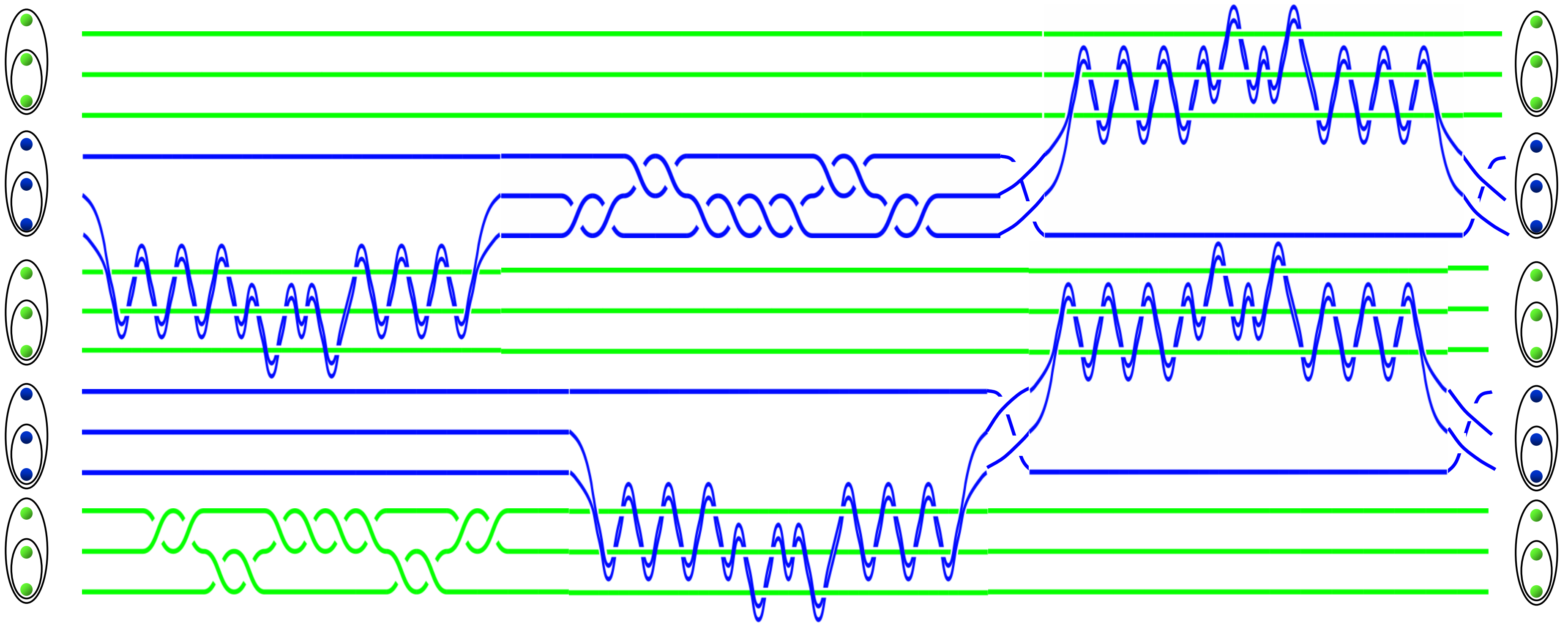


NEB, L. Hormozi, G. Zikos, & S. Simon, PRL 2005
L. Hormozi, NEB, & S. Simon, PRL 2009

Quantum Circuit



Braid



“Surface Code” Approach

Key Idea: Use a quantum computer to *simulate* a theory of anyons.

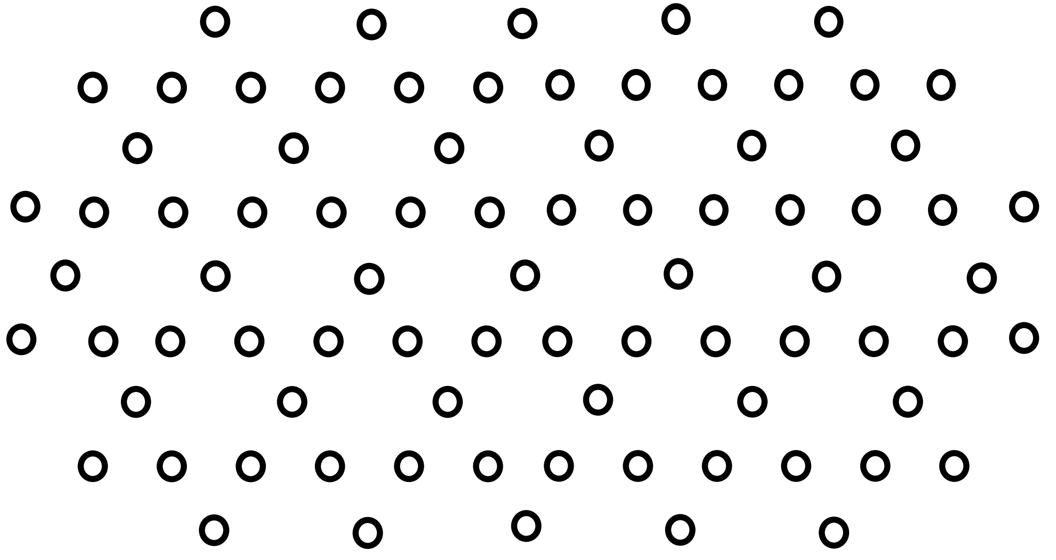
The simulation “inherits” the fault-tolerance of the anyon theory.

Most promising approach is based on “Abelian anyons.” High error thresholds, but can’t compute purely by braiding.

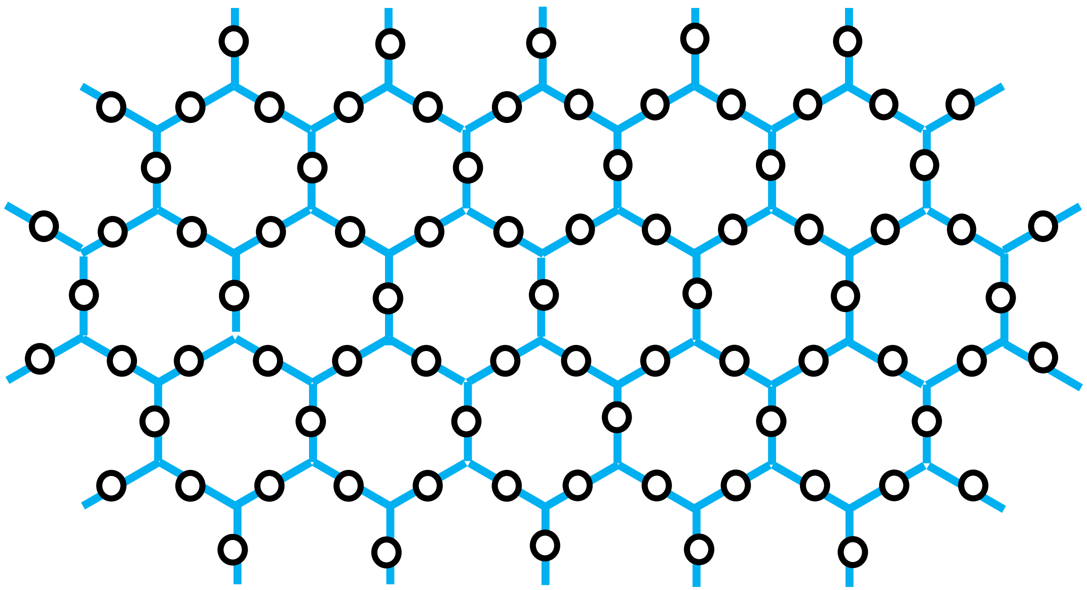
Bravyi & Kitaev, 2003
Raussendorf & Harrington, PRL 2007
Fowler, Stephens, and Groszkowski, PRA 2009

More speculative idea: simulate Fibonacci anyons.

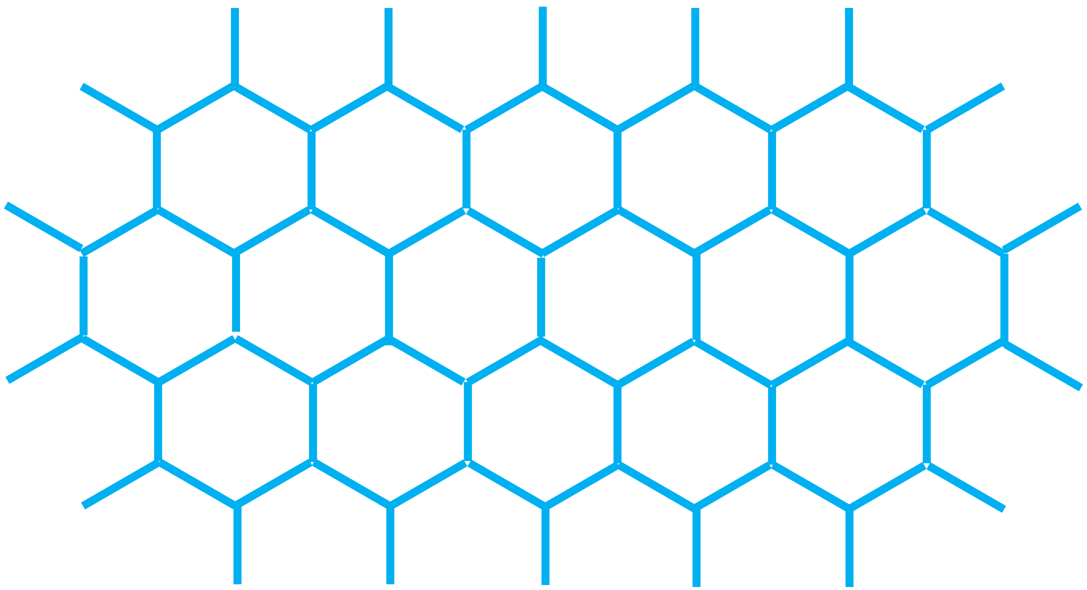
Konig, Kuperberg, Reichardt, Ann. Phys. 2010
NEB, D.P. DiVincenzo, PRB 2012
W. Feng, NEB, D.P. DiVincenzo, In preparation



2D Array of Qubits



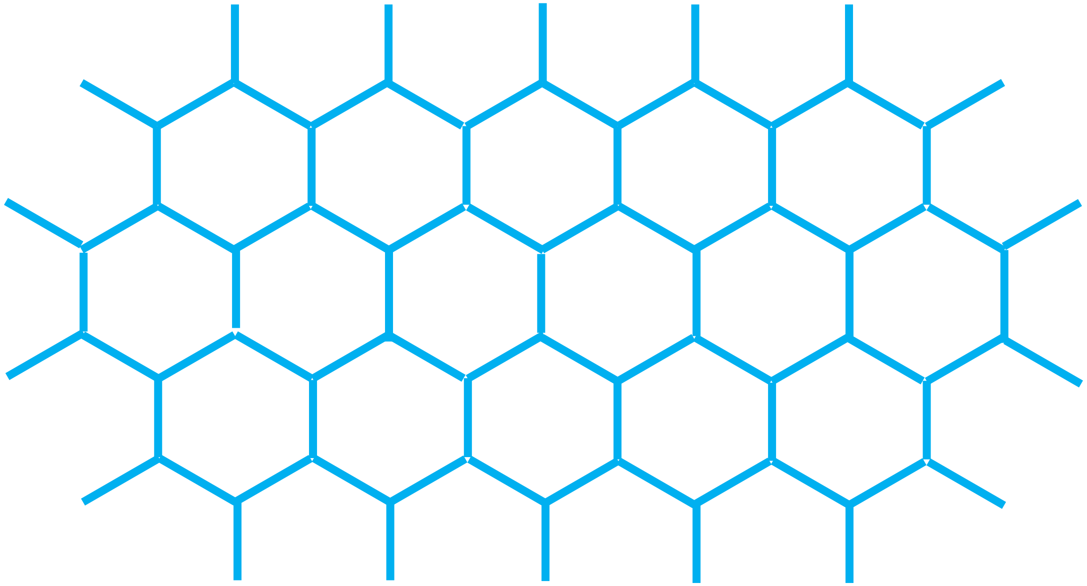
Trivalent Lattice



Trivalent Lattice

“Fibonacci” Levin-Wen Model

Levin & Wen, PRB 2005



Trivalent Lattice

$$H = -\sum_{v} Q_v - \sum_{p} B_p$$

"Fibonacci" Levin-Wen Model

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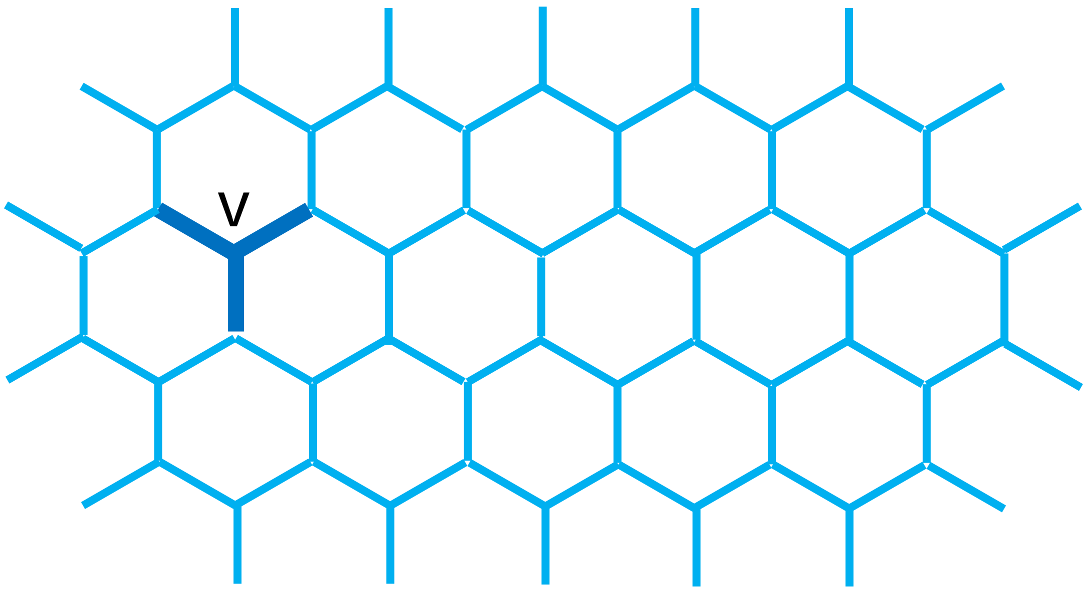


Trivalent Lattice

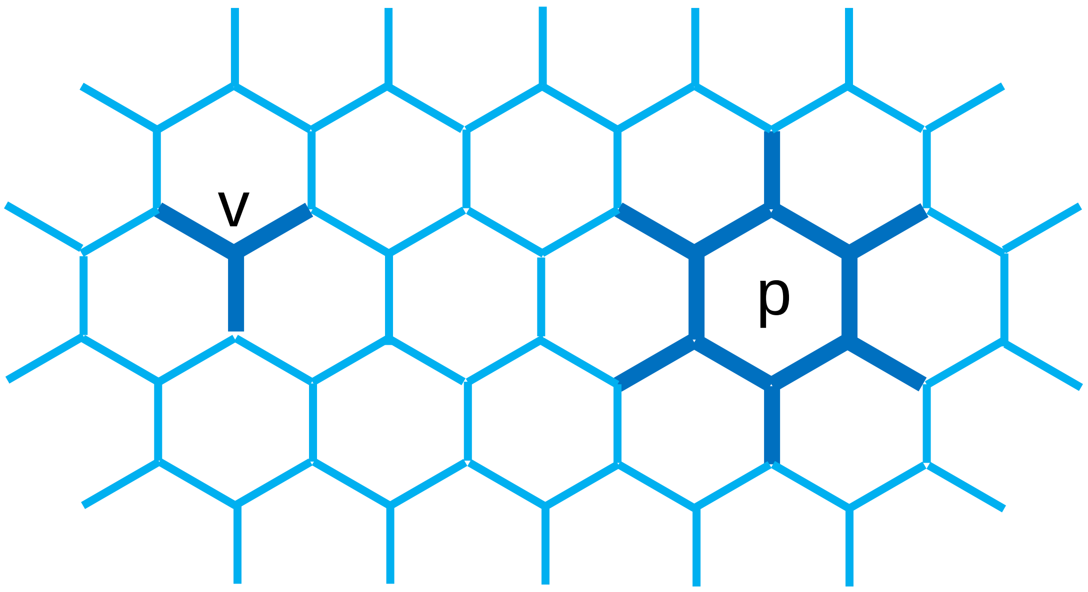
$$H = -\sum_{v} Q_{v} - \sum_{p} B_{p}$$

Vertex
Operator

$Q_v = 0{,}1$

“Fibonacci” Levin-Wen Model

Levin & Wen, PRB 2005



Trivalent Lattice

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$

Vertex Operator

Plaquette Operator

$Q_{\mathrm{v}} = 0,1$

$B_{\mathrm{p}} = 0,1$

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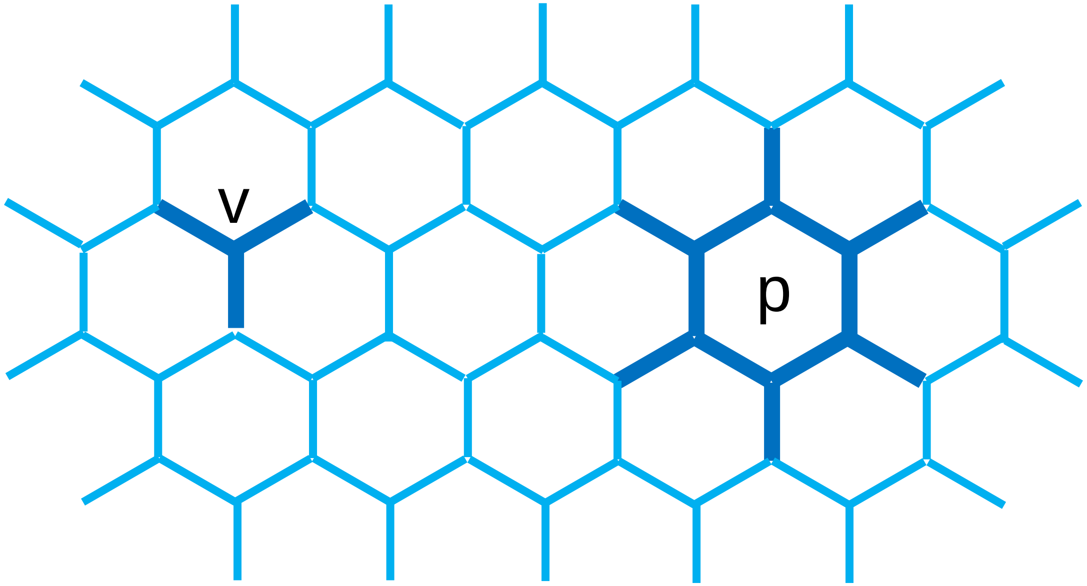
Levin & Wen, PRB 2005



Trivalent Lattice

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$

Vertex Operator

Q_{v} = 0,1

Plaquette Operator

B_{p} = 0,1

$$[Q_{\mathbf{v}}, B_{\mathbf{p}}] = 0$$

$$[Q_{\mathbf{v}}, Q_{\mathbf{v}'}] = 0$$

$$[B_{\mathbf{p}}, B_{\mathbf{p}'}] = 0$$

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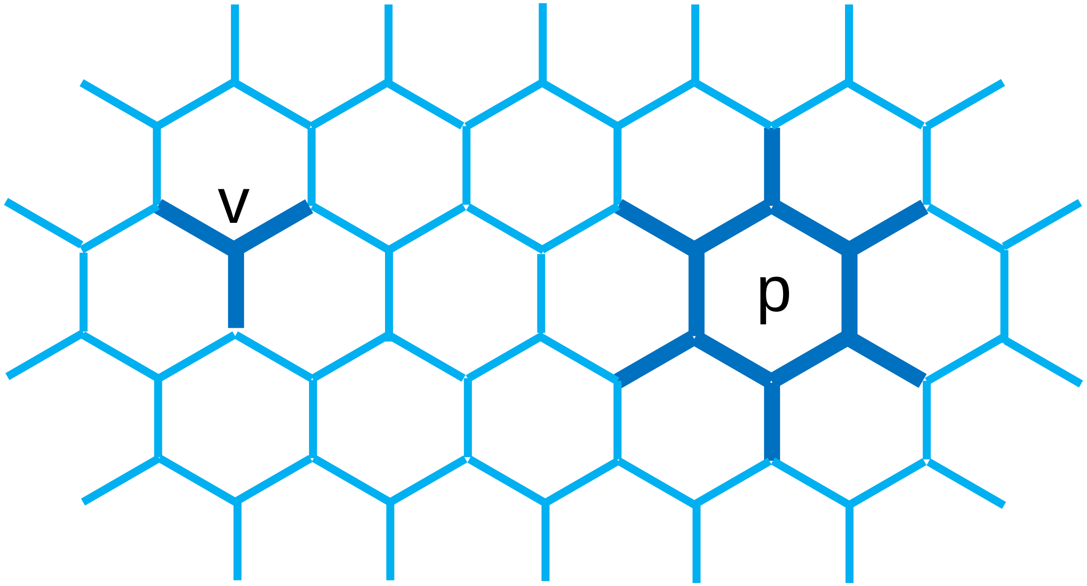

Trivalent Lattice

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$

Vertex Operator

$Q_{\mathrm{v}} = 0,1$

Plaquette Operator

$B_{\mathrm{p}} = 0,1$

Ground State

$Q_{\mathrm{v}} = 1$ on each vertex

$B_{\mathrm{p}} = 1$ on each plaquette

"Fibonacci" Levin-Wen Model

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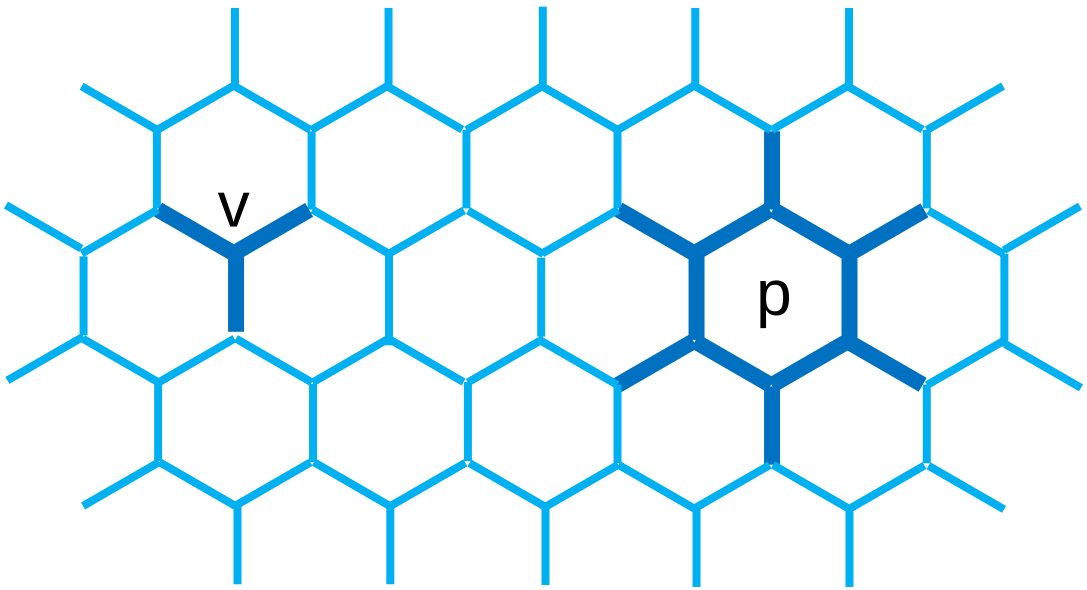

Trivalent Lattice

$$H = -\sum_{v} Q_v - \sum_{p} B_p$$

Vertex Operator

$Q_v = 0,1$

Plaquette Operator

$B_p = 0,1$

Ground State

$Q_v = 1$ on each vertex

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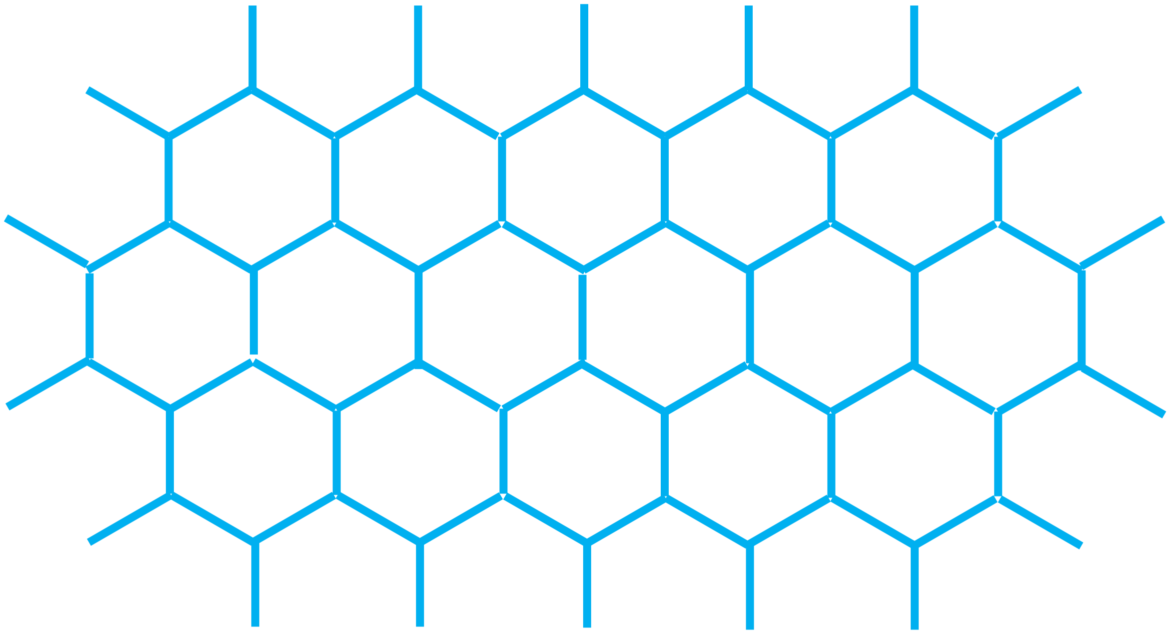
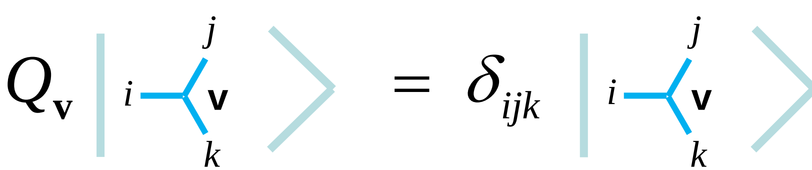
Excited States are Fibonacci Anyons

Vertex Operator: Q_v

$$H = -\sum_v Q_v - \sum_p B_p$$





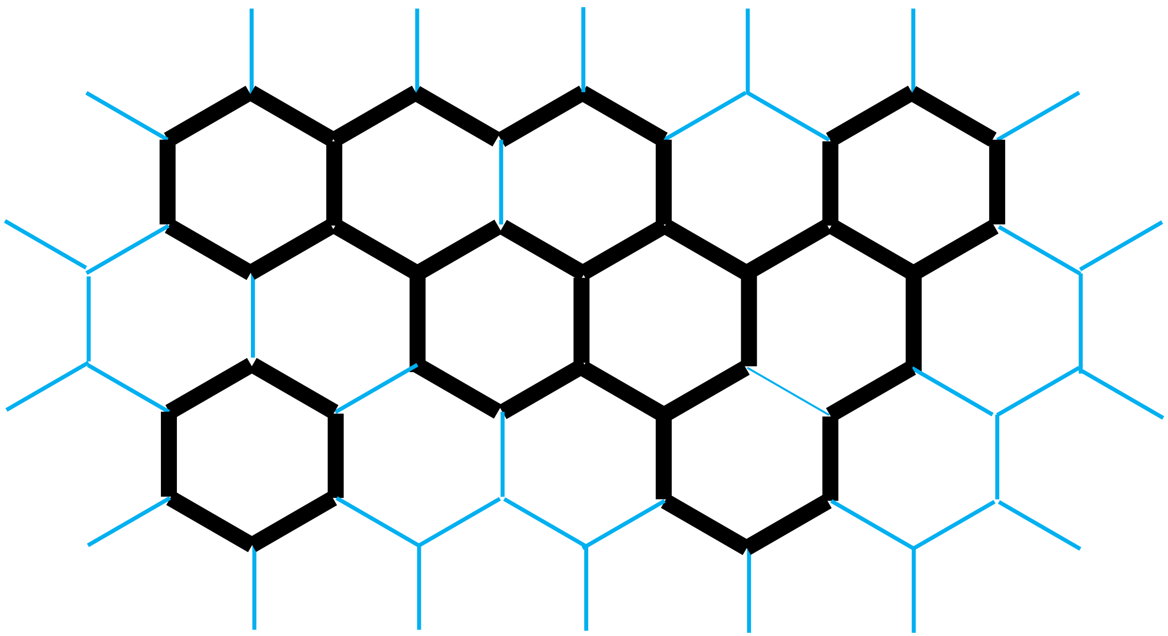
“Fibonacci” Levin-Wen Model

$$\delta_{100} = \delta_{010} = \delta_{001} = 0$$

All other $\delta_{ijk} = 1$

Vertex Operator: Q_v

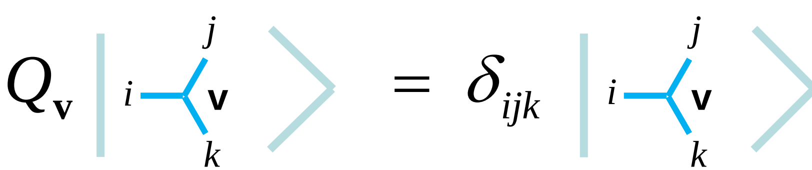
$$H = -\sum_v Q_v - \sum_p B_p$$



$= |0\rangle$

$= |1\rangle$

$Q_v = 1$
on each vertex
→ "branching" loop states.



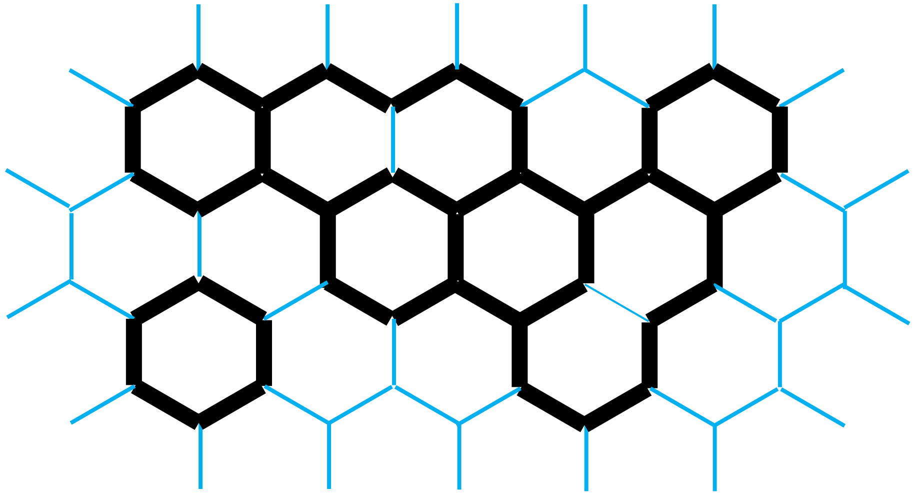
"Fibonacci" Levin-Wen Model

$$\delta_{100} = \delta_{010} = \delta_{001} = 0$$

All other $\delta_{ijk} = 1$

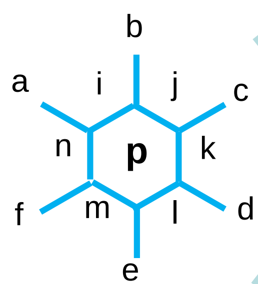
Plaquette Operator: B_{p}

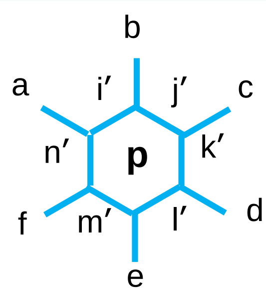
$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$



$B_{\mathrm{p}} = 1$
on each plaquette
➡ superposition
of loop states

$$B^{s}_{\mathbf{p}} \left| \right\rangle = \sum_{i'j'k'l'm'n'} B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef) \left| \right\rangle$$





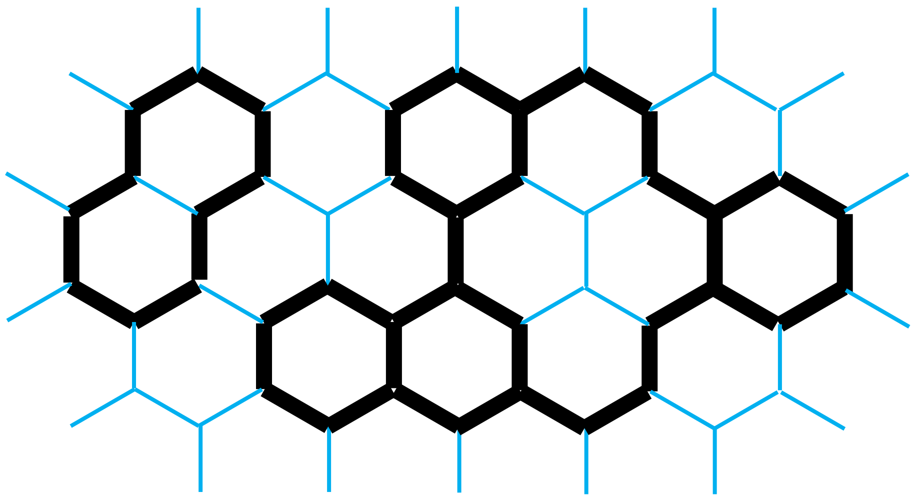
$$B_{\mathbf{p}} = \frac{B^{0}_{\mathbf{p}} + \varphi B^{1}_{\mathbf{p}}}{1+\varphi^{2}}$$

$$B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef) = F^{ani}_{si'n'} F^{bij}_{sj'i'} F^{cjk}_{sk'j'} F^{dkl}_{sl'k'} F^{elm}_{sm'l'} F^{fmn}_{sn'm'}$$

Very Complicated 12-qubit Interaction!

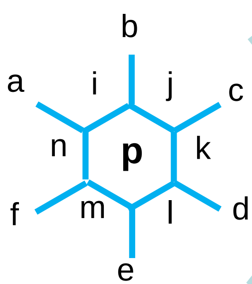
Plaquette Operator: B_p

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$

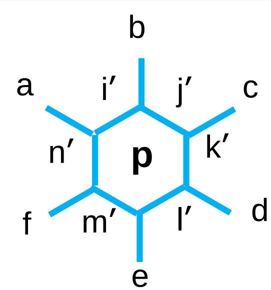


$B_\mathrm{p} = 1$
on each plaquette
➡ superposition
of loop states

$$B^{s}_{\mathbf{p}} \Big|$$



$$\Big\rangle = \sum_{i'j'k'l'm'n'} B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef) \Big|$$



$$\Big\rangle$$

$$B_{\mathbf{p}} = \frac{B^0_{\mathbf{p}} + \varphi B^1_{\mathbf{p}}}{1+\varphi^2}$$

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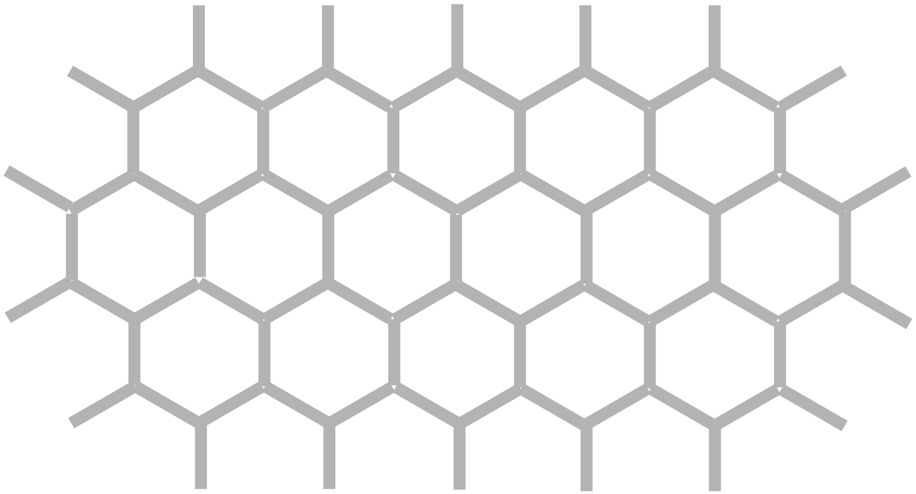
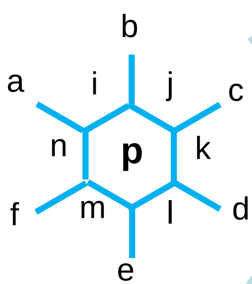
Plaquette Operator: B_{p}

$$H = -\sum_{\mathrm{v}} Q_{\mathrm{v}} - \sum_{\mathrm{p}} B_{\mathrm{p}}$$



$B_{\mathrm{p}} = 1$
on each plaquette
➔ superposition
of loop states

$B^{s}_{\mathbf{p}} \Big|$


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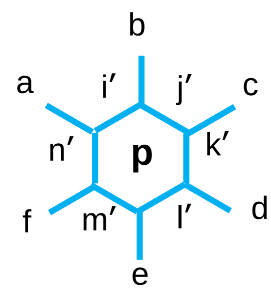


$\Big\rangle$

$$B_{\mathbf{p}} = \frac{B^0_{\mathbf{p}} + \varphi B^1_{\mathbf{p}}}{1+\varphi^2}$$

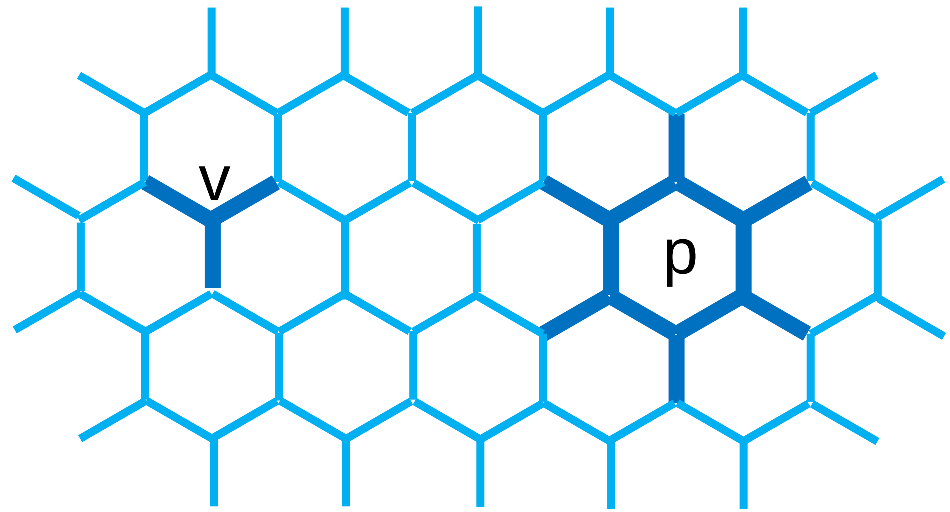
$$B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef) = F^{ani}_{si'n'} F^{bij}_{sj'i'} F^{cjk}_{sk'j'} F^{dkl}_{sl'k'} F^{elm}_{sm'l'} F^{fmn}_{sn'm'}$$

Very Complicated 12-qubit Interaction!

"Surface Code" Approach

Konig, Kuperberg, Reichardt, Ann. Phys. 2010

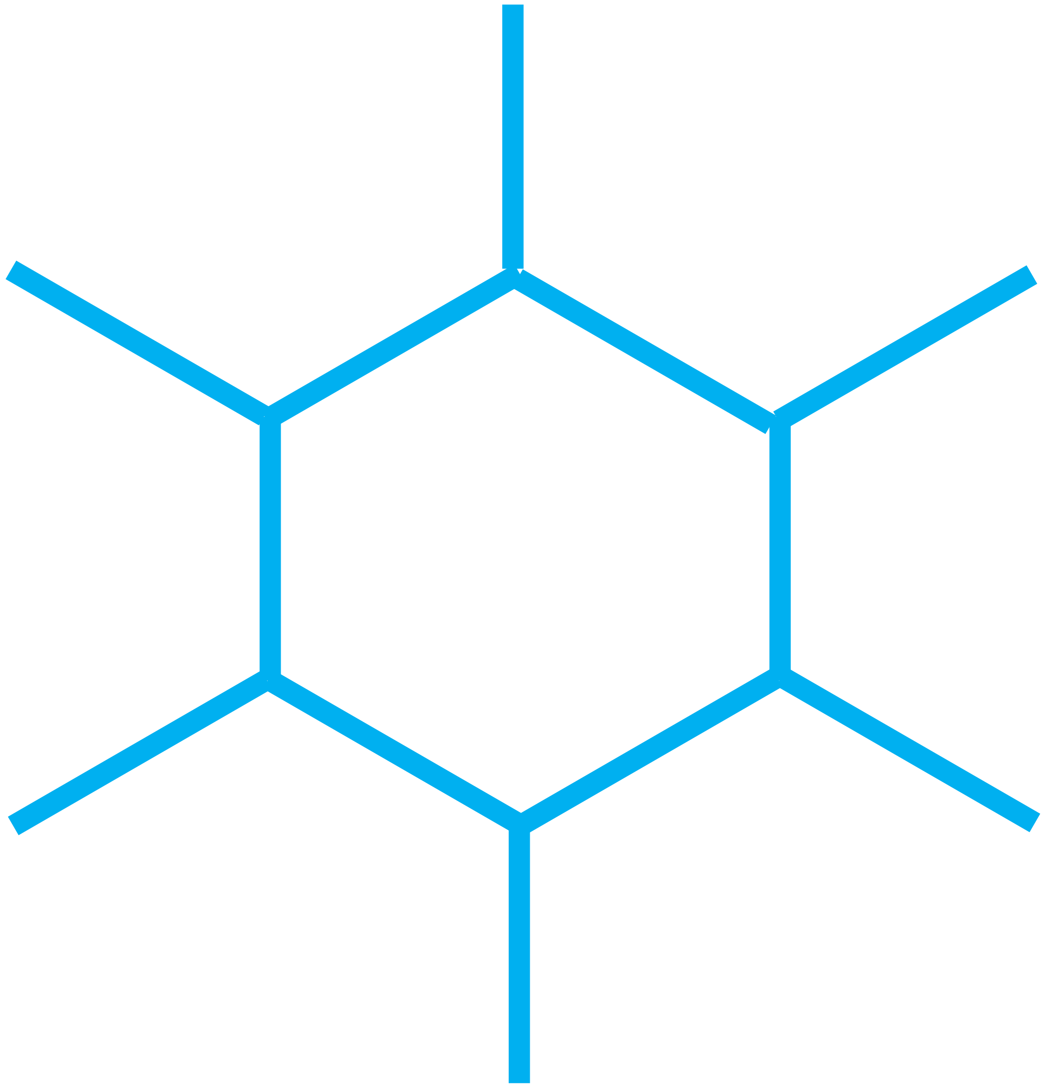
Use quantum computer to repeatedly measure Q_v and B_p.



If Q_v=1 on each vertex and B_p=1 on each plaquette, good. If not, treat as an "error."

How hard is it to measure Q_v and B_p with a quantum computer?

6-sided Plaquette: B_p is a 12-qubit Operator



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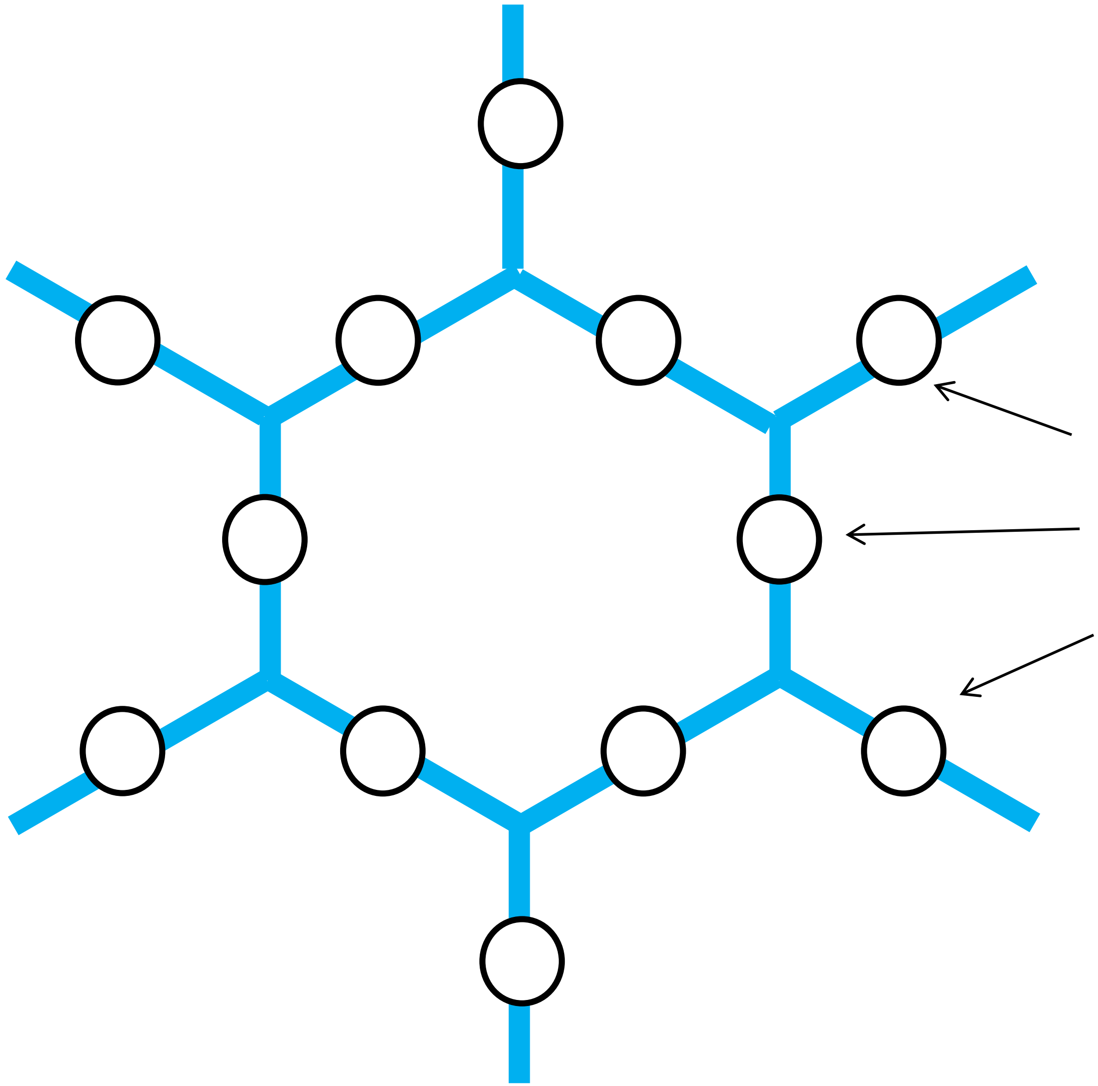
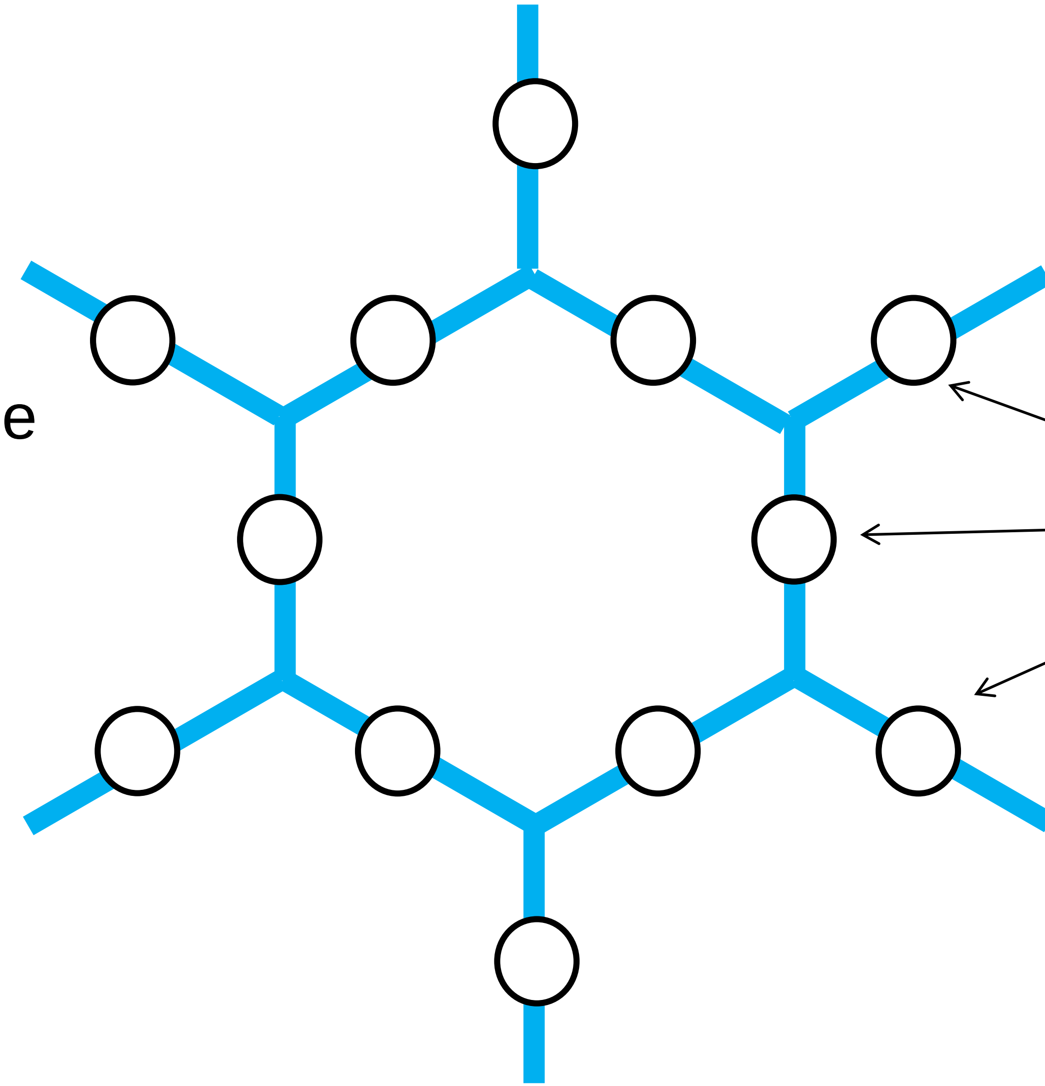


Physical qubits are fixed in space

6-sided Plaquette: B_p is a 12-qubit Operator

Trivalent lattice is abstract and can be "redrawn"



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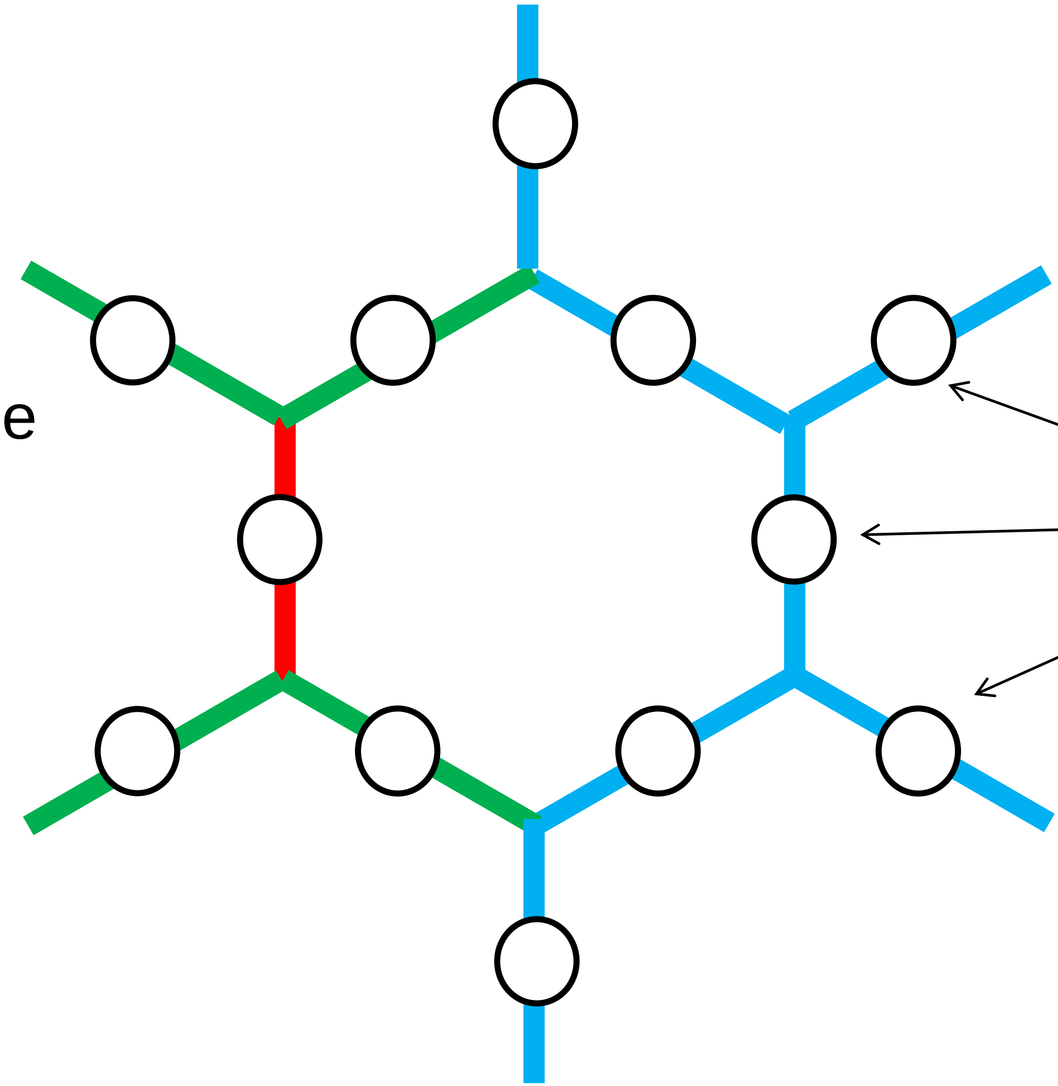

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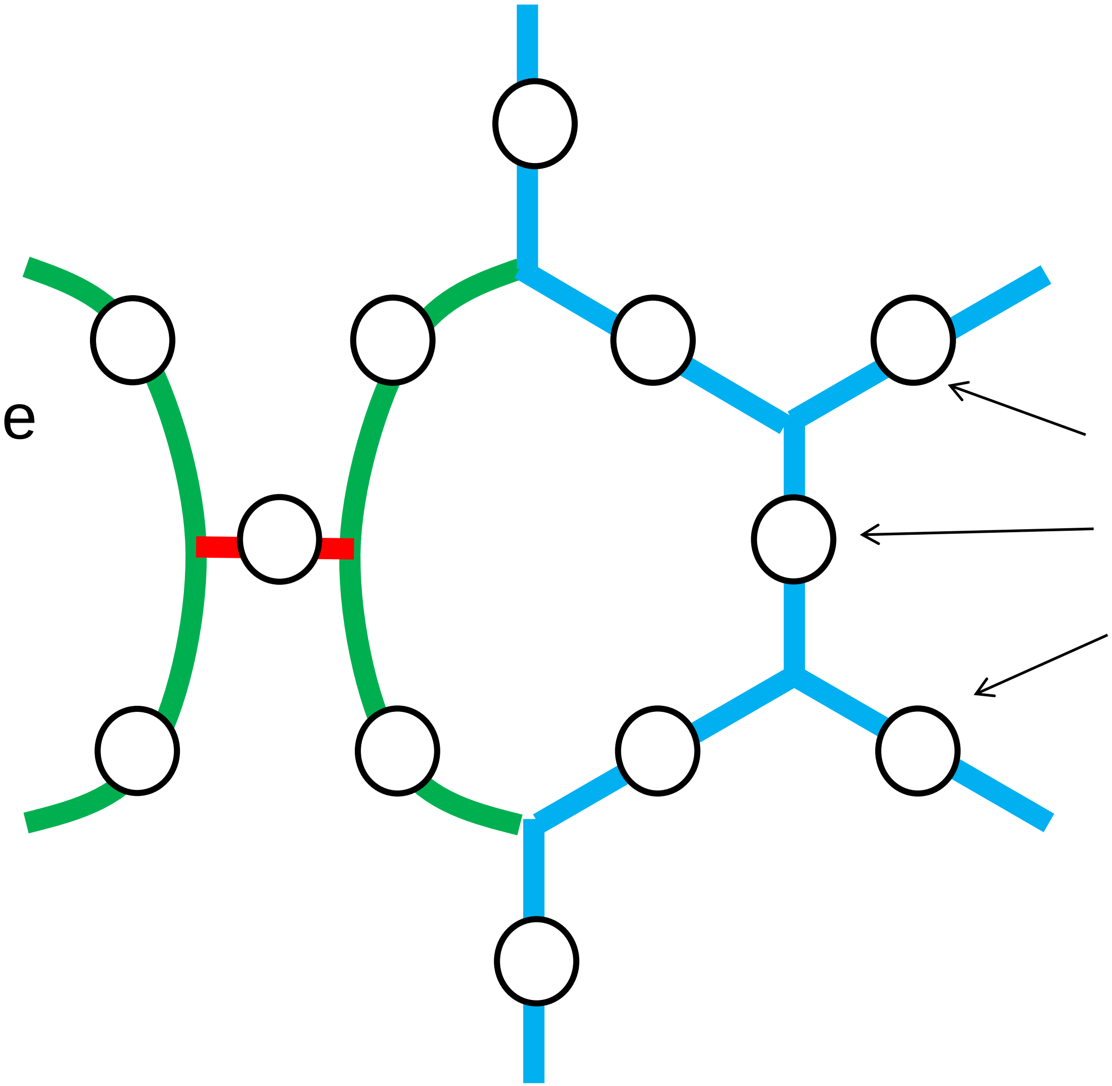

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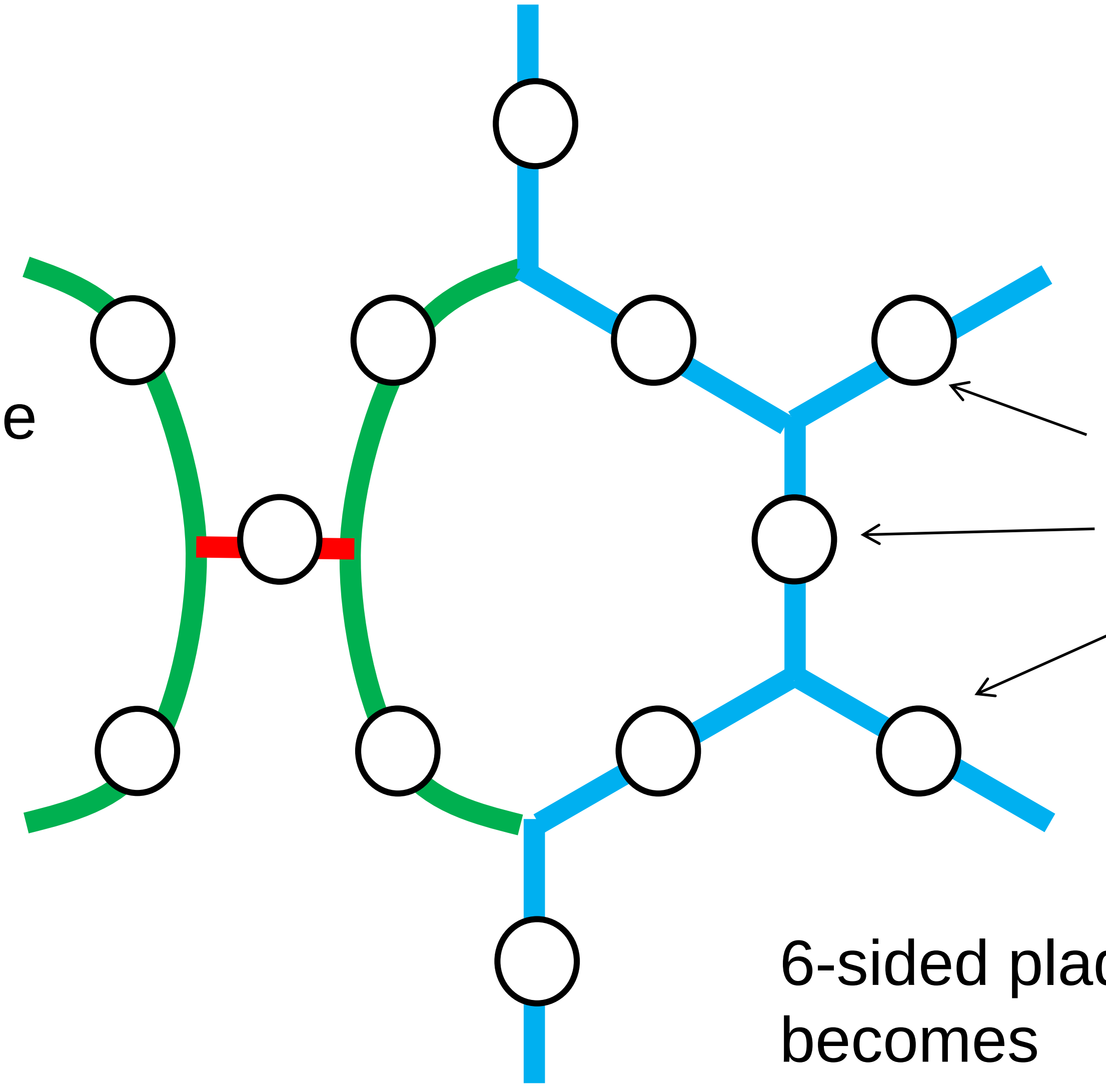
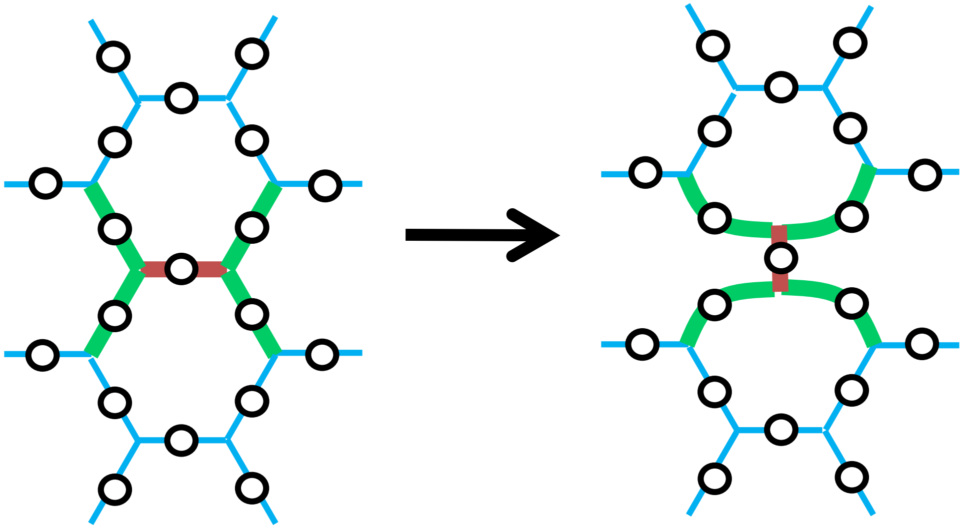


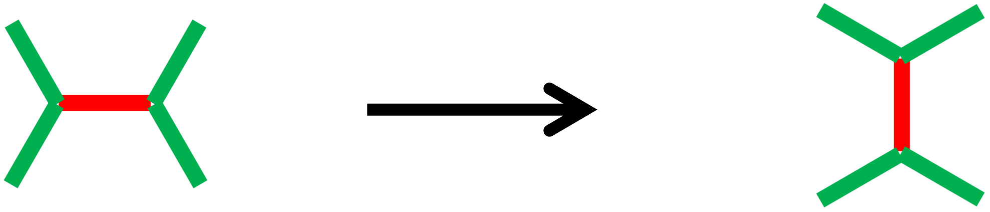
Physical qubits are fixed in space

6-sided plaquette becomes 5-sided plaquette!

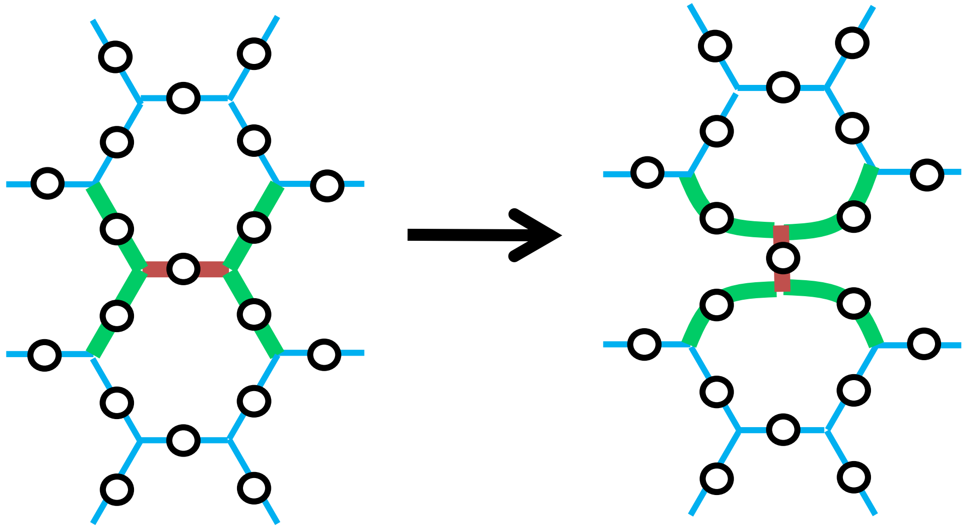
The F–Move



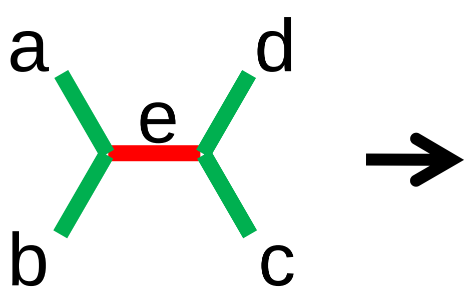
Locally redraw the lattice five qubits at a time.



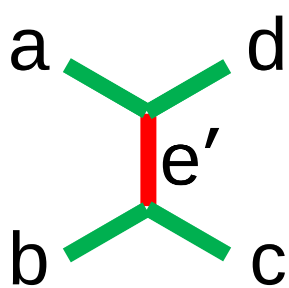
The F–Move



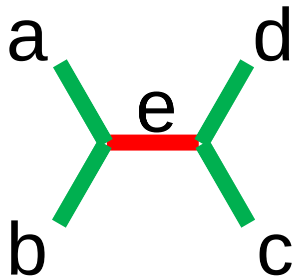
Apply a unitary operation on these five qubits to stay in the Levin-Wen ground state.



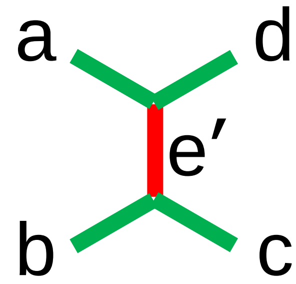
$$\sum_{e'} F^{abe}_{cde'}$$

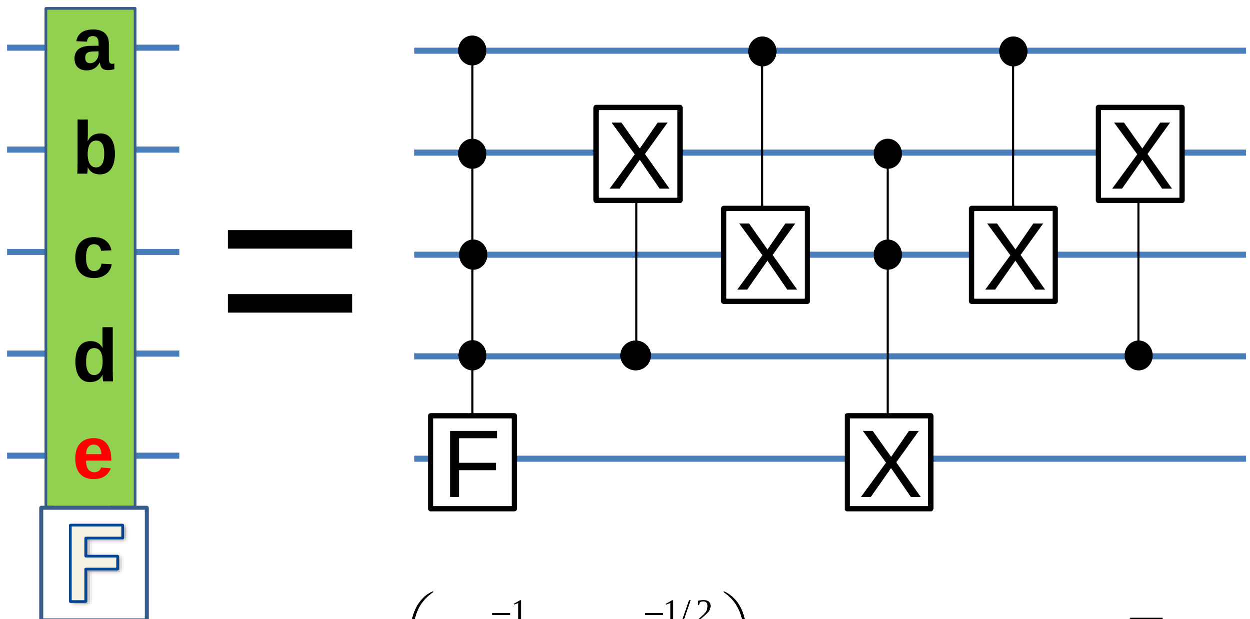


F Quantum Circuit



$$\rightarrow \sum_{e'} F^{abe}_{cde'}$$

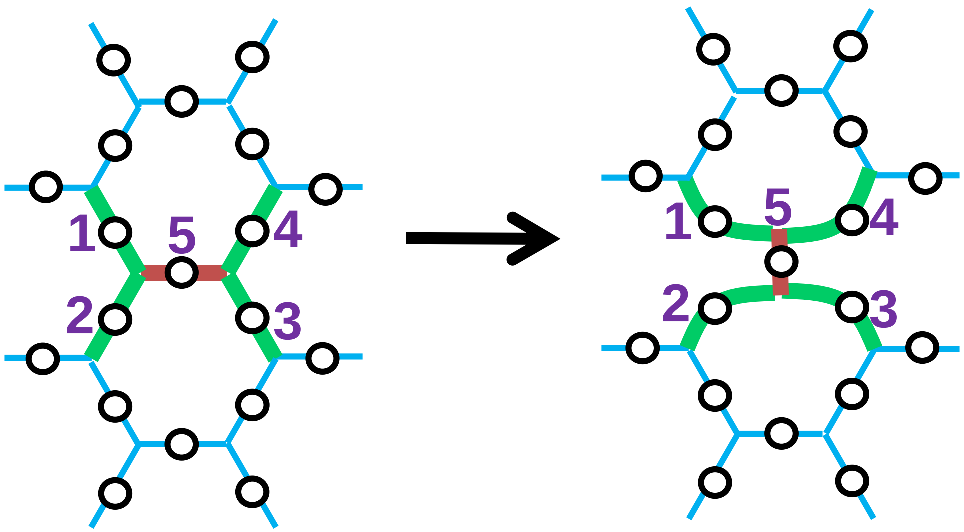


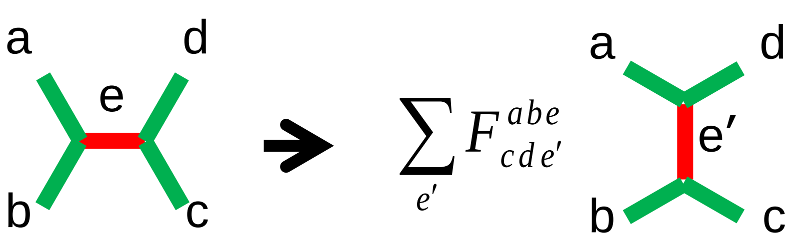


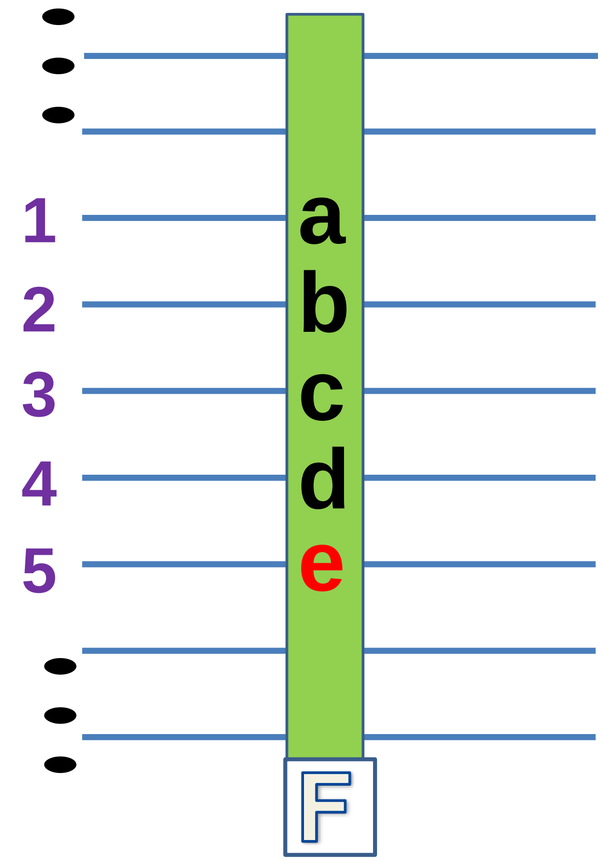
$$F = \begin{pmatrix} \varphi^{-1} & \varphi^{-1/2} \\ \varphi^{-1/2} & -\varphi^{-1} \end{pmatrix} \qquad \varphi = \frac{\sqrt{5}+1}{2}$$

NEB, D.P. DiVincenzo, PRB 2012

F Quantum Circuit







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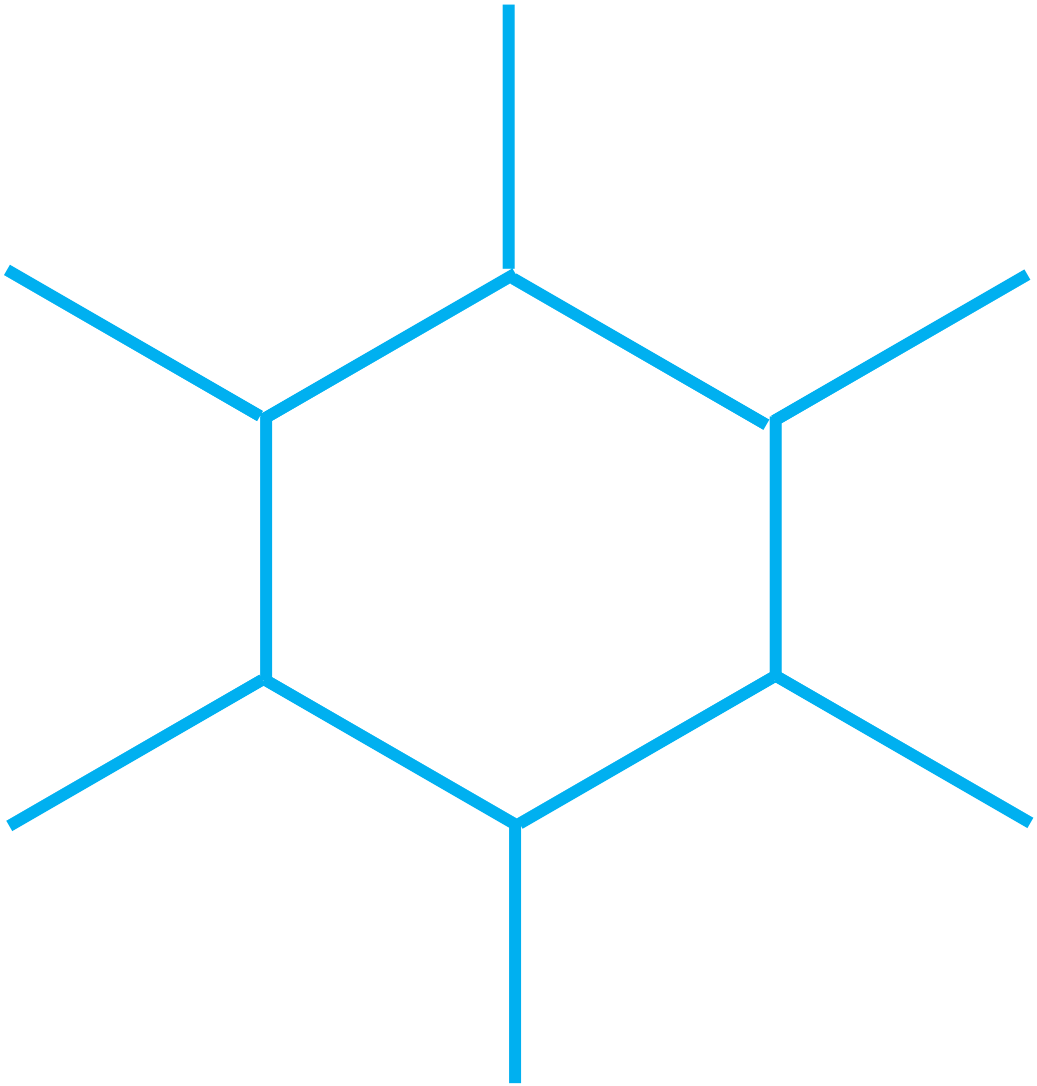
Plaquette Reduction



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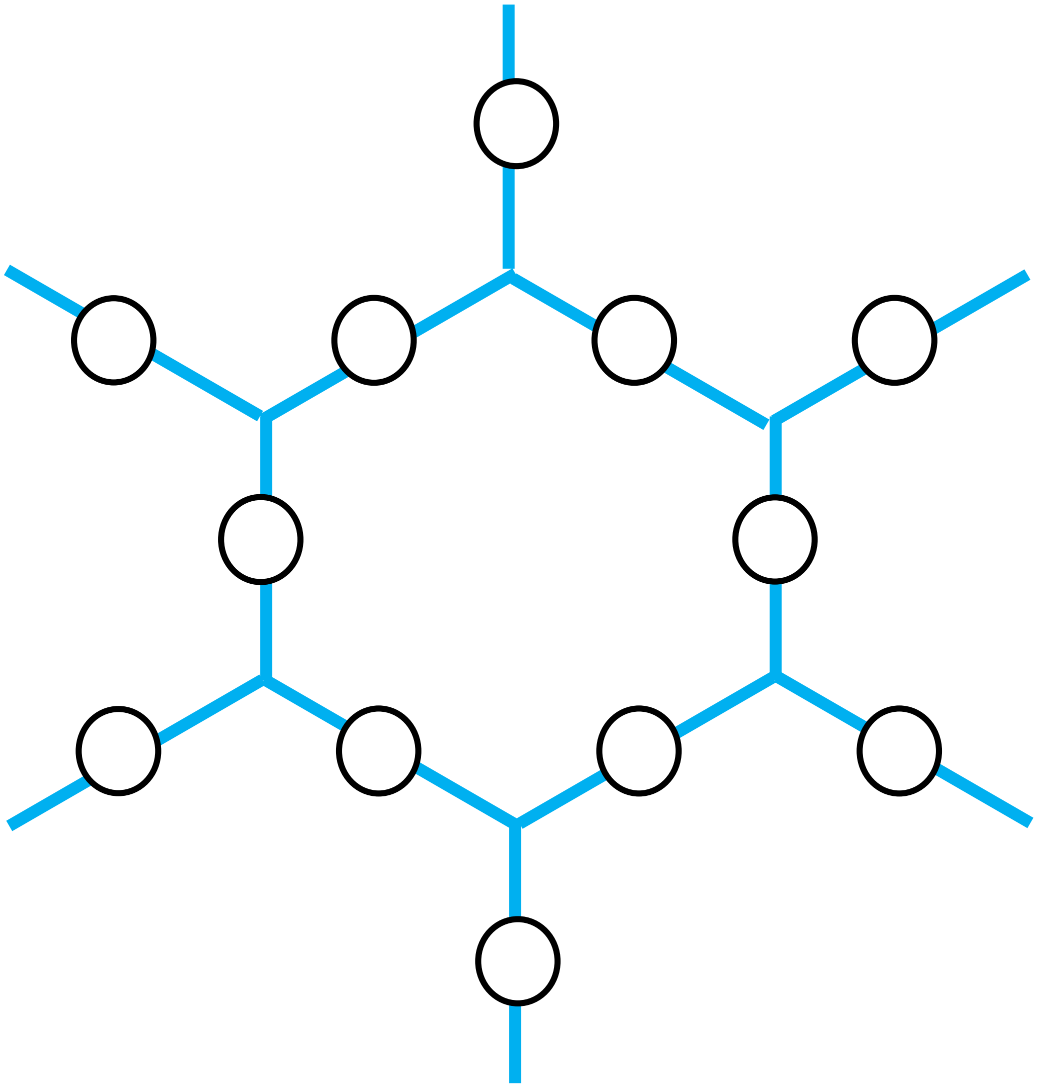

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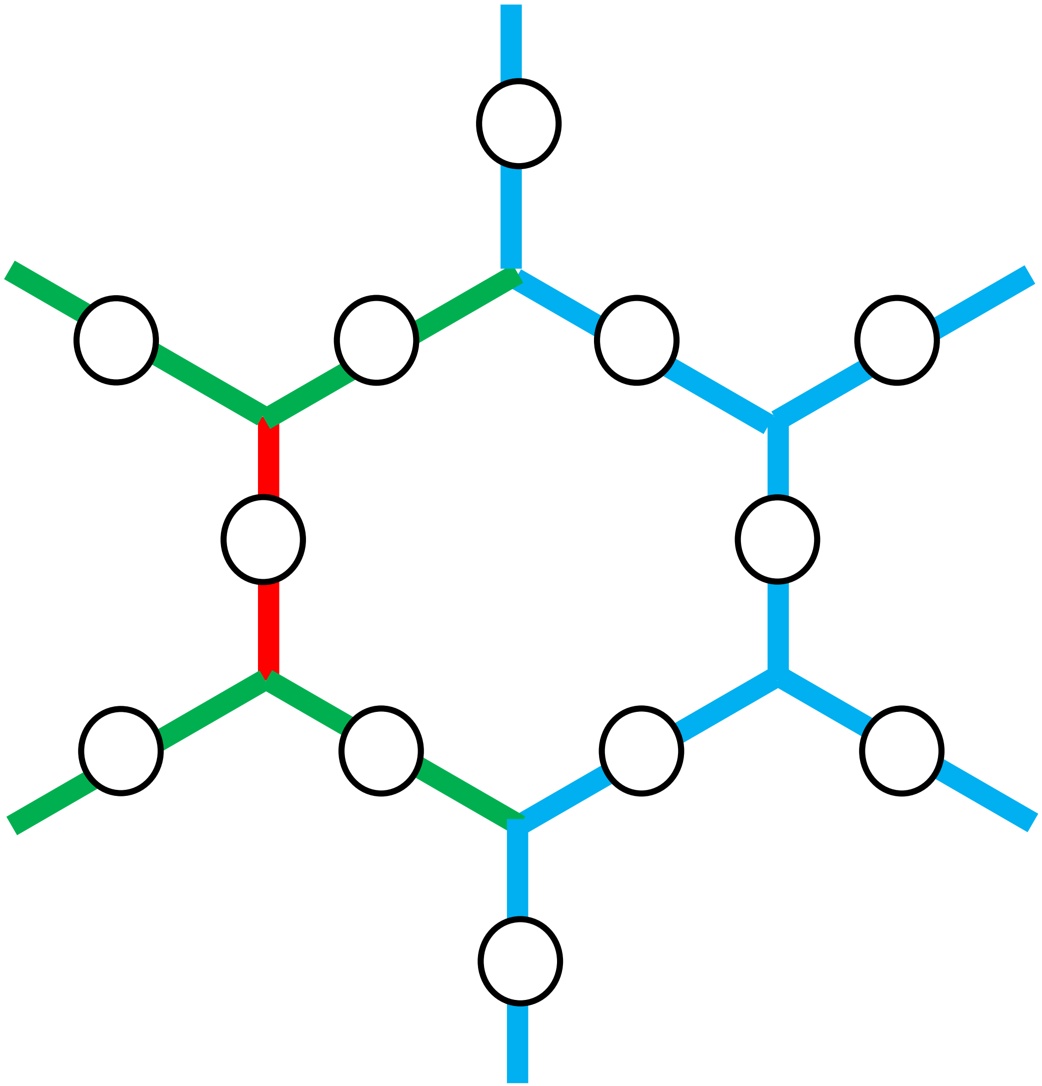

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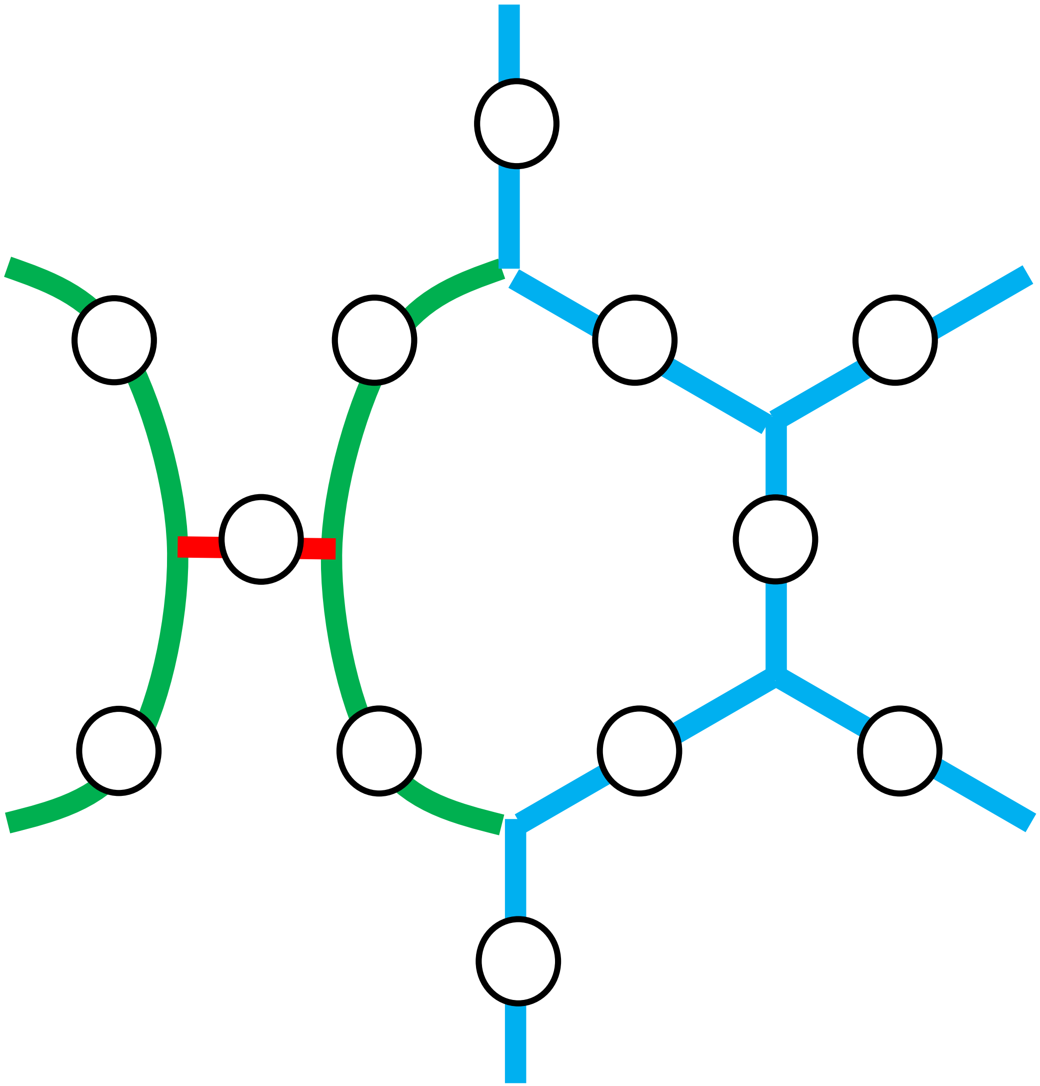

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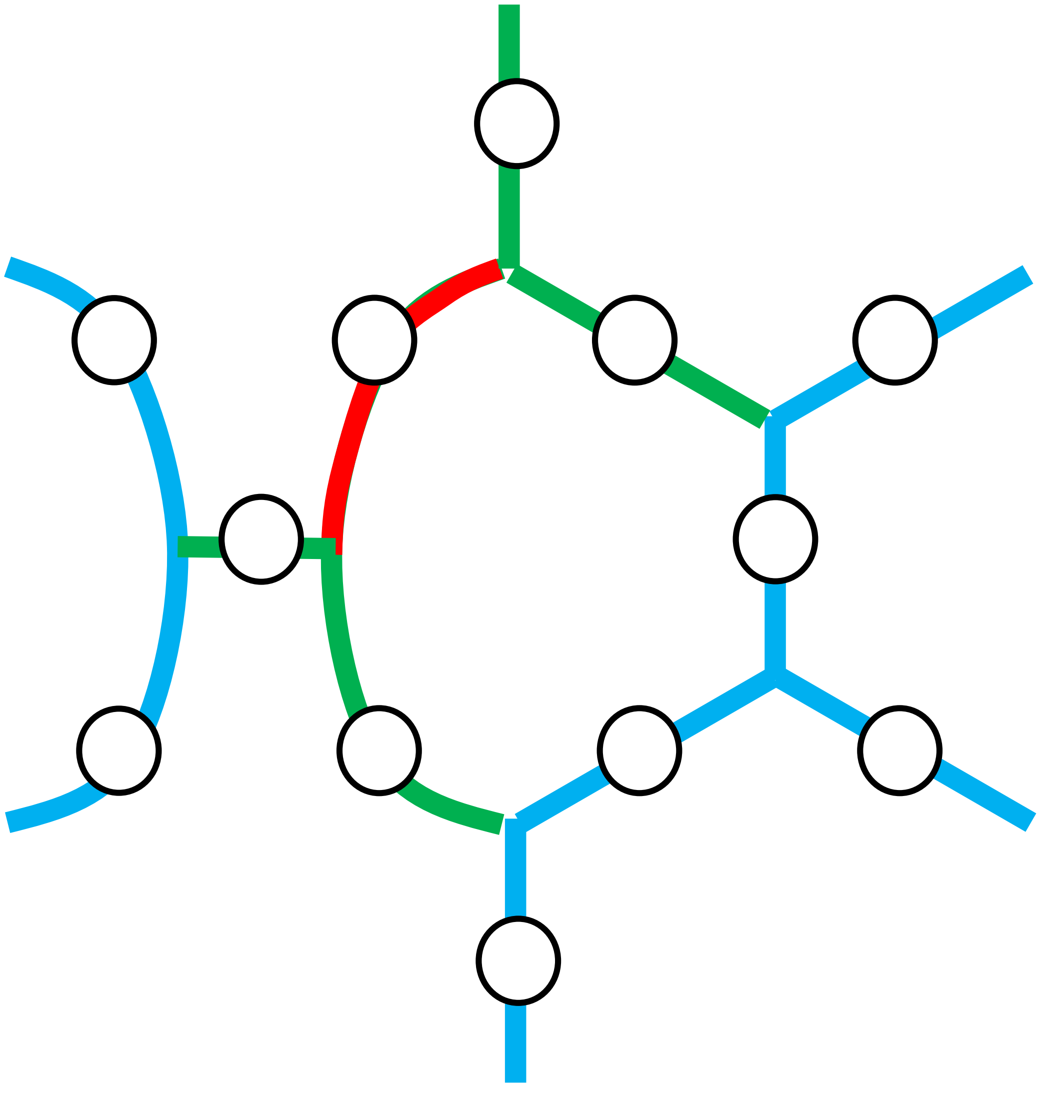

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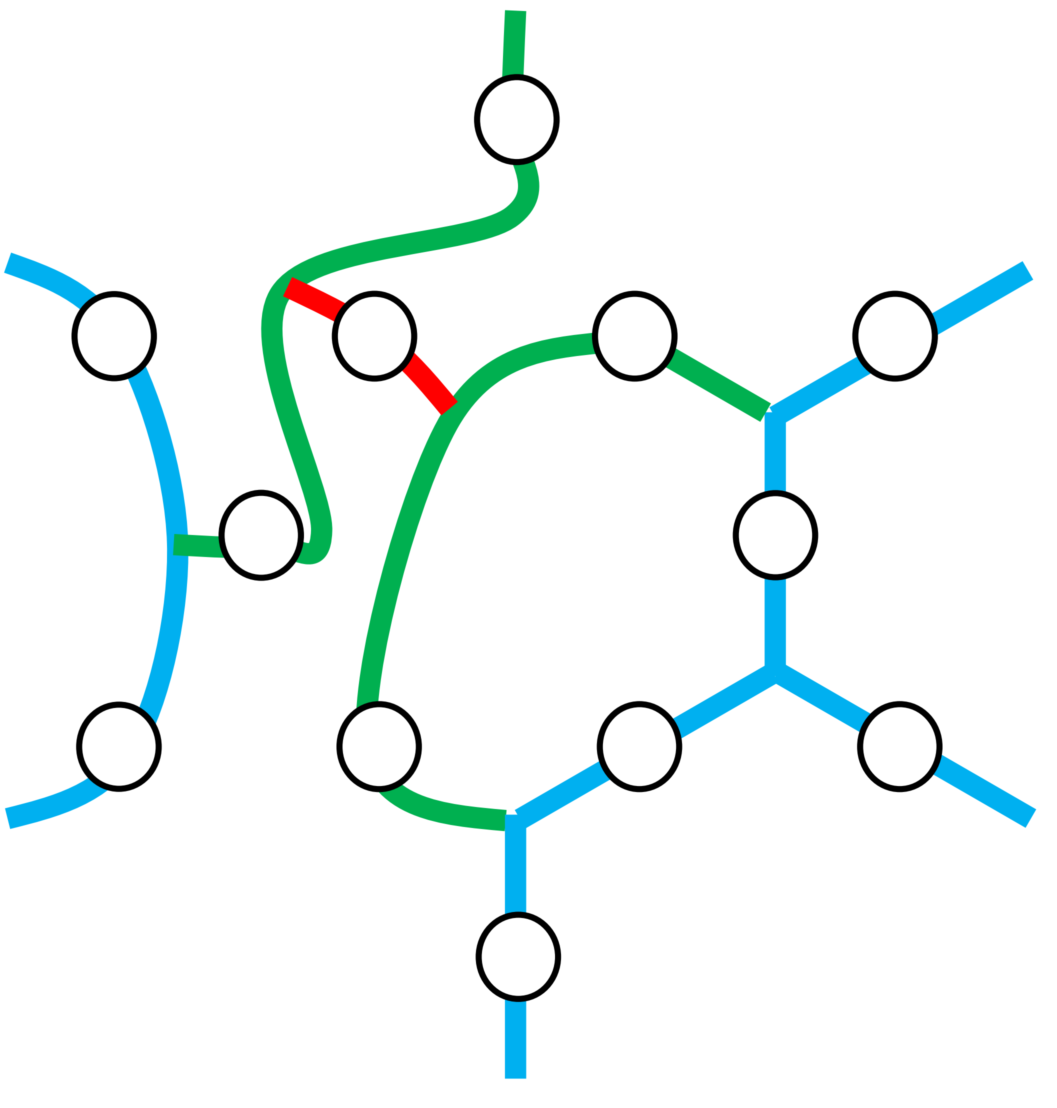

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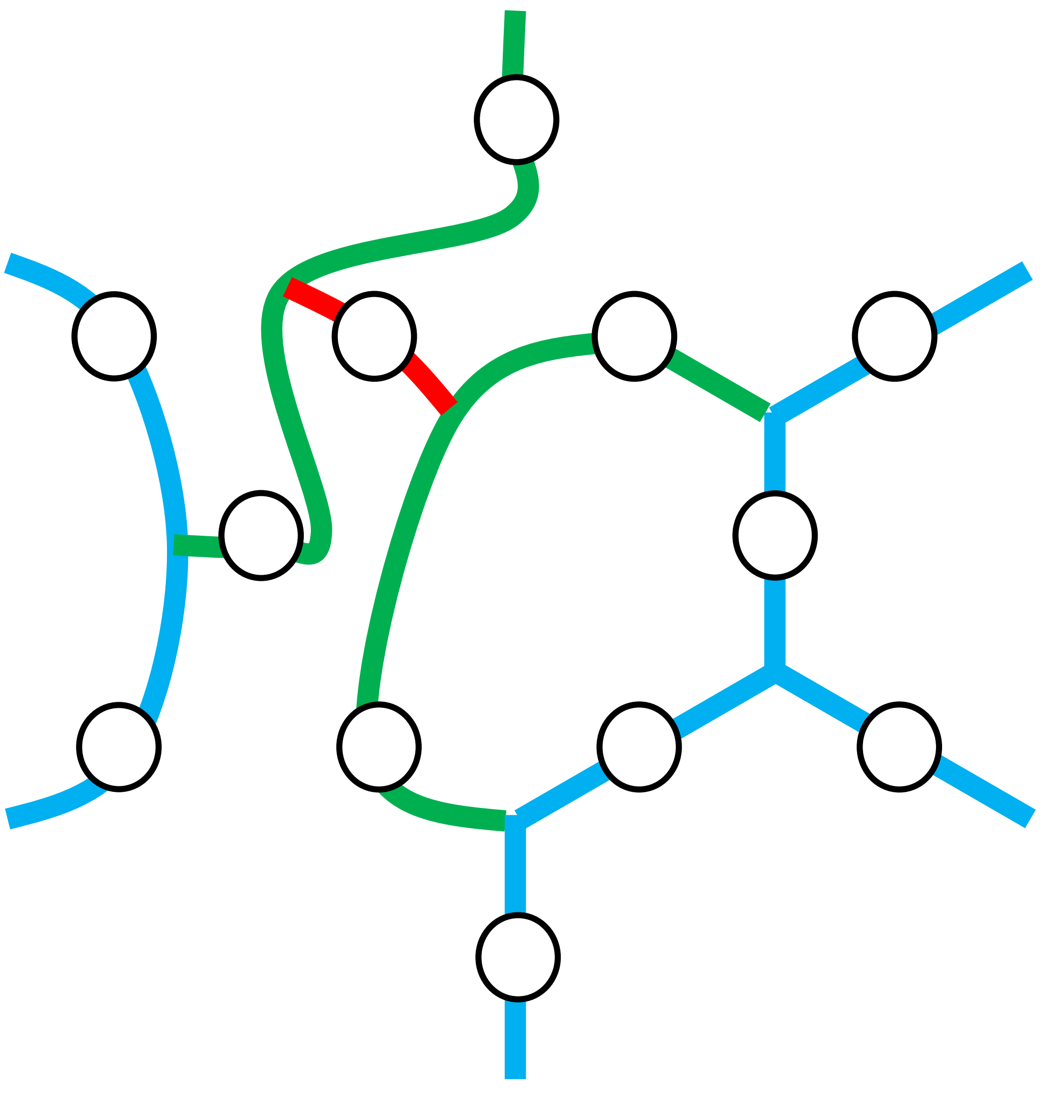

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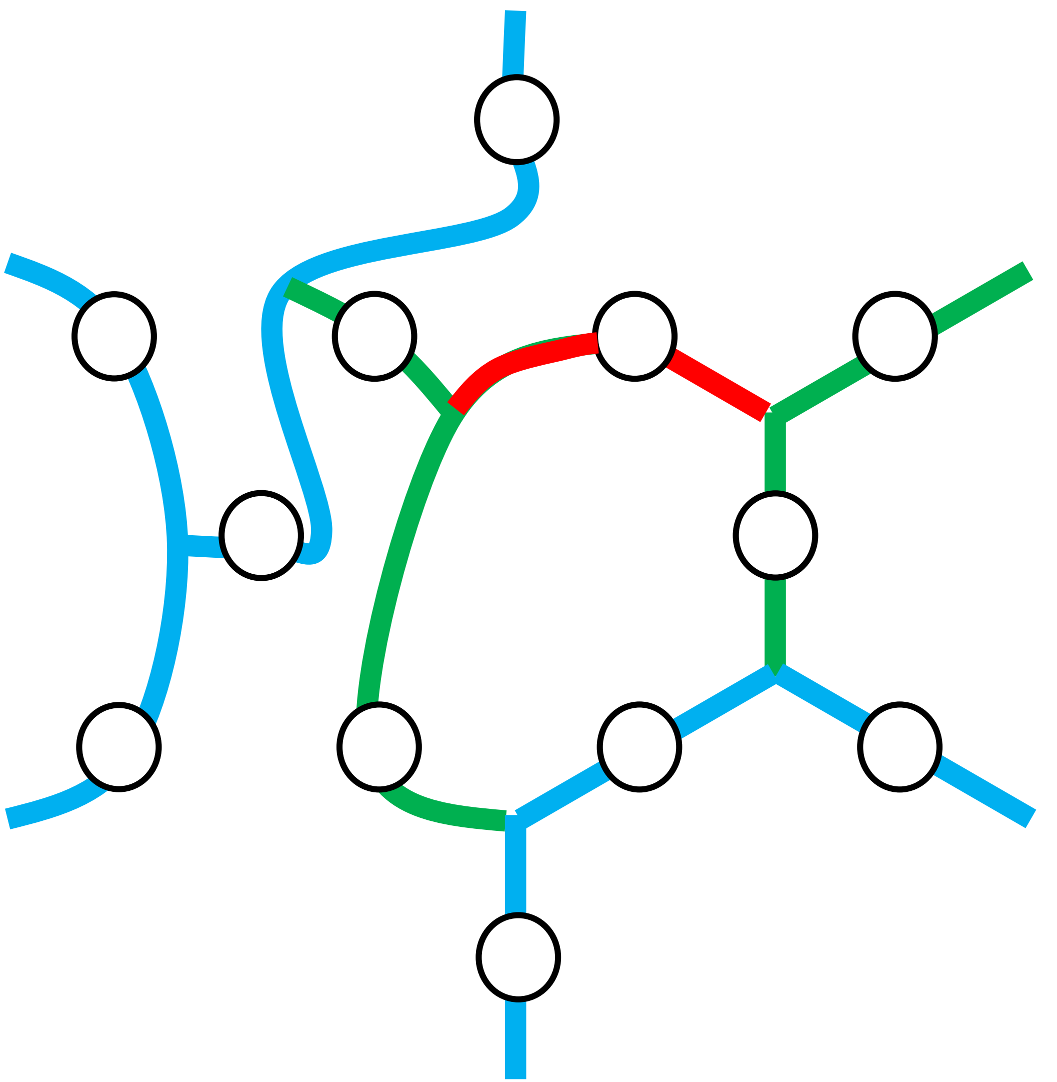

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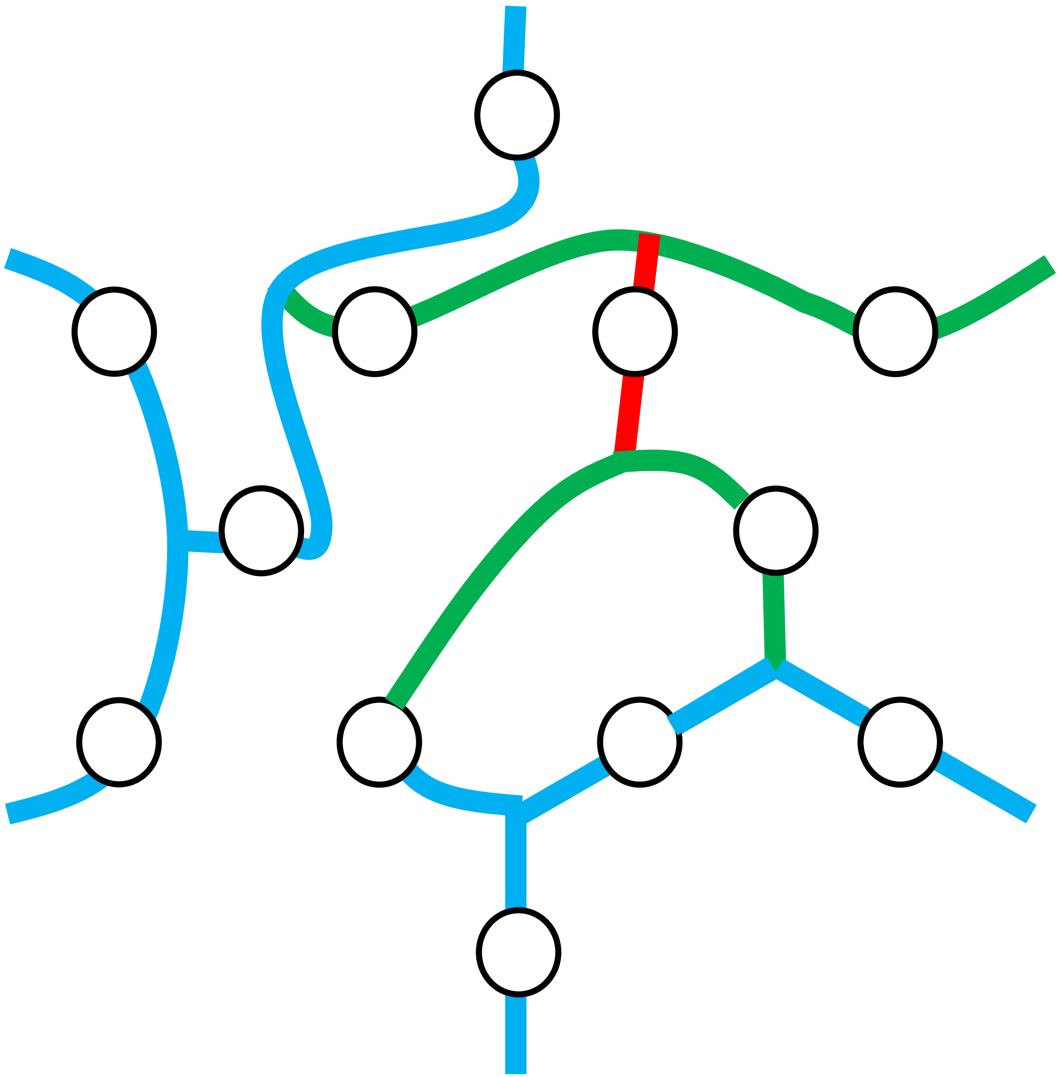

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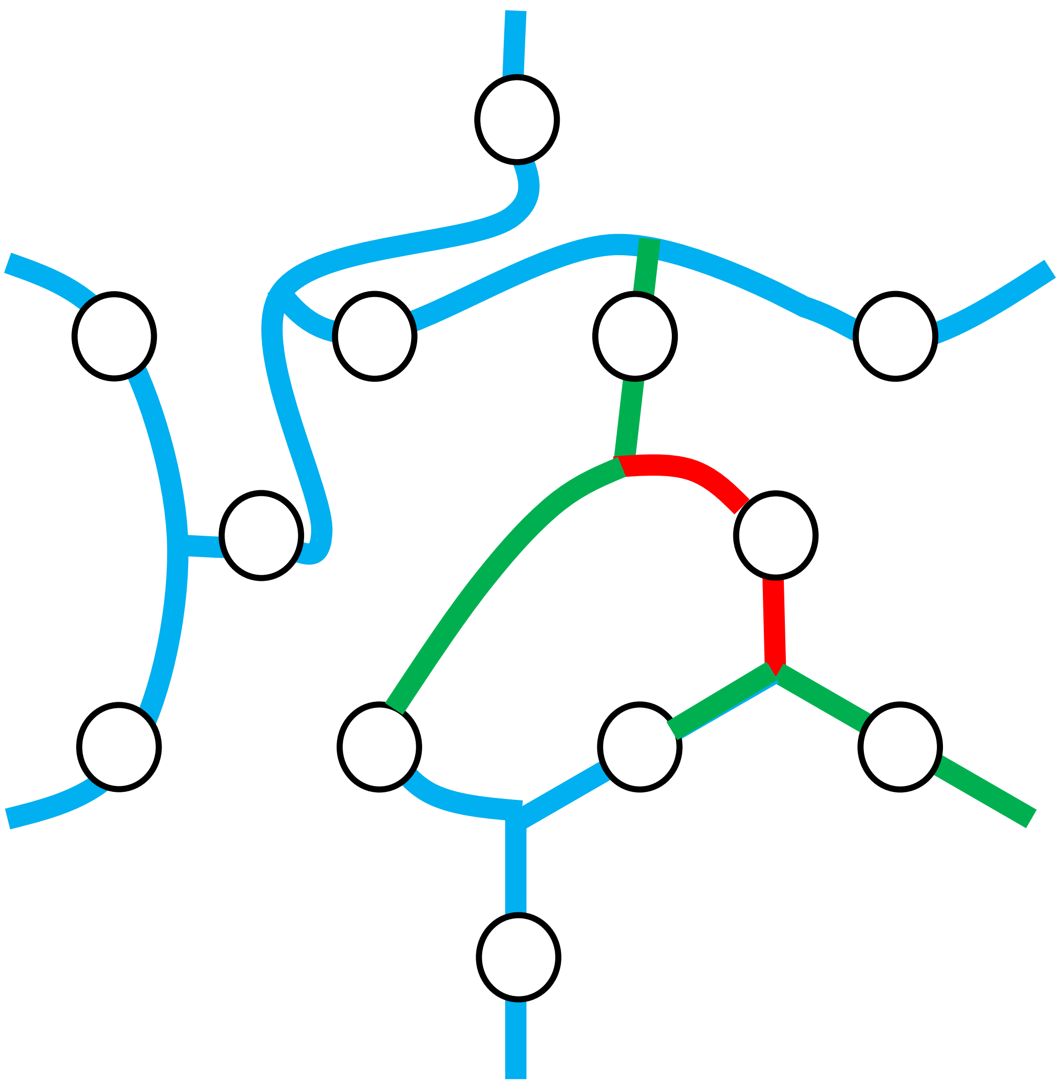

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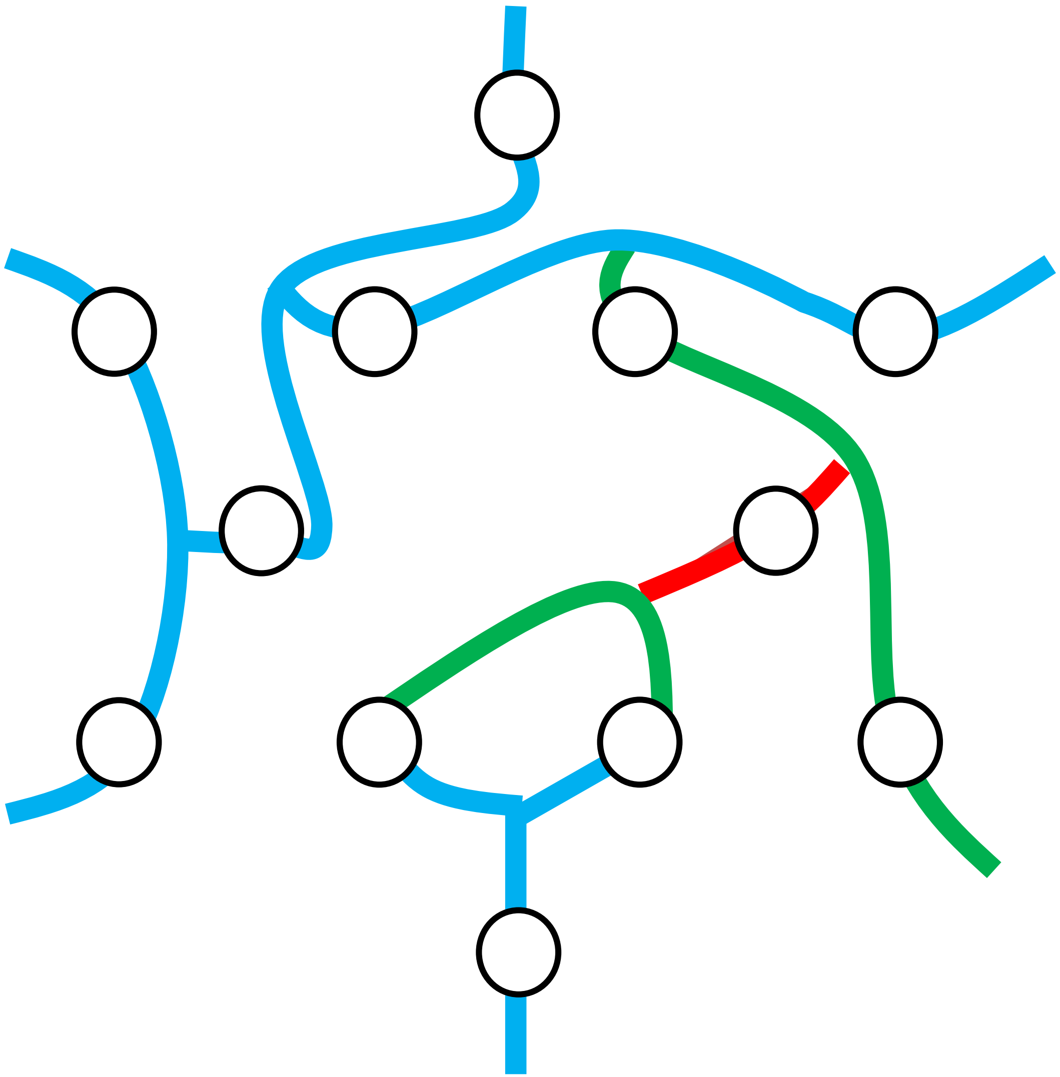

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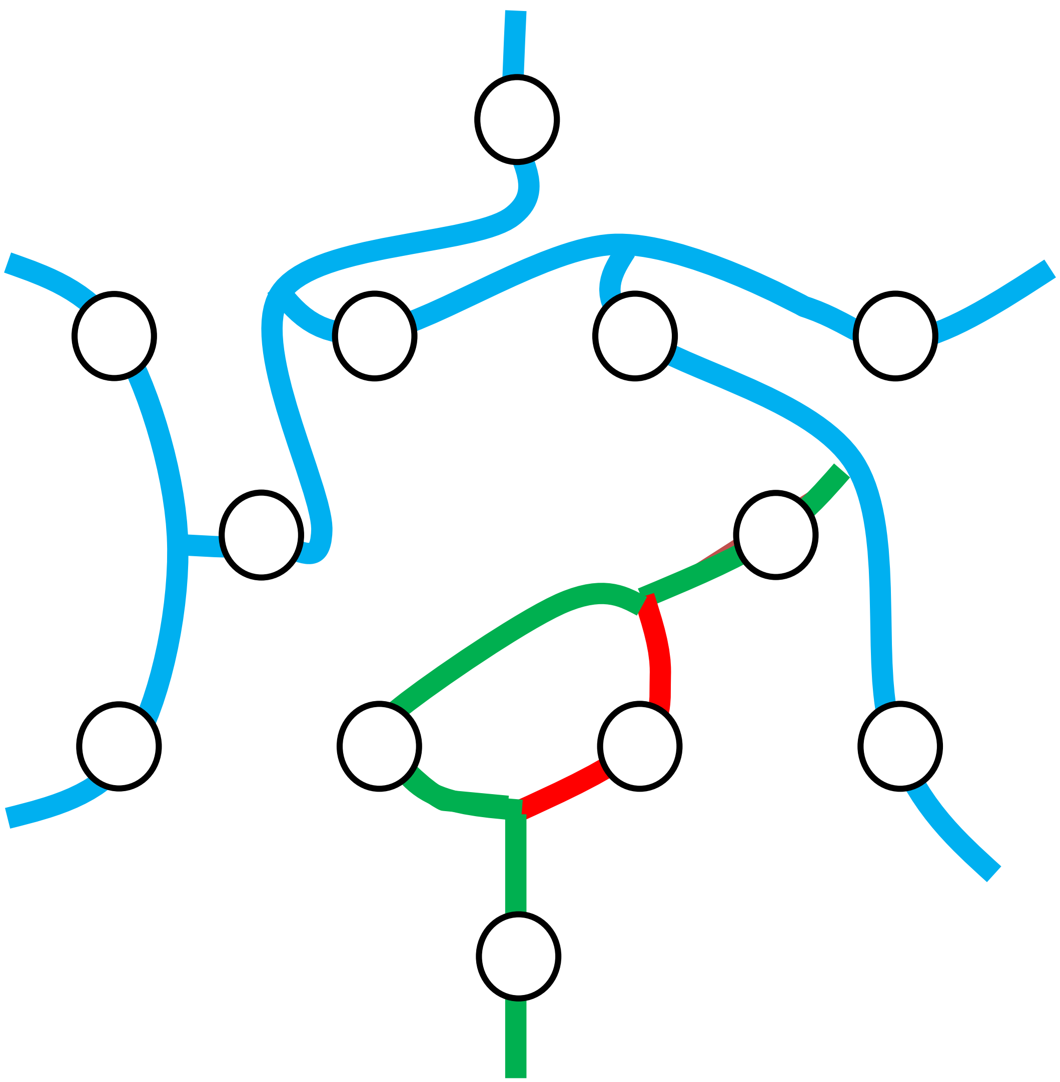

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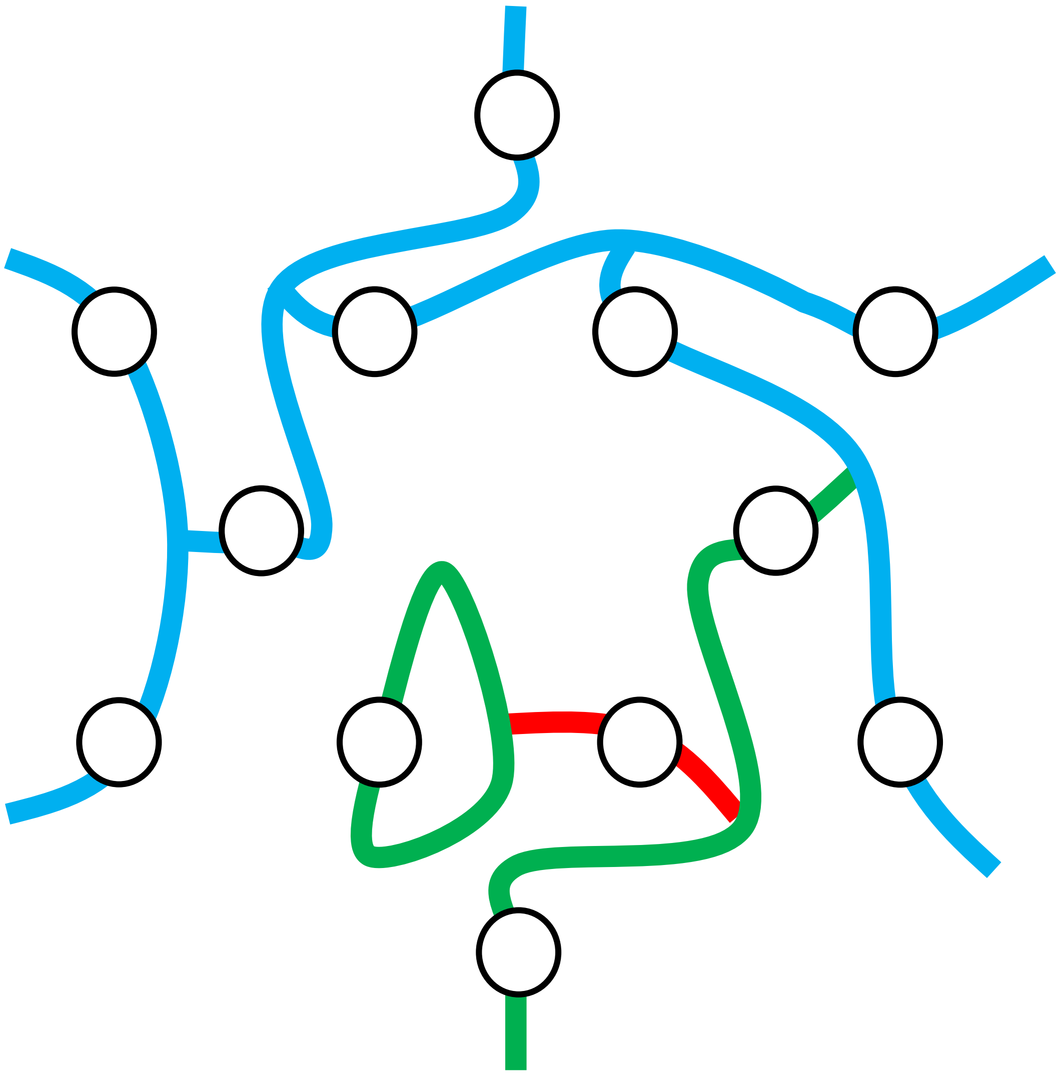

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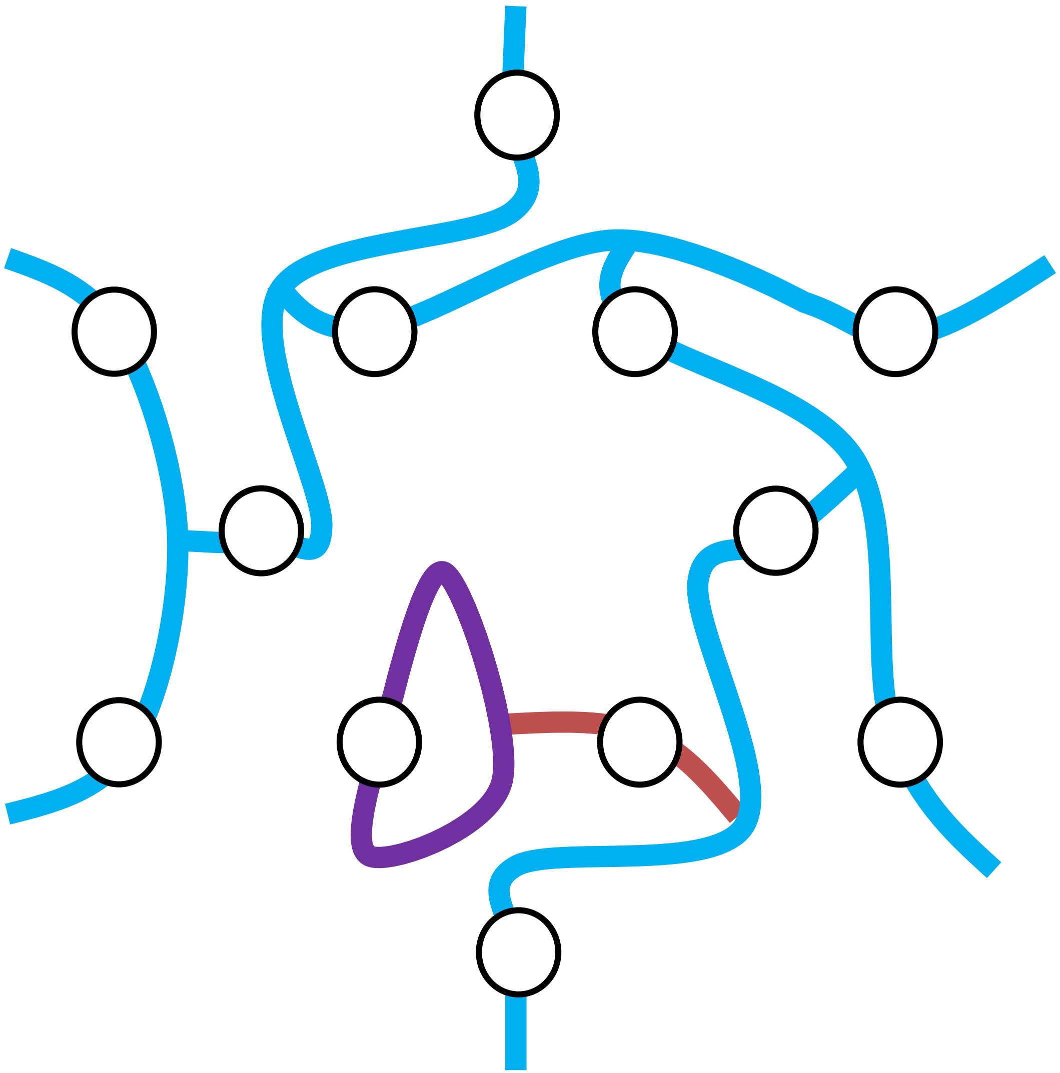

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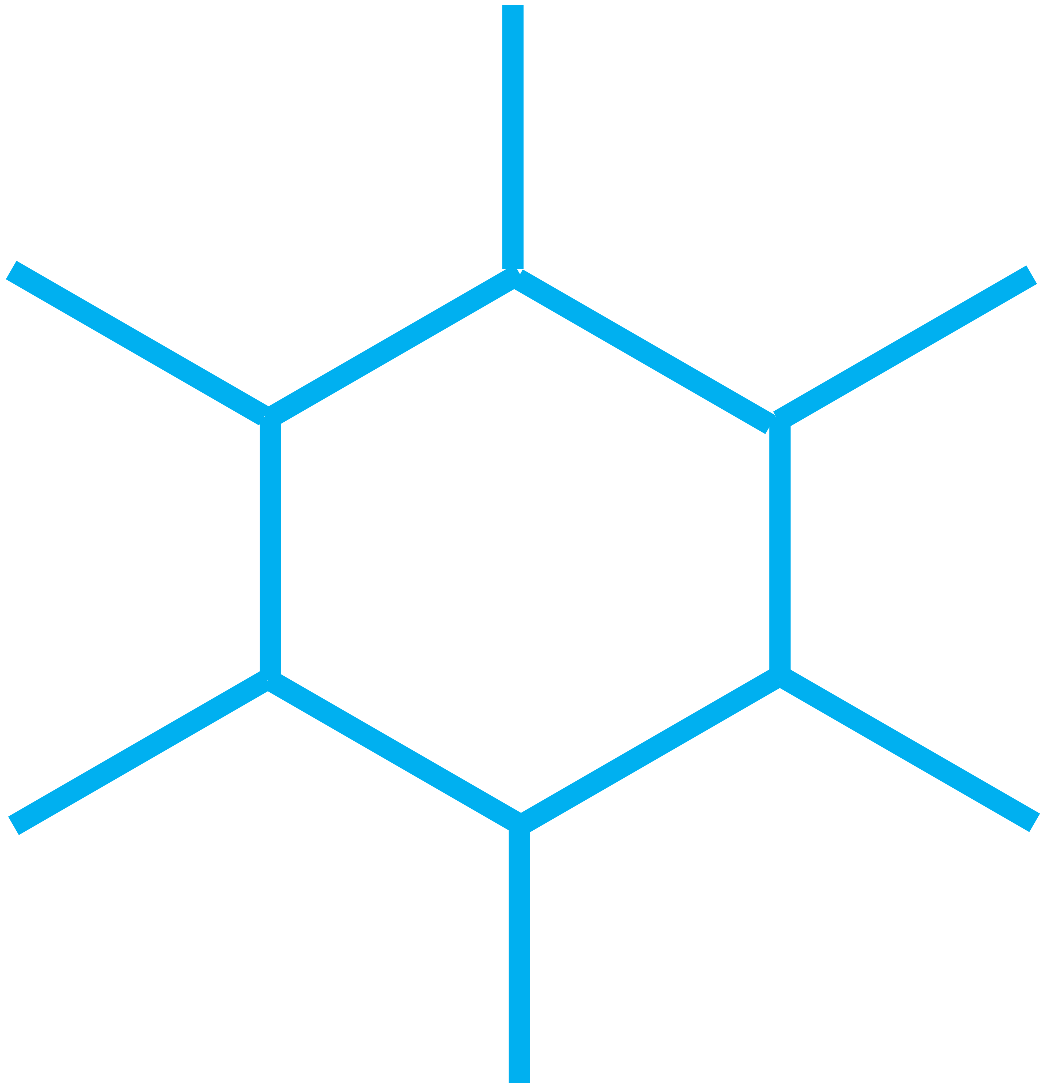

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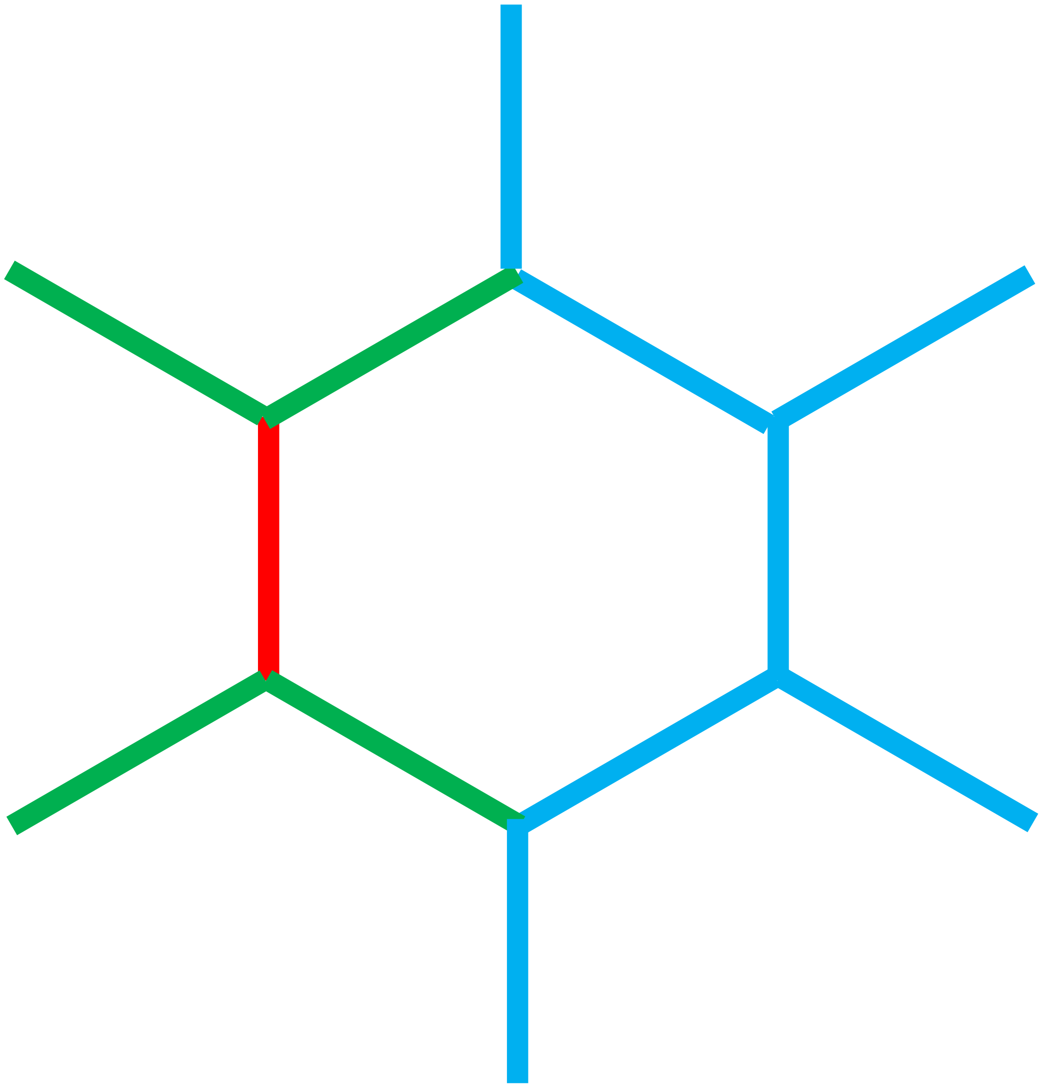

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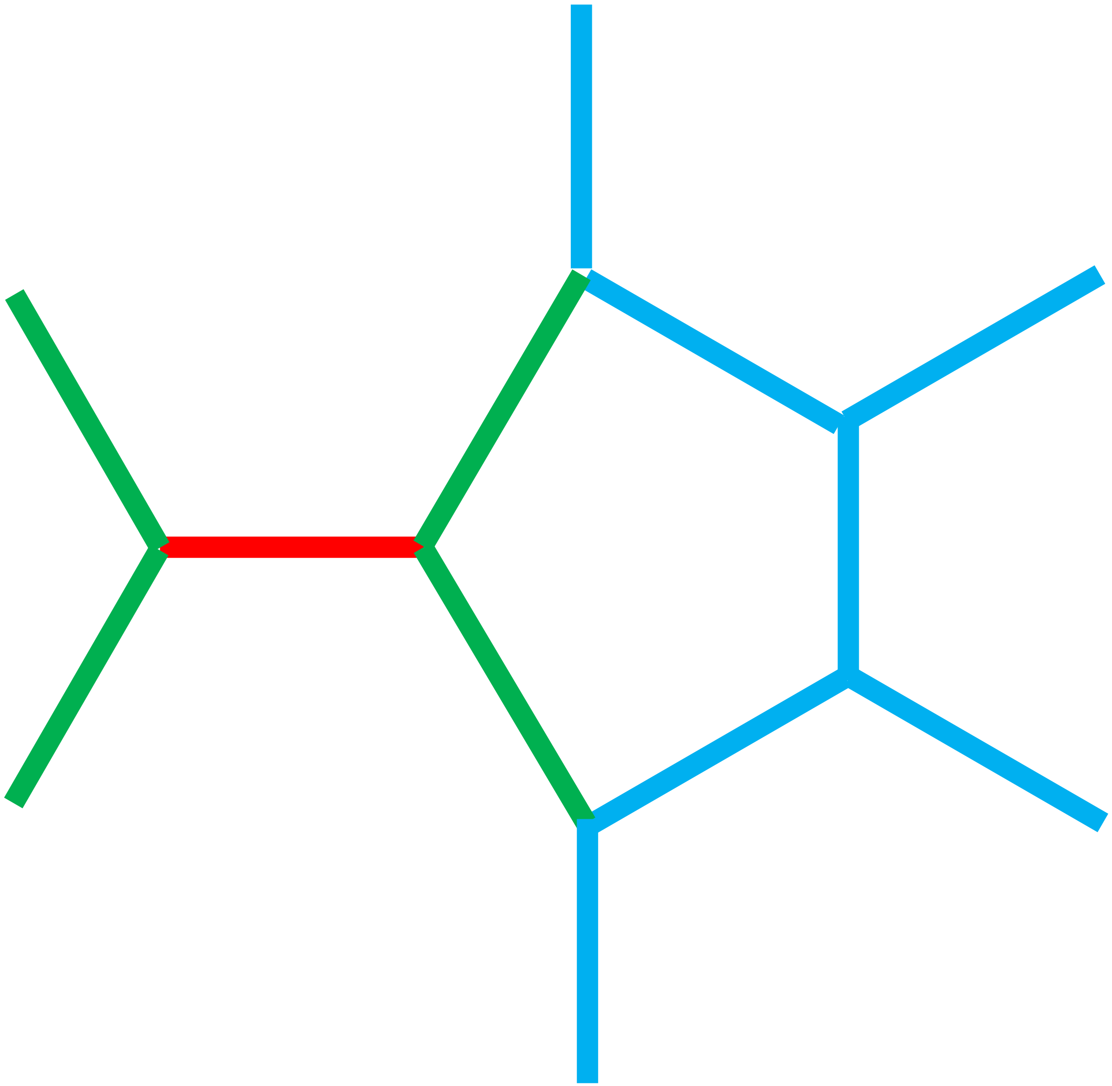

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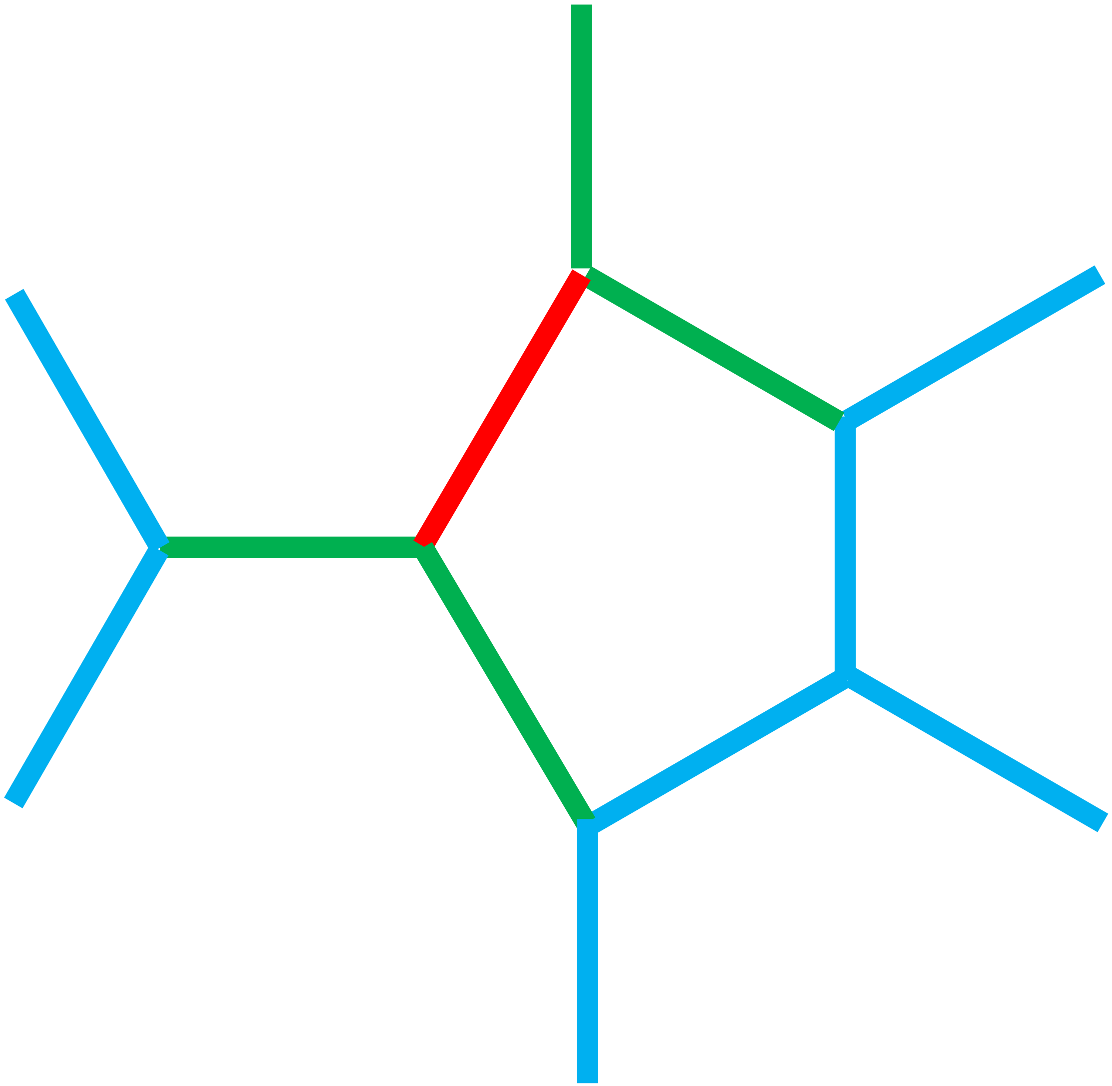

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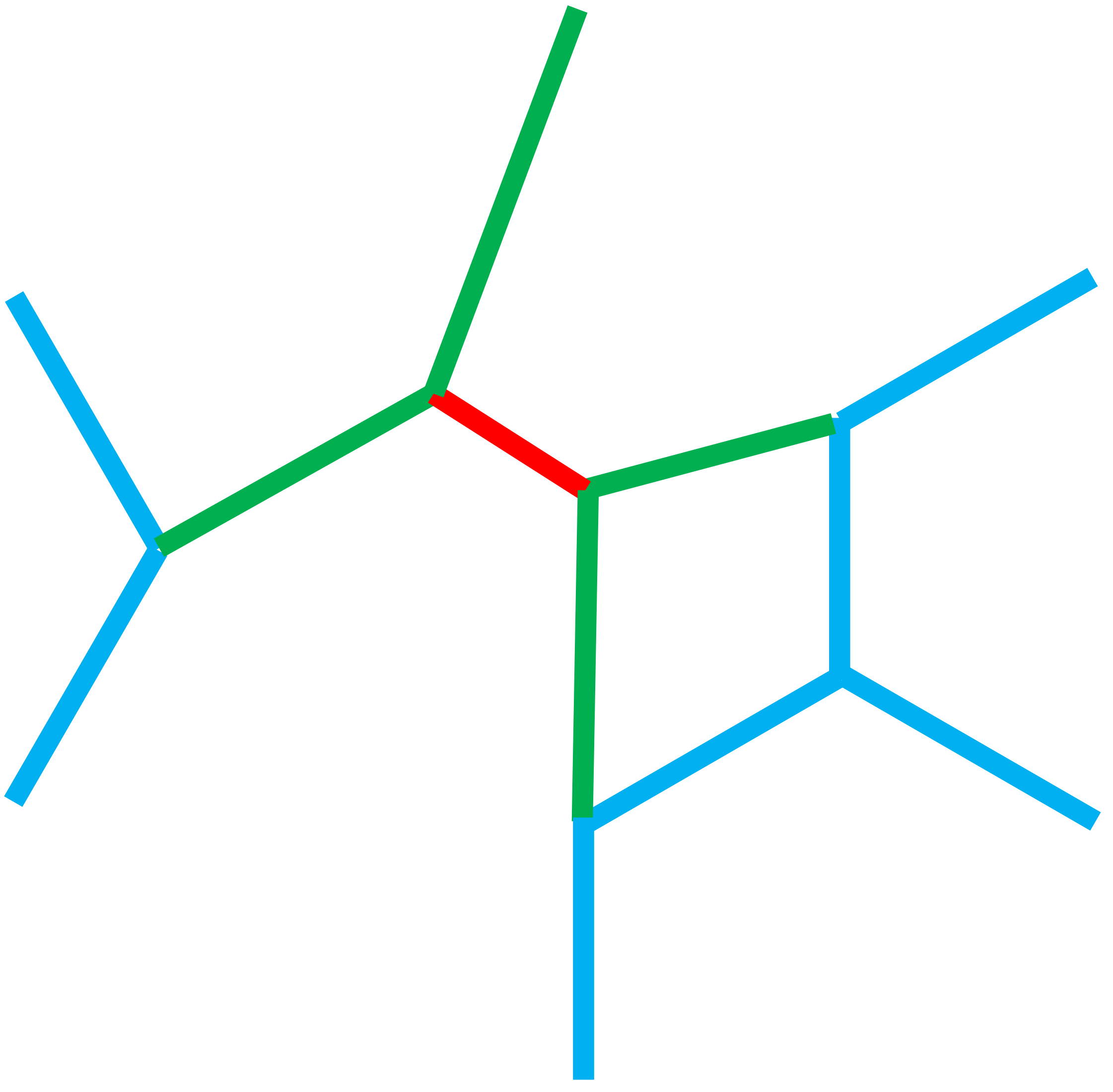

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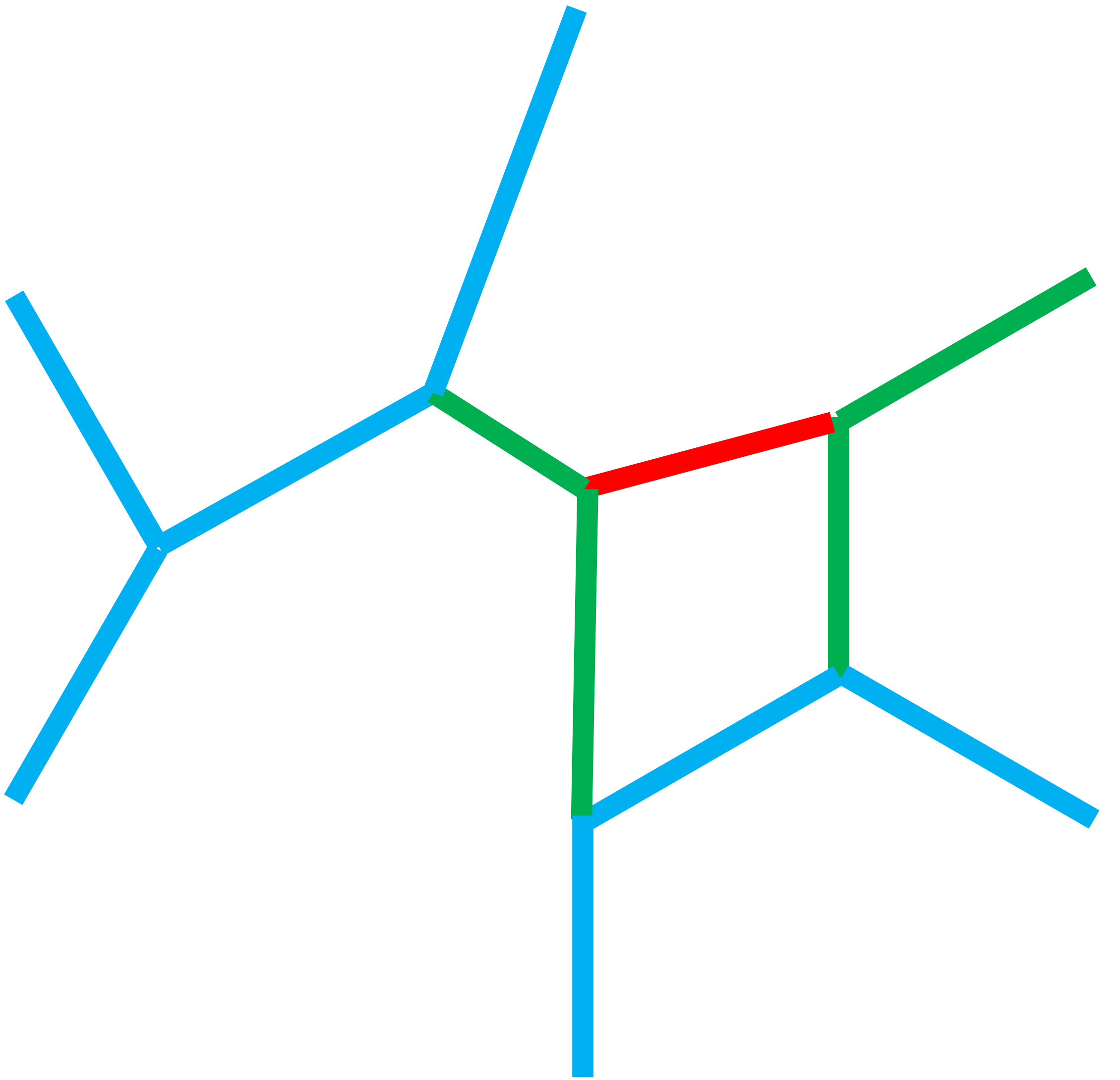

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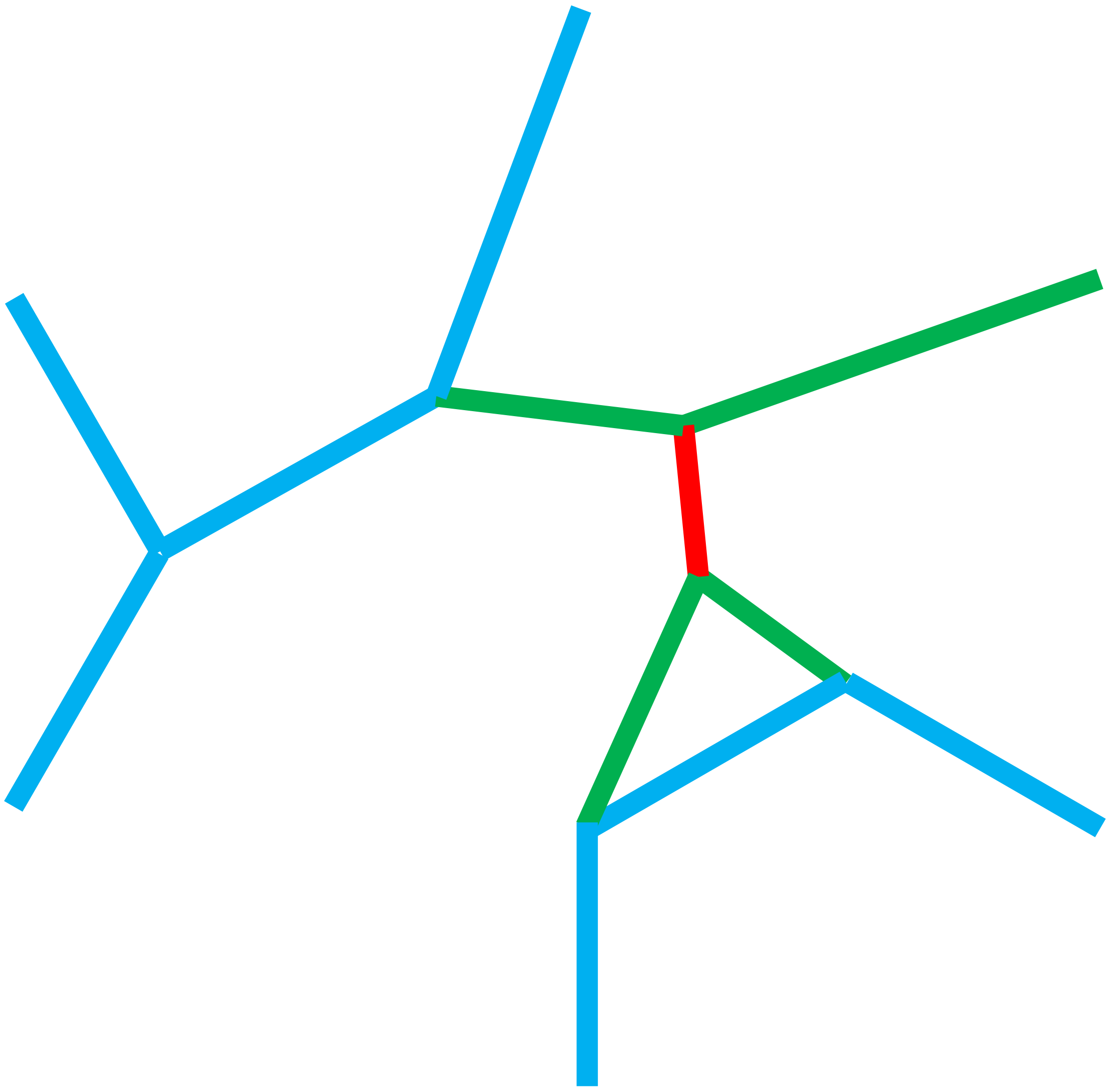

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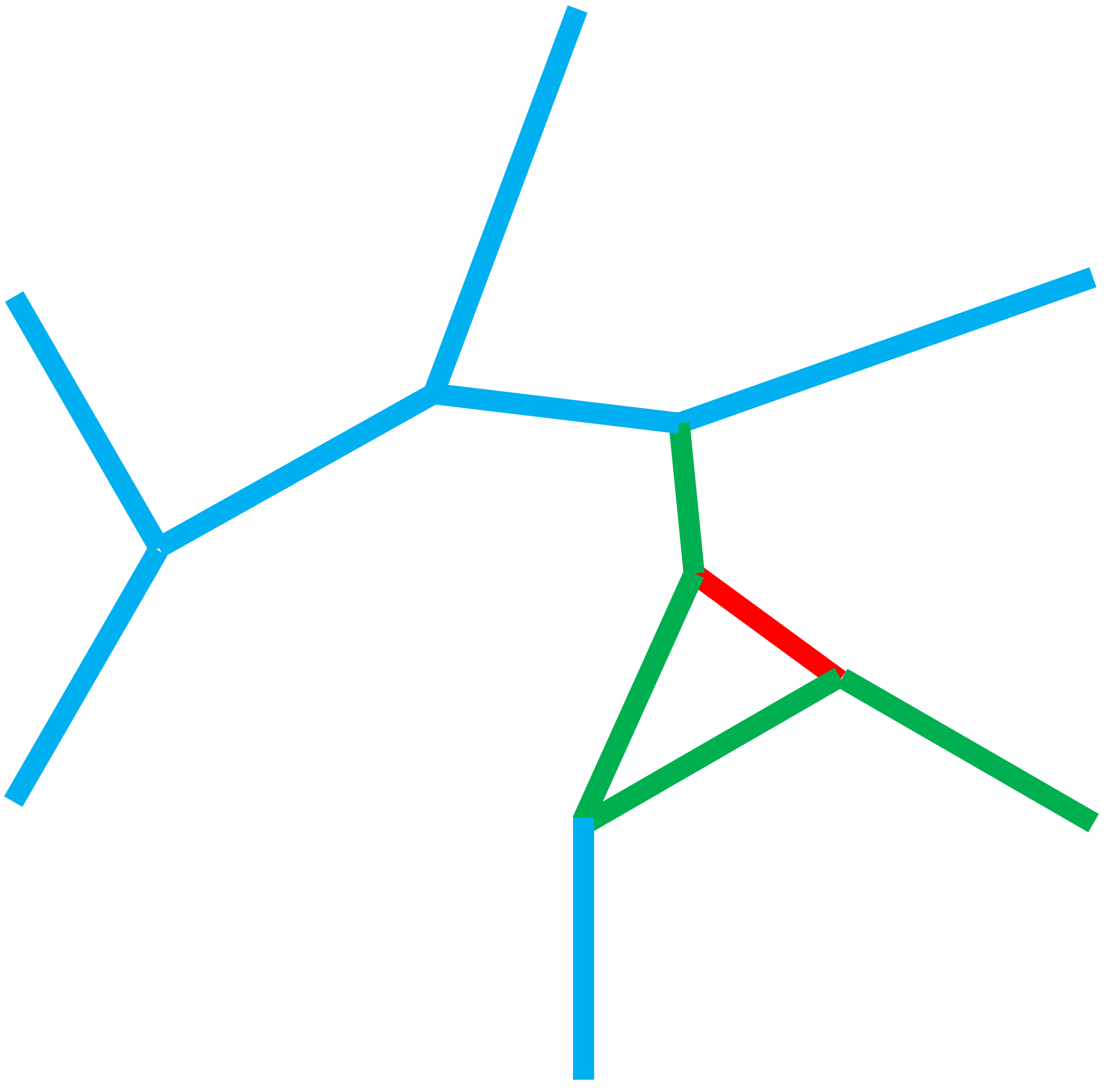

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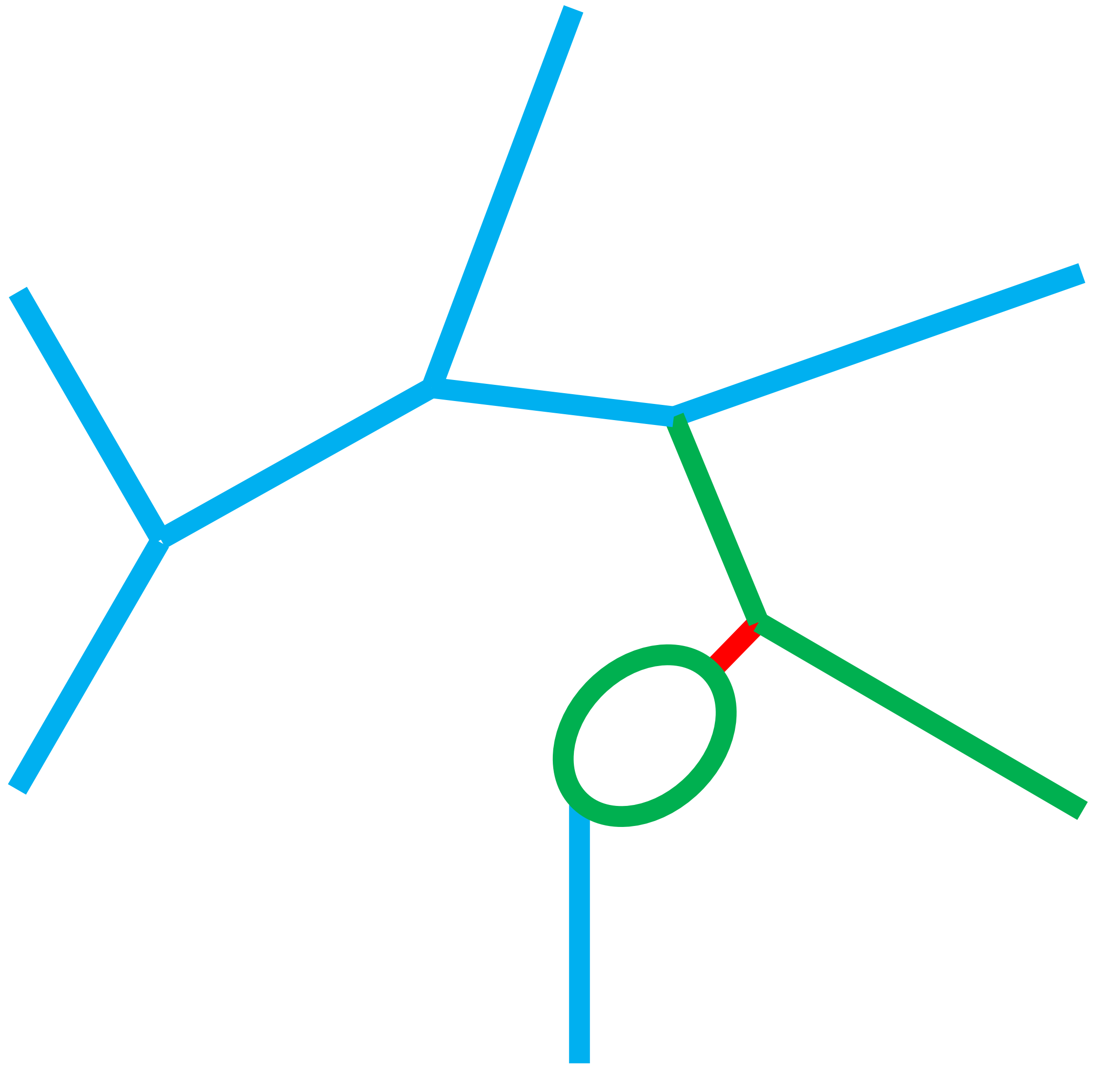

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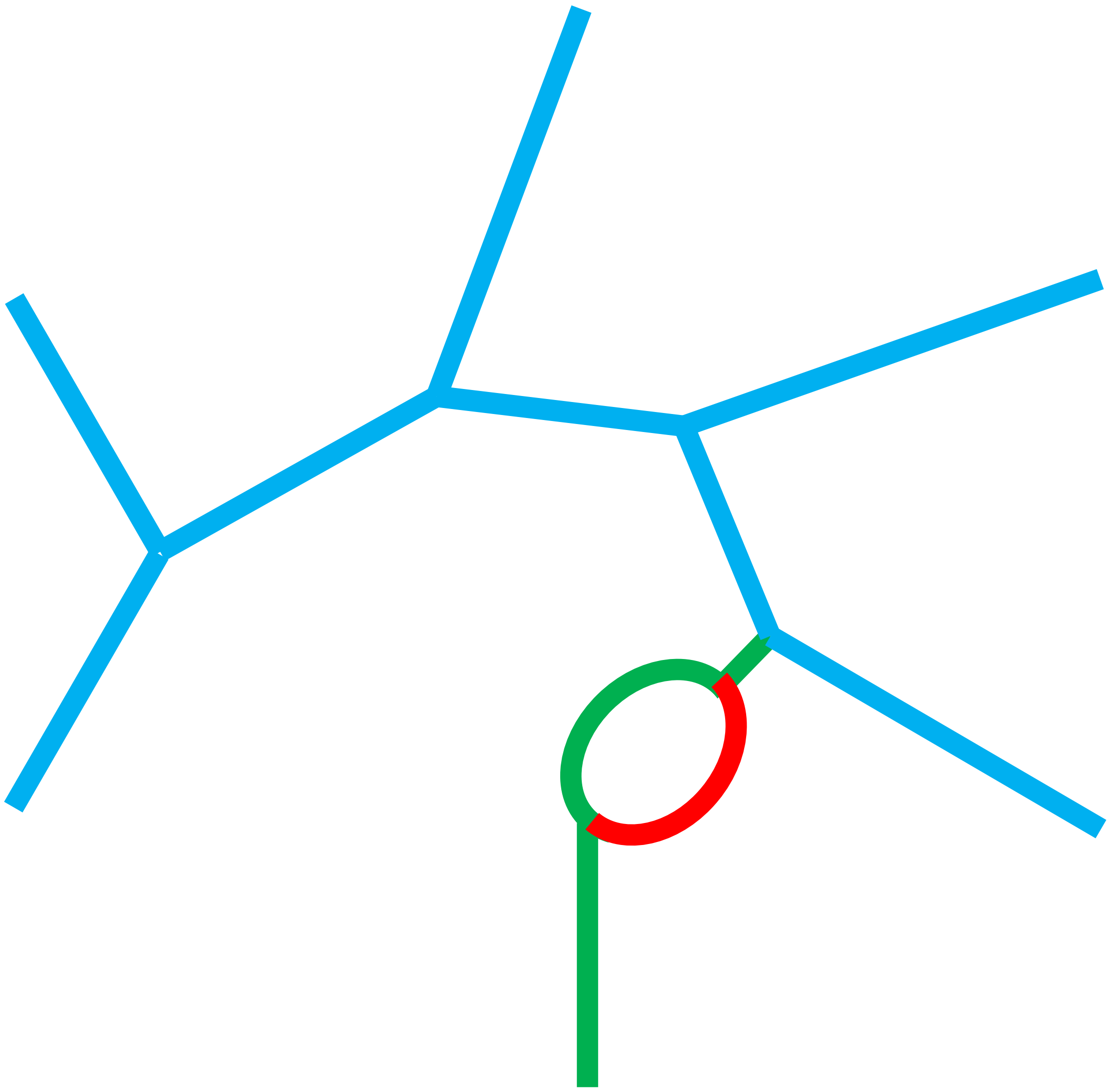

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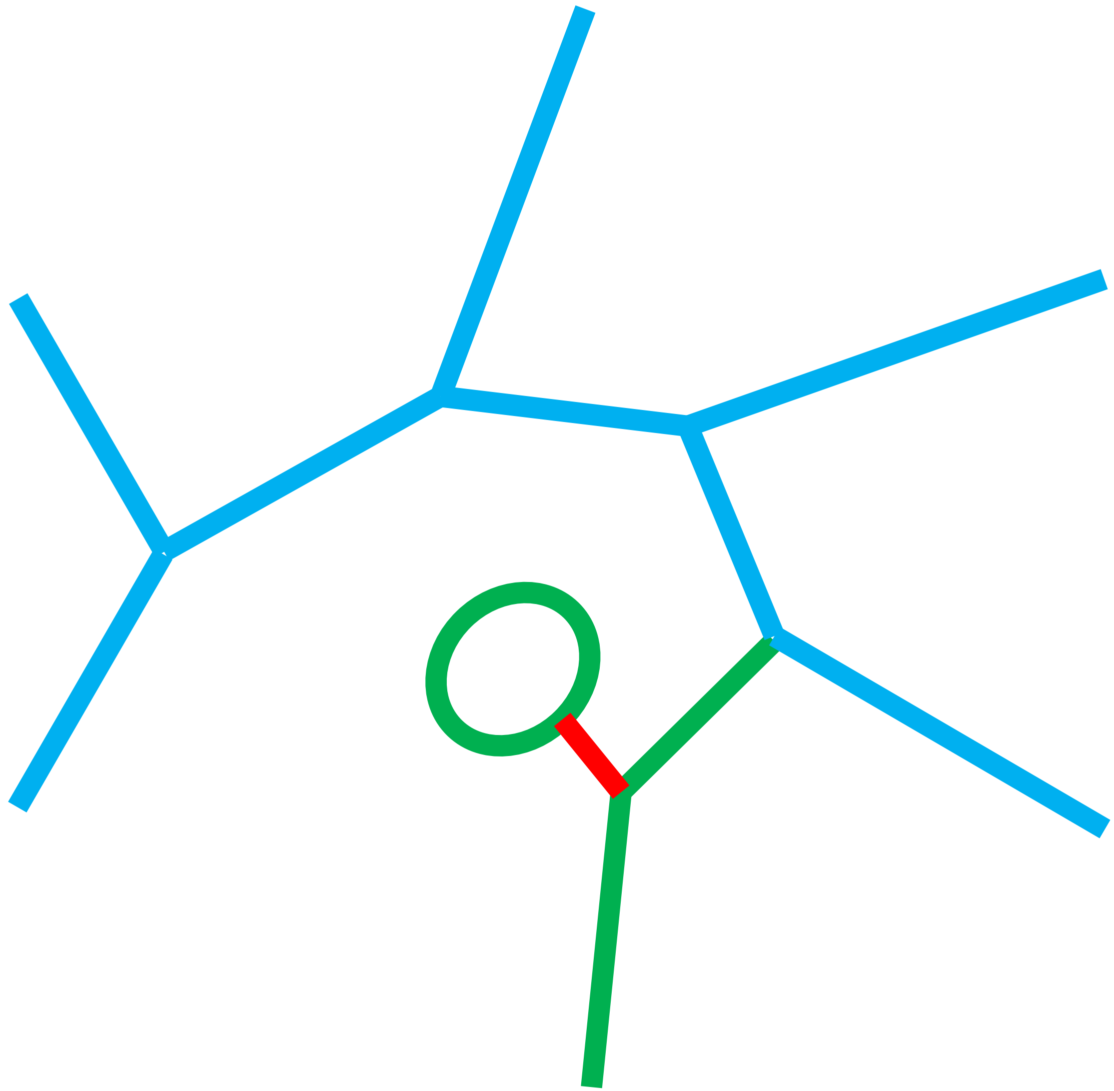

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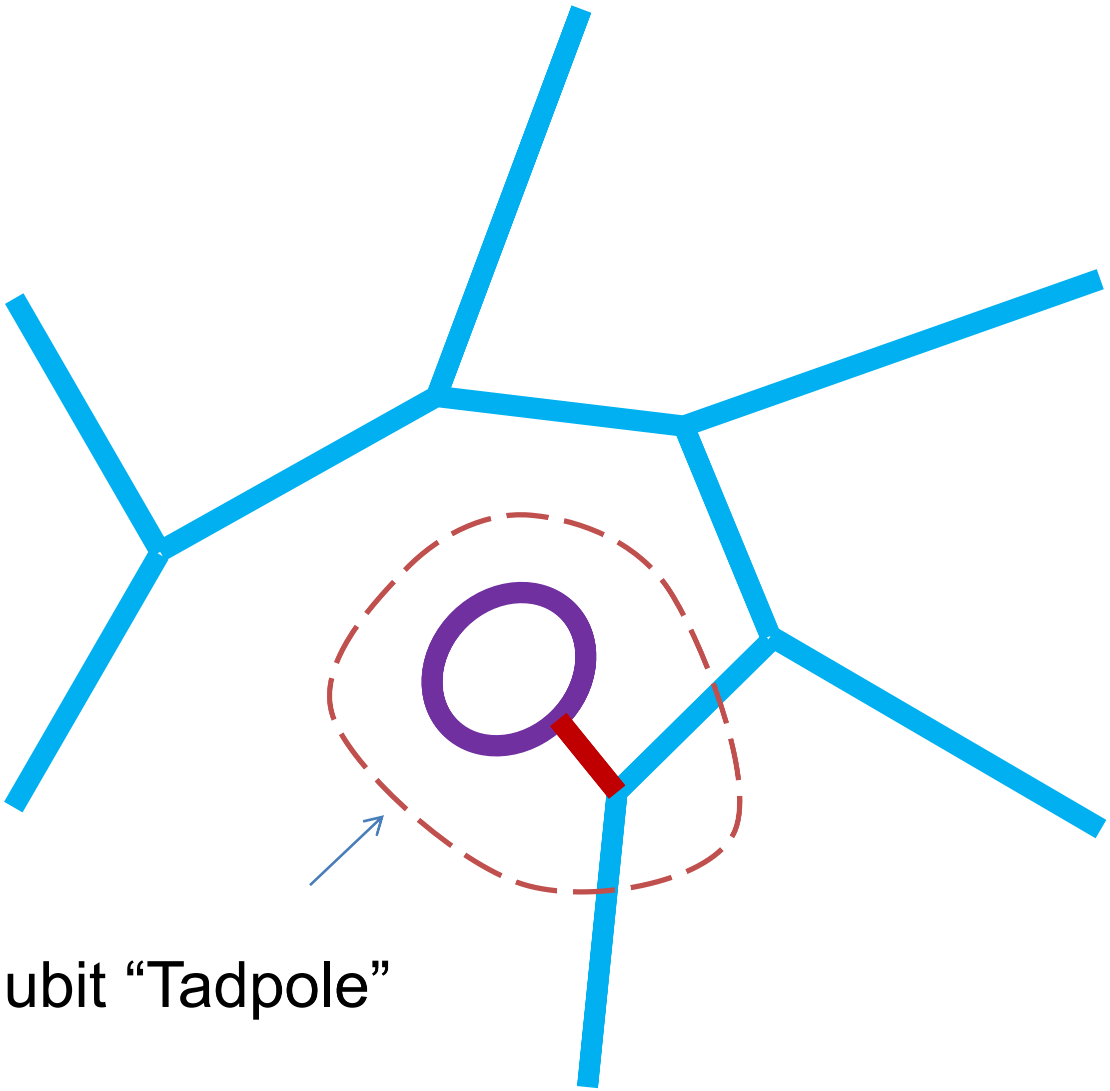
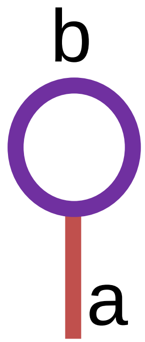
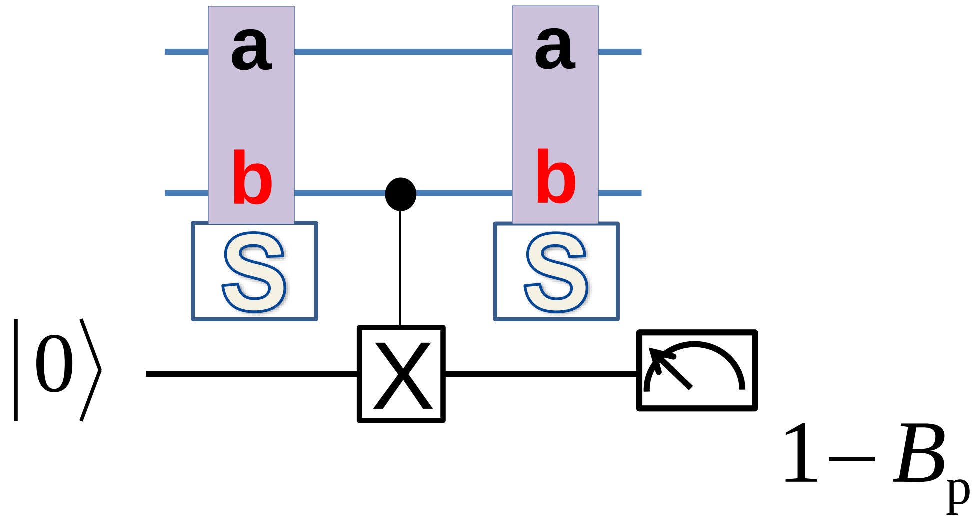


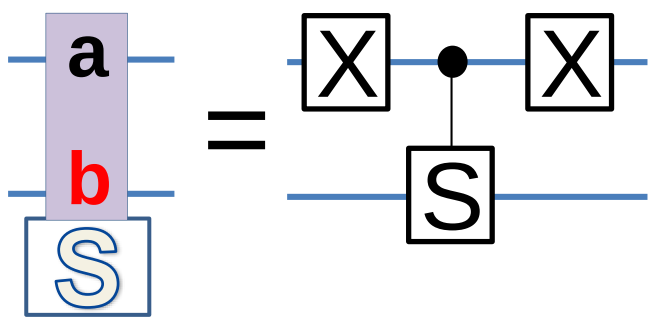
2-qubit “Tadpole”

Measuring B_p for a Tadpole is Easy!



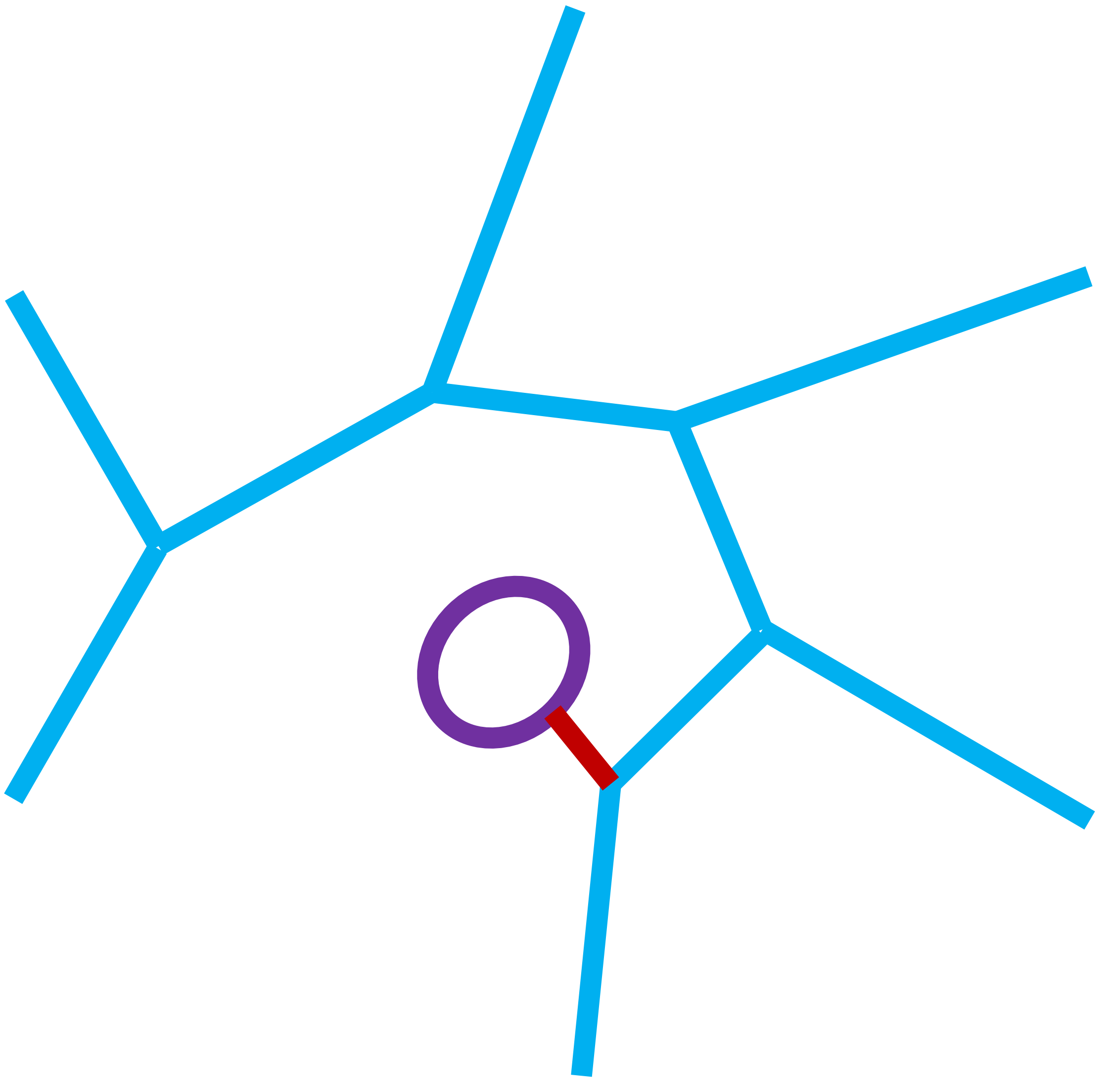


S Quantum Circuit

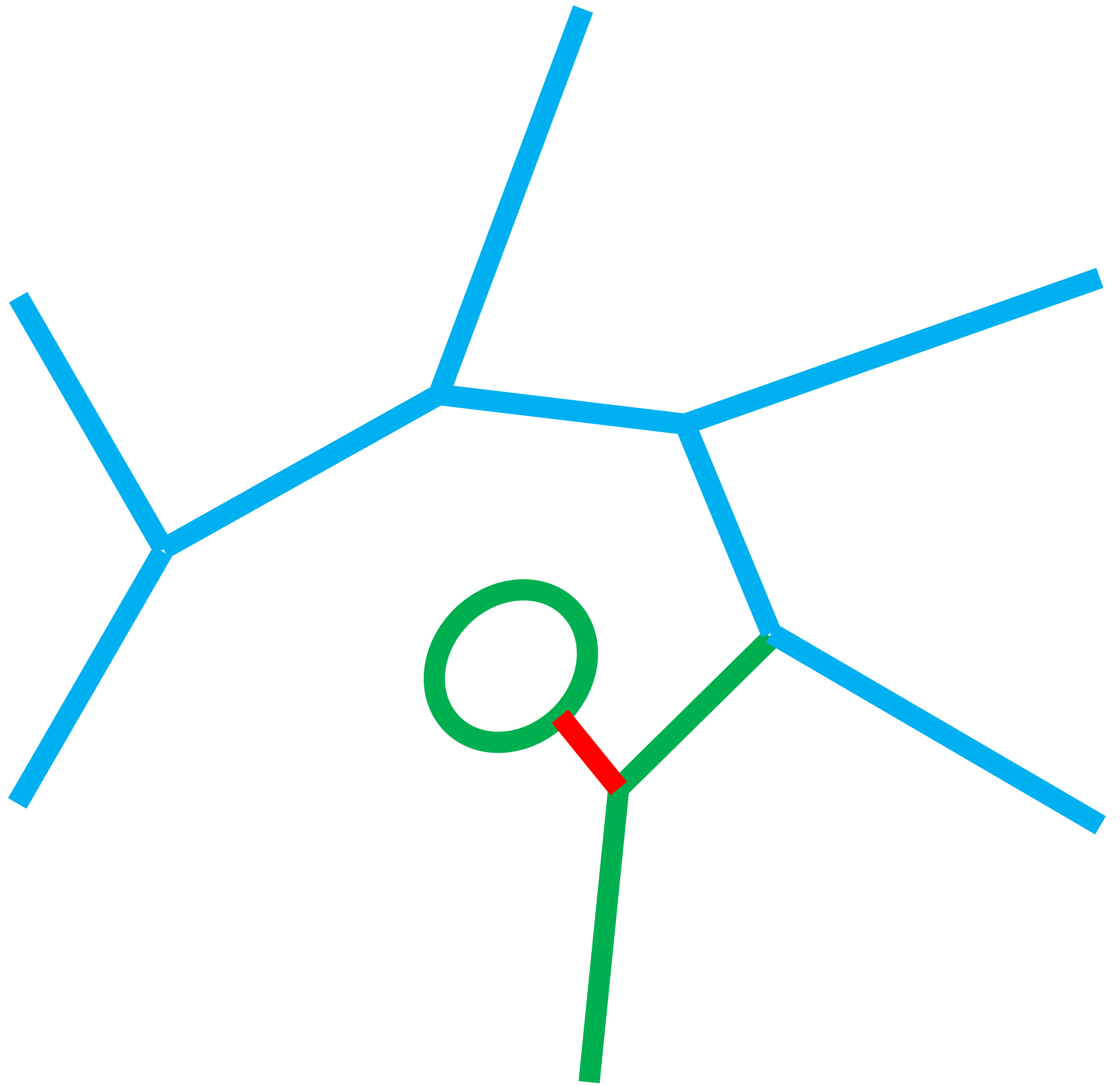


$$S = \frac{1}{\sqrt{1+\varphi^2}} \begin{pmatrix} 1 & \varphi \\ \varphi & -1 \end{pmatrix}$$

Plaquette Restoration



Plaquette Restoration



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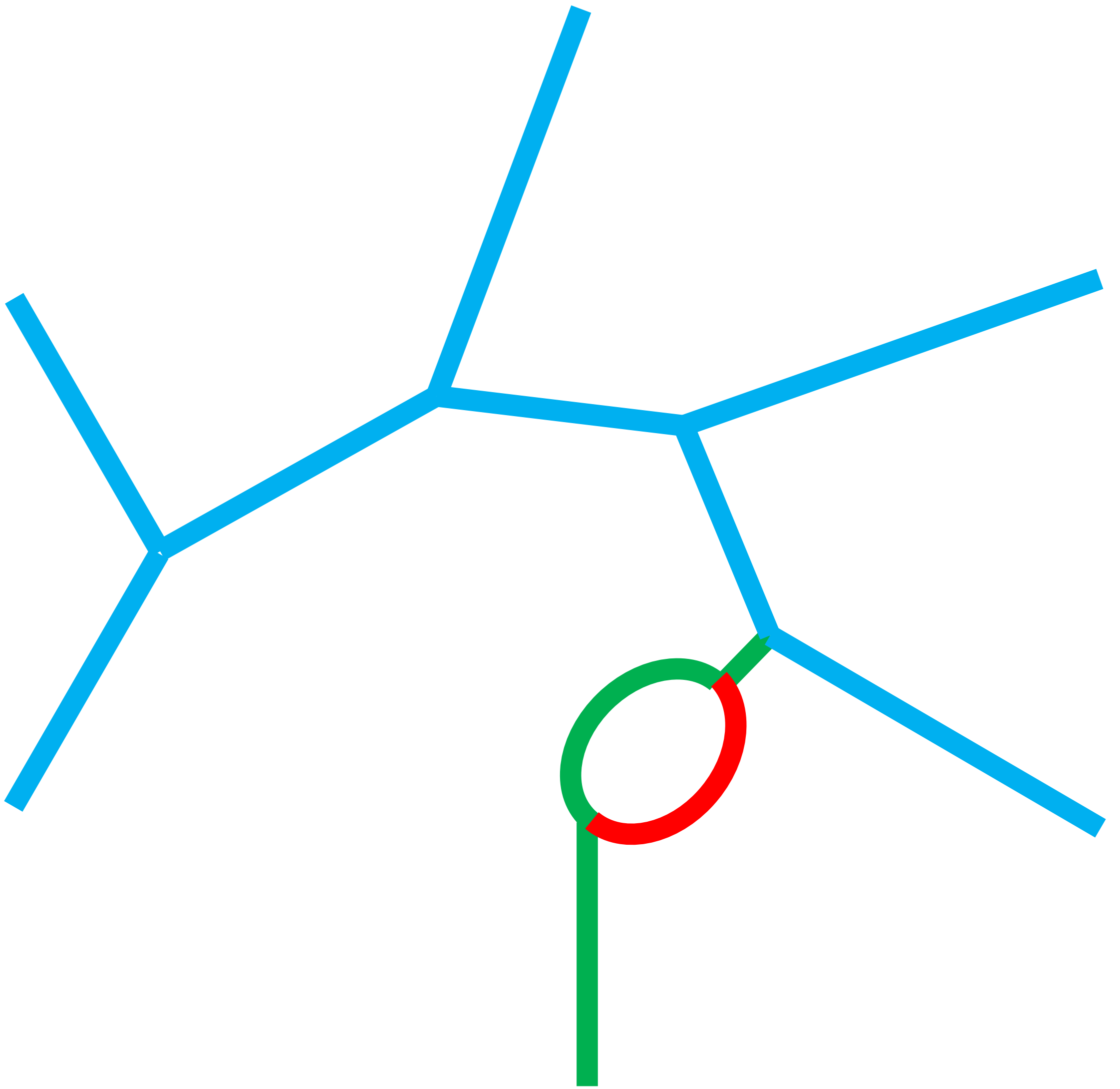

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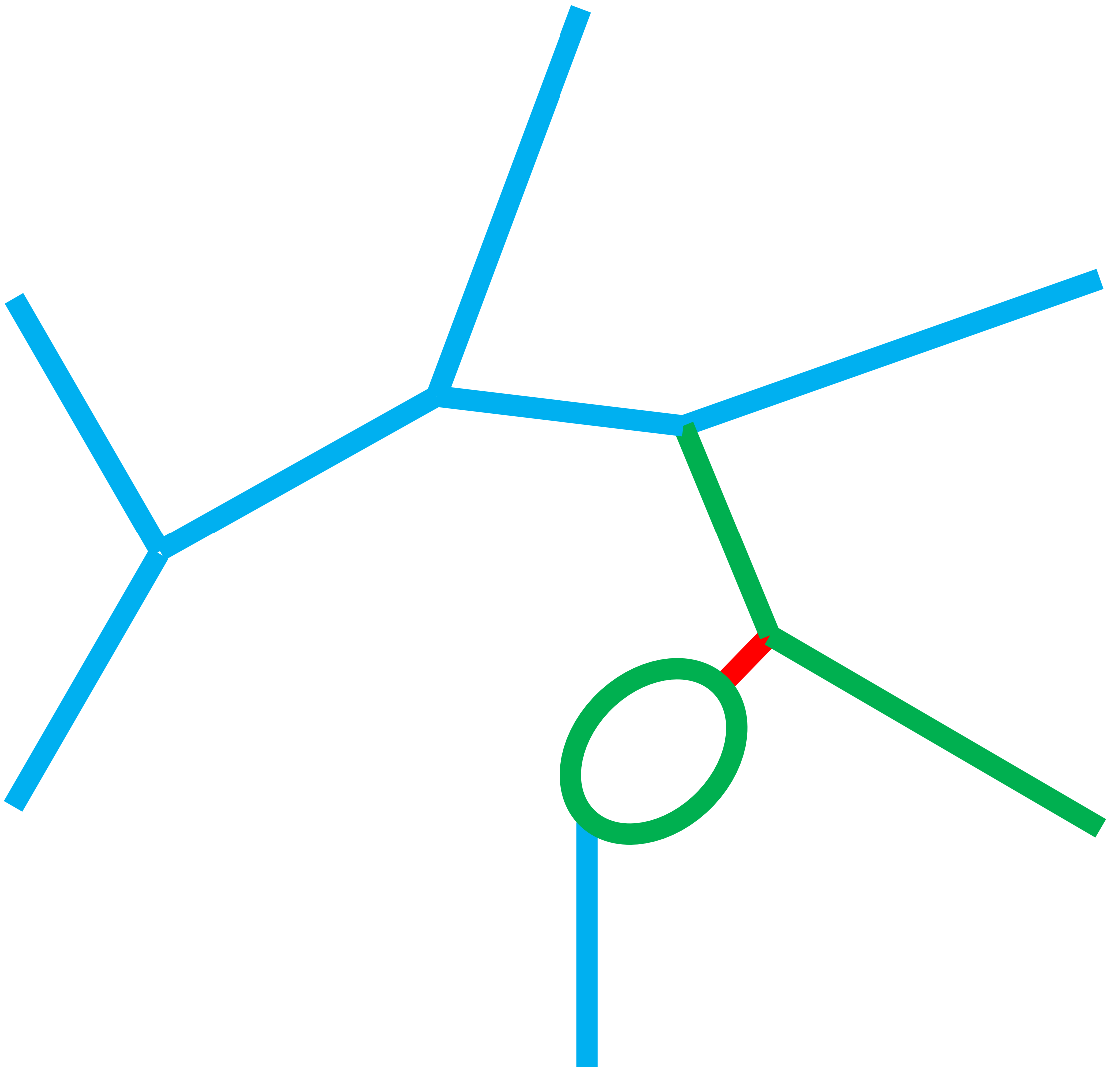

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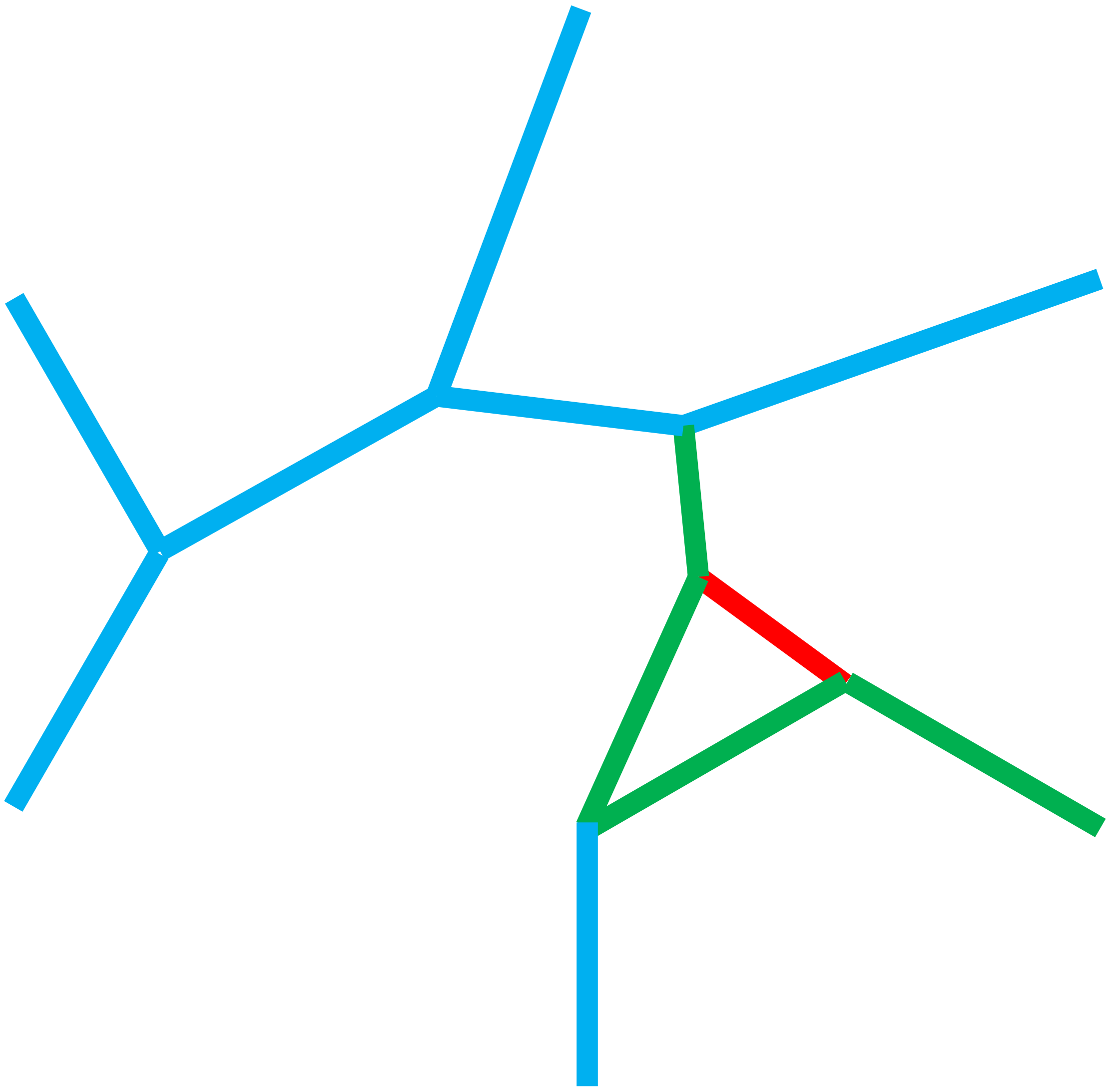

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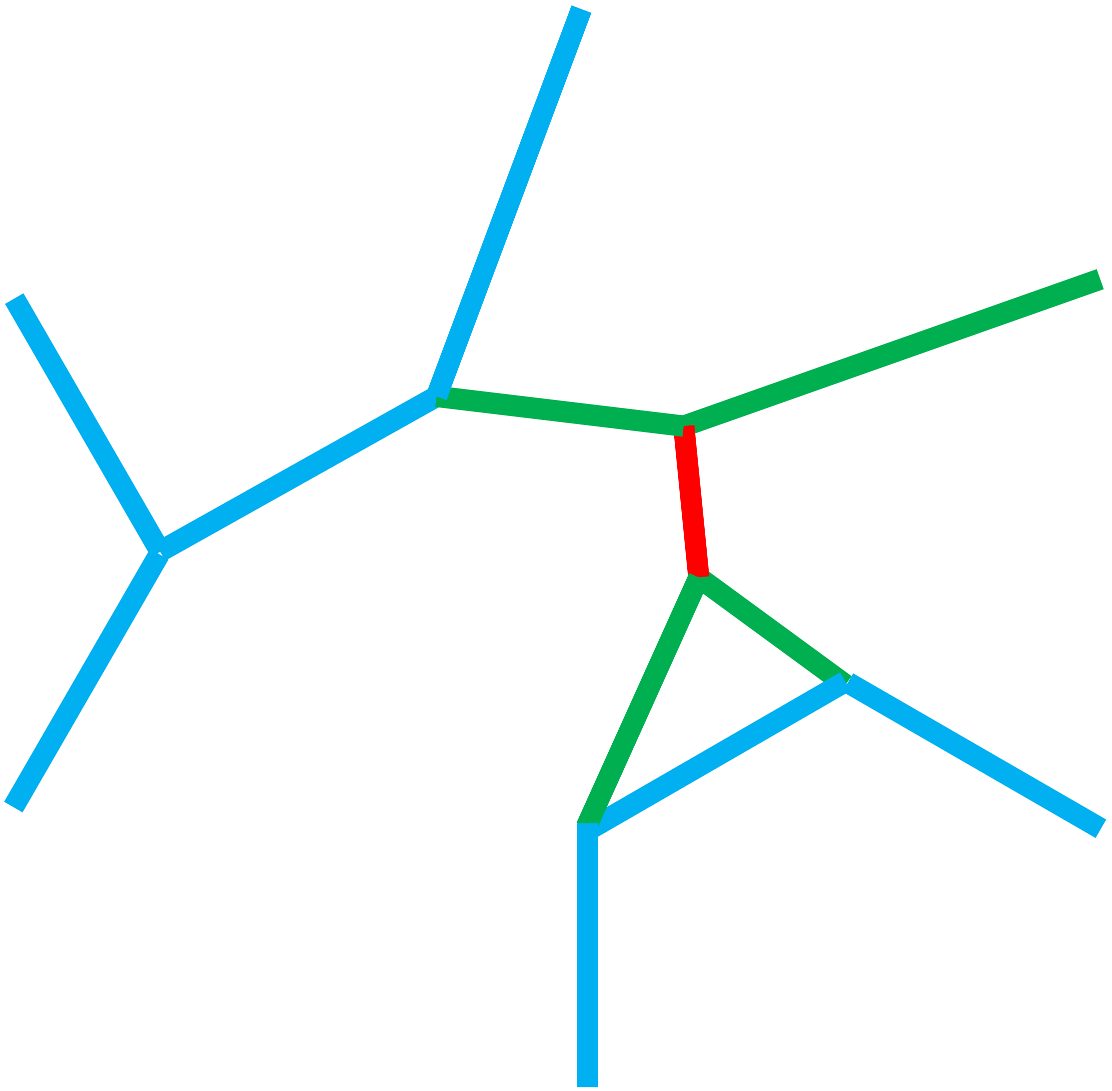

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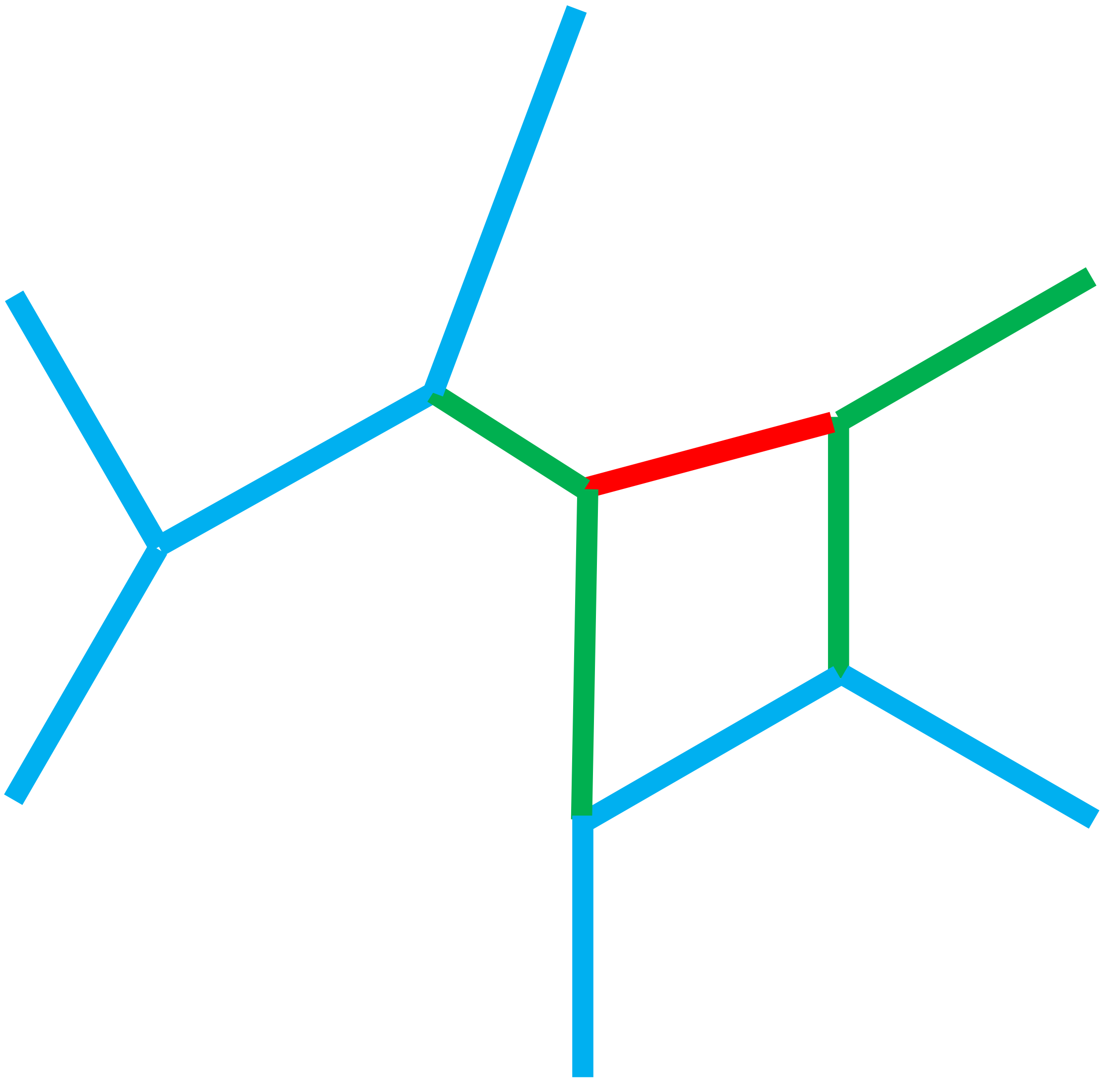

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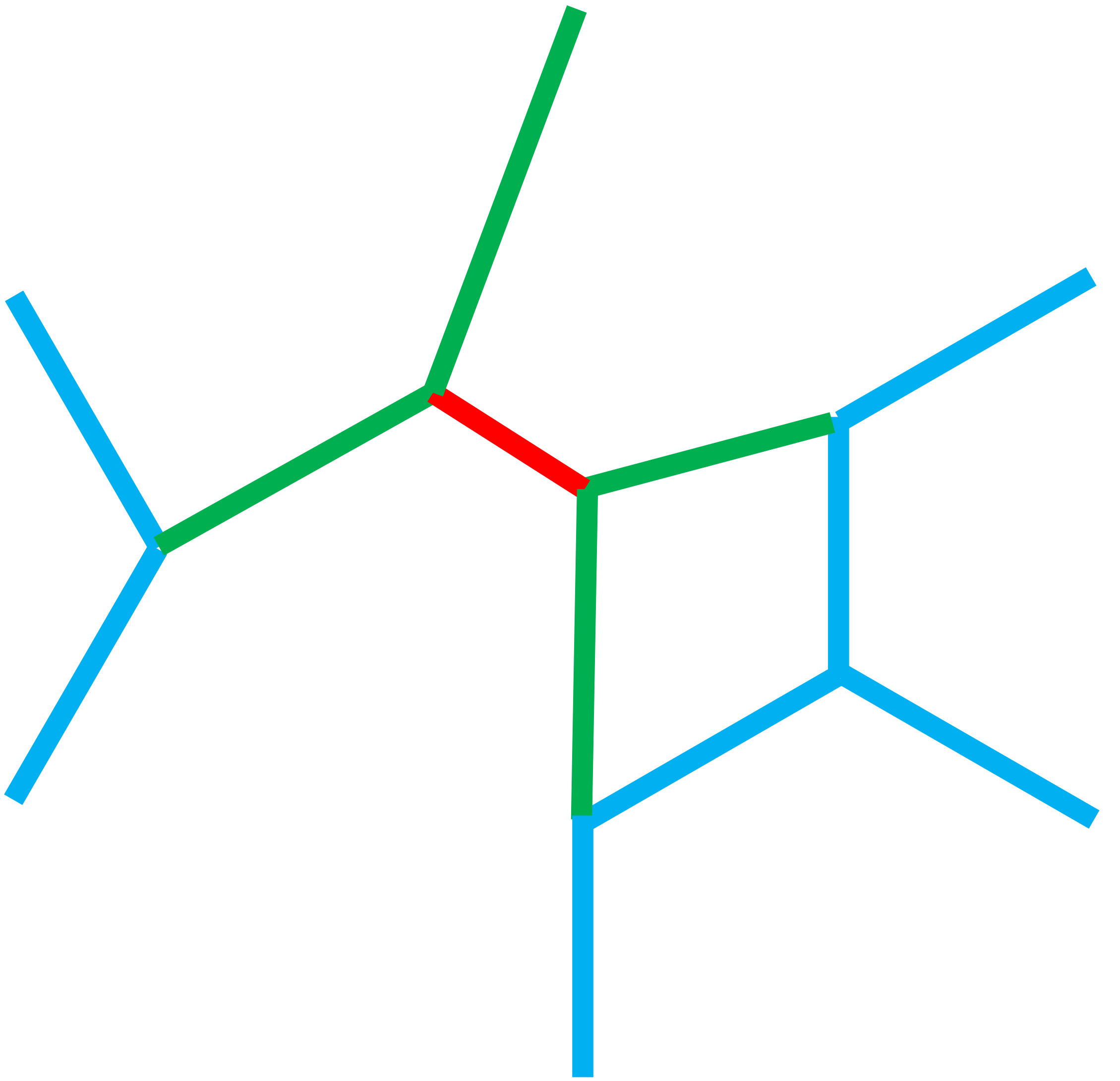

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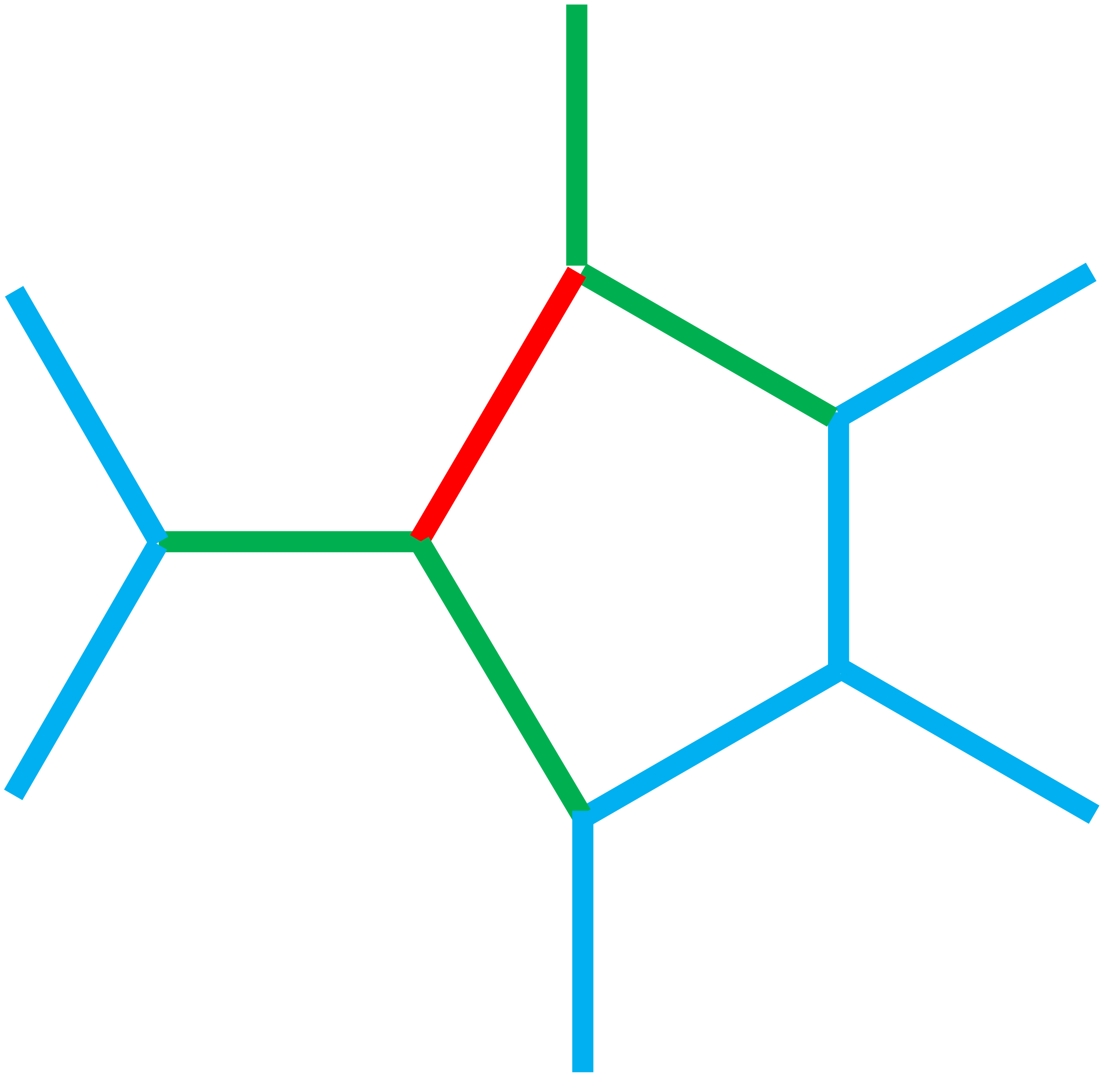

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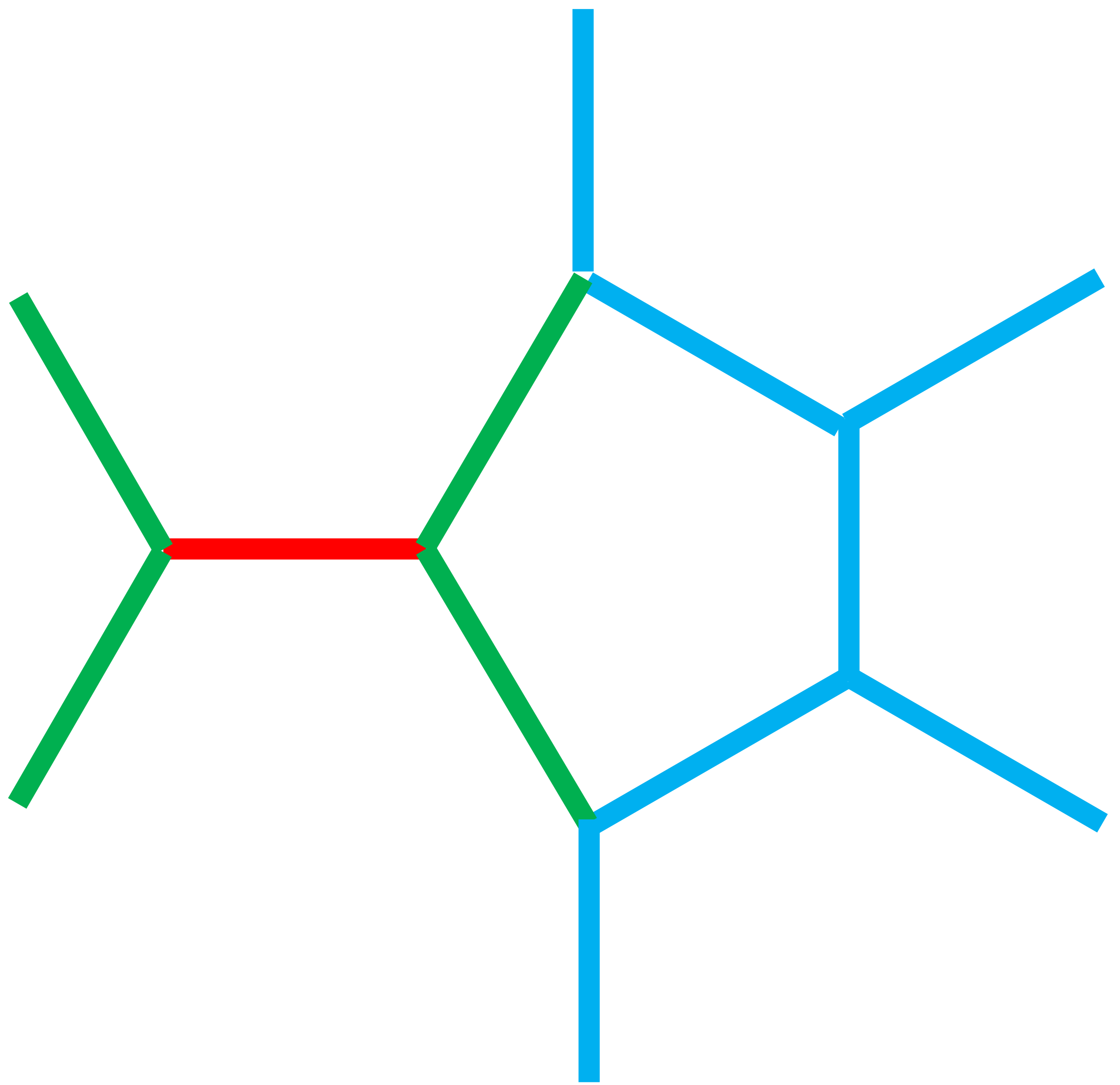

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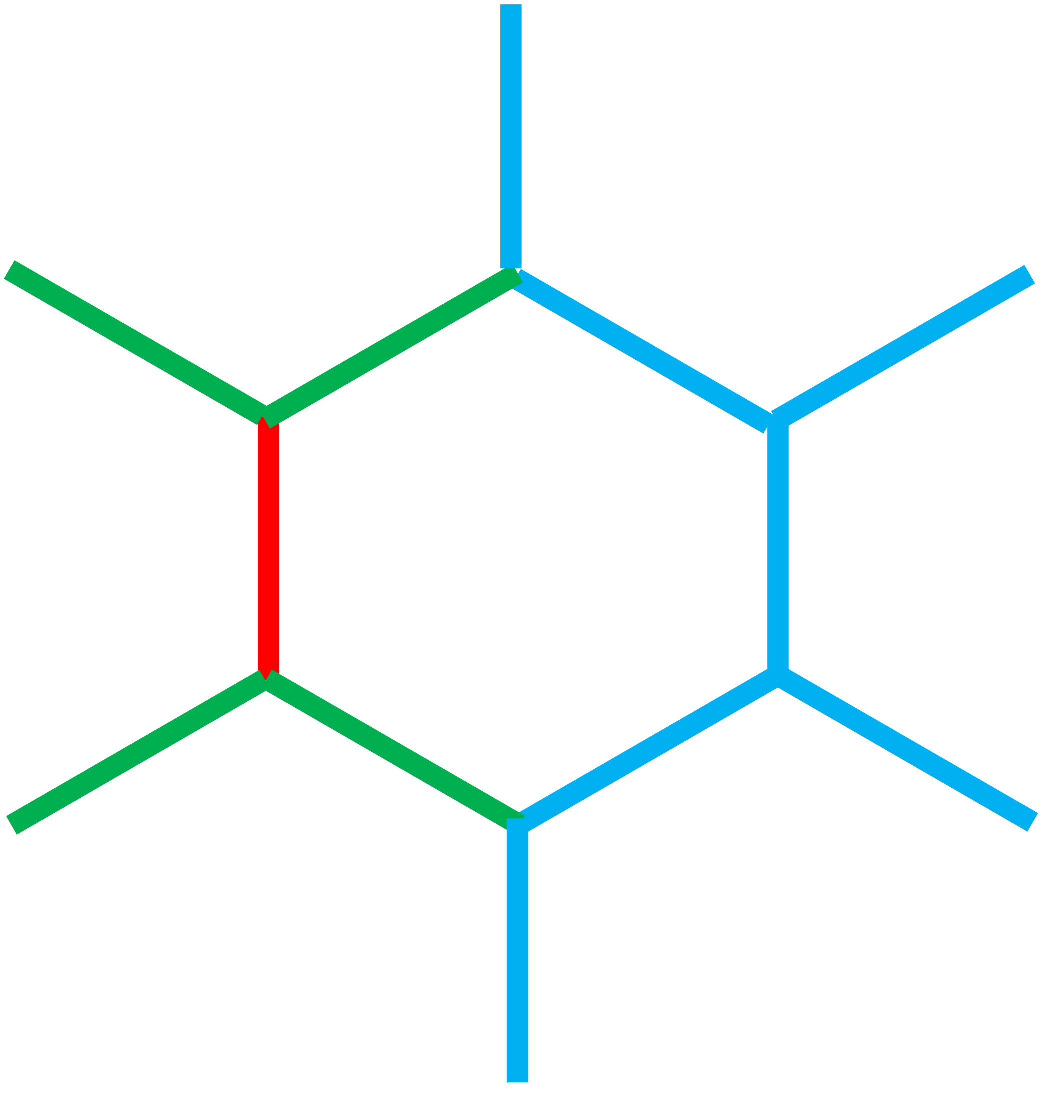
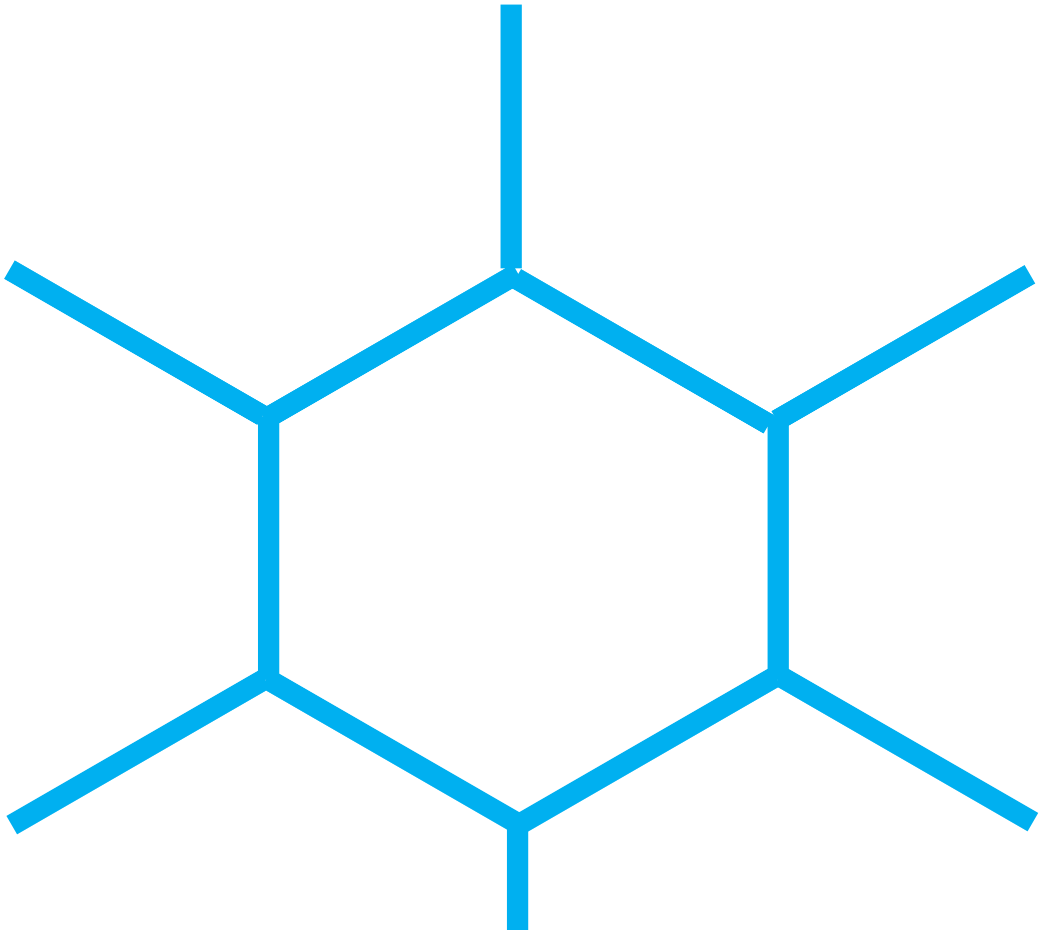
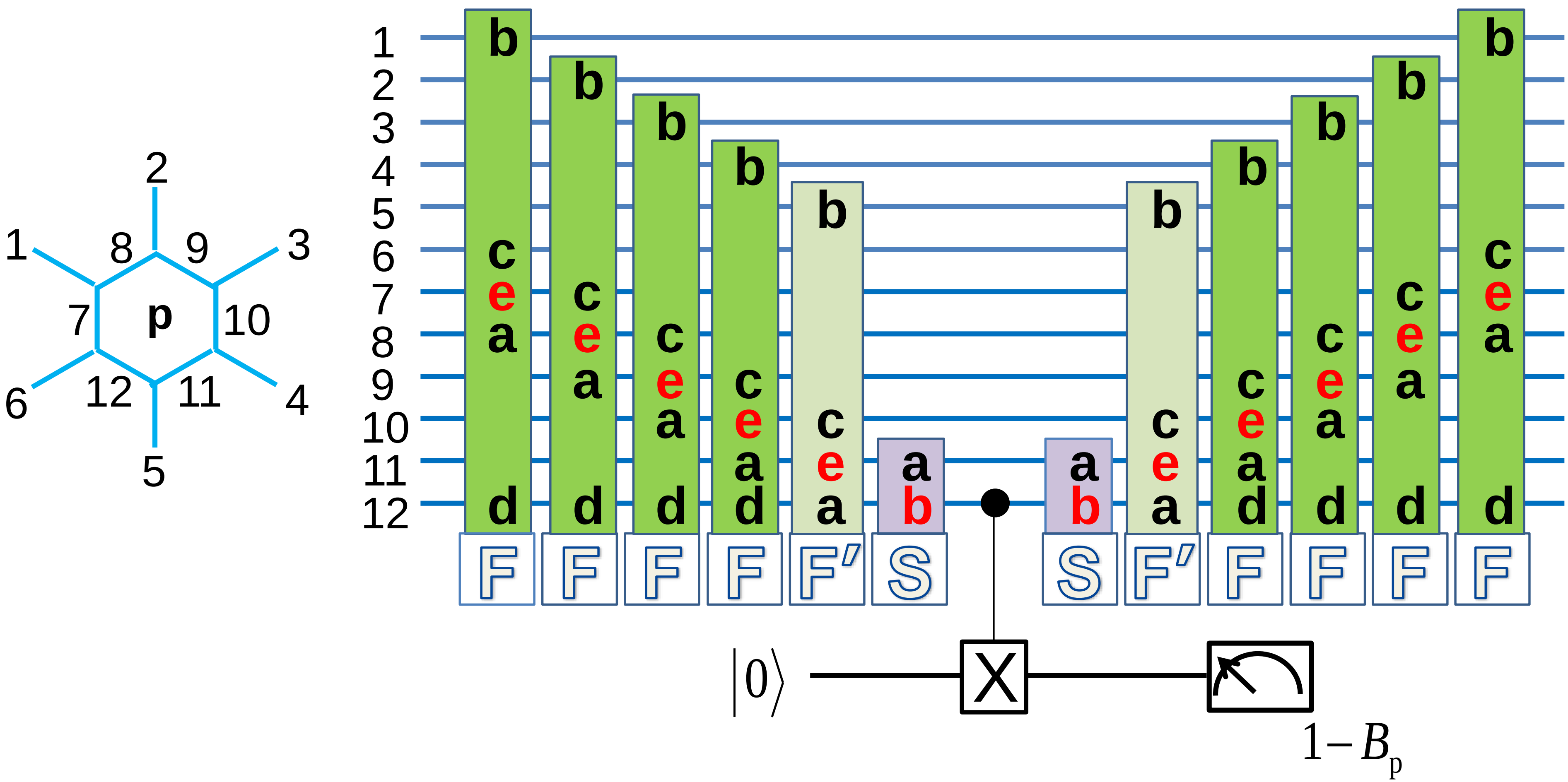


Plaquette Restoration



Quantum Circuit for Measuring B_p



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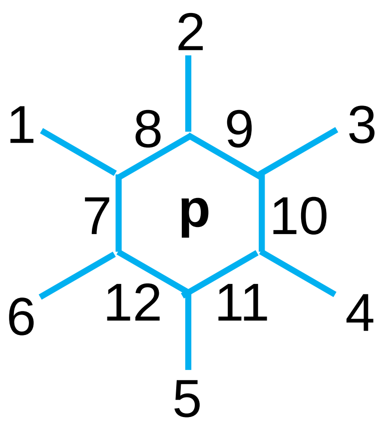
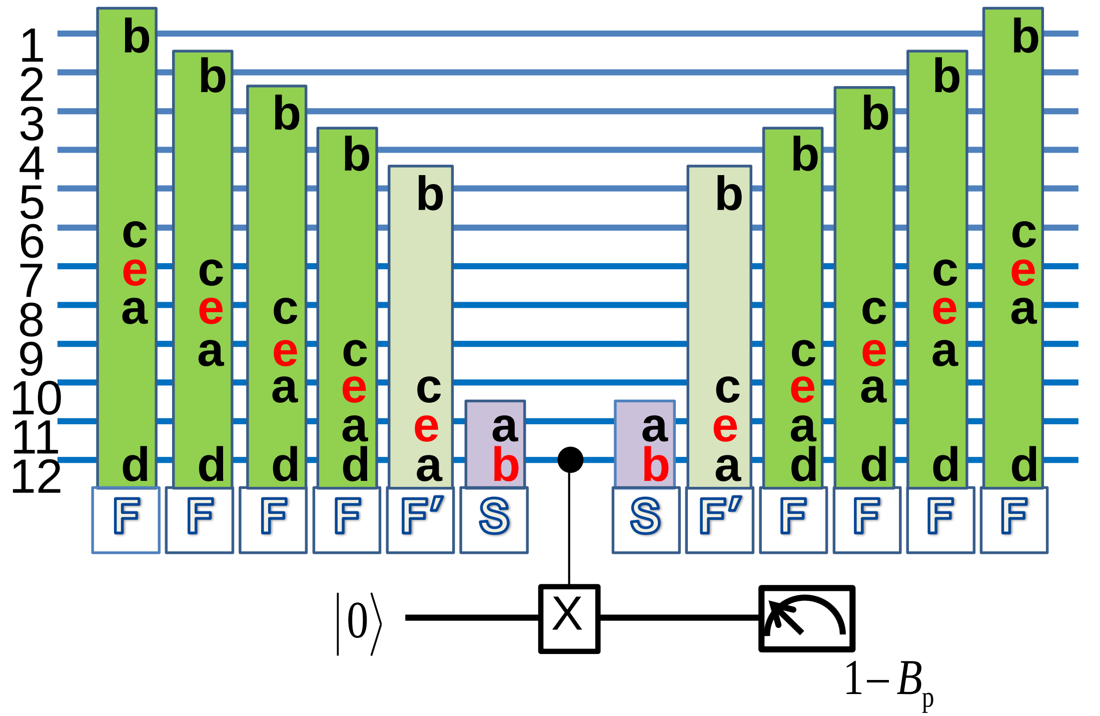
Gate Count





8	5-qubit Toffoli Gates
2	4-qubit Toffoli Gates
10	3-qubit Toffoli Gates
43	CNOT Gates
24	Single Qubit Gates

371	CNOT Gates
392	Single Qubit Rotations

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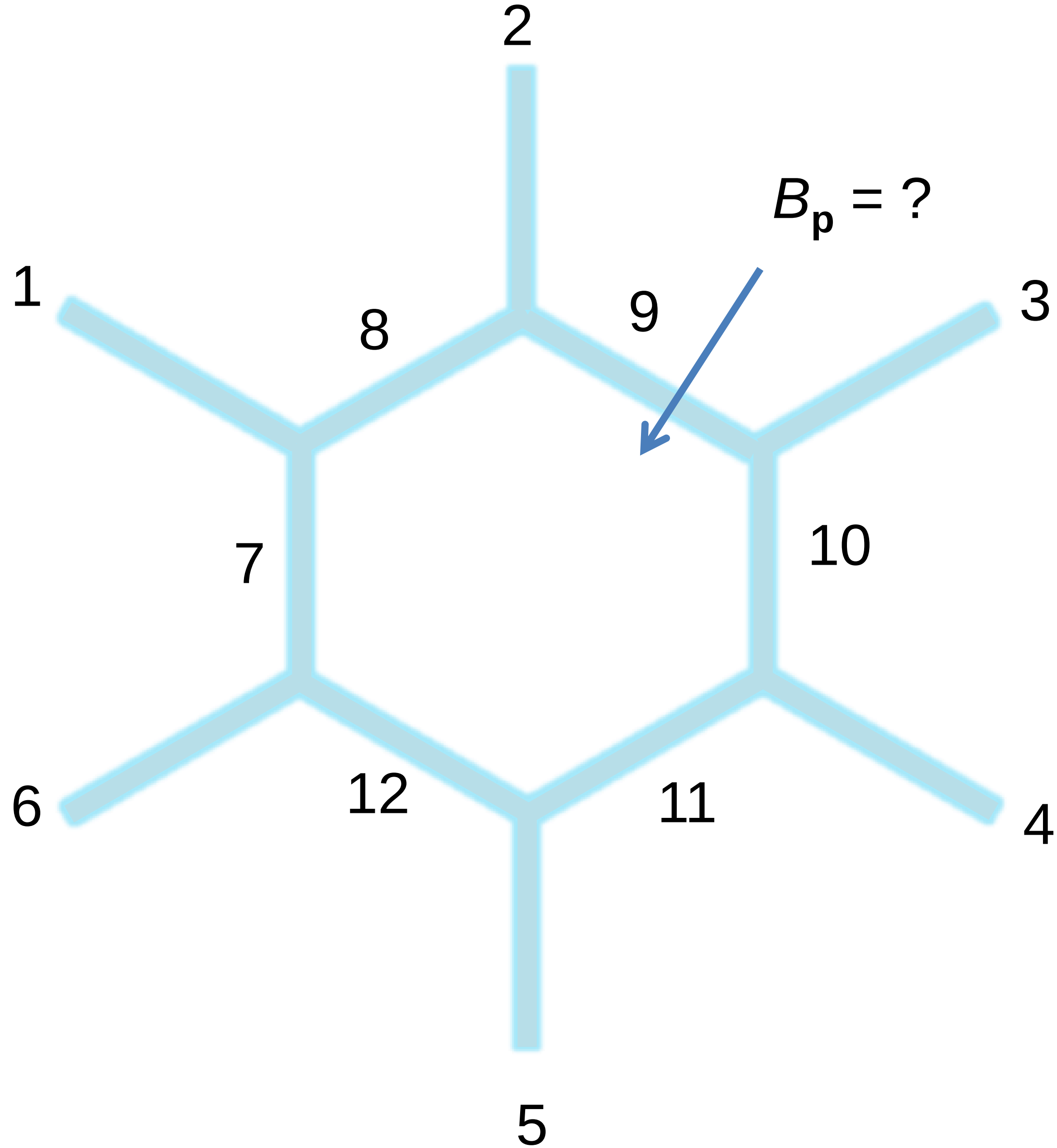
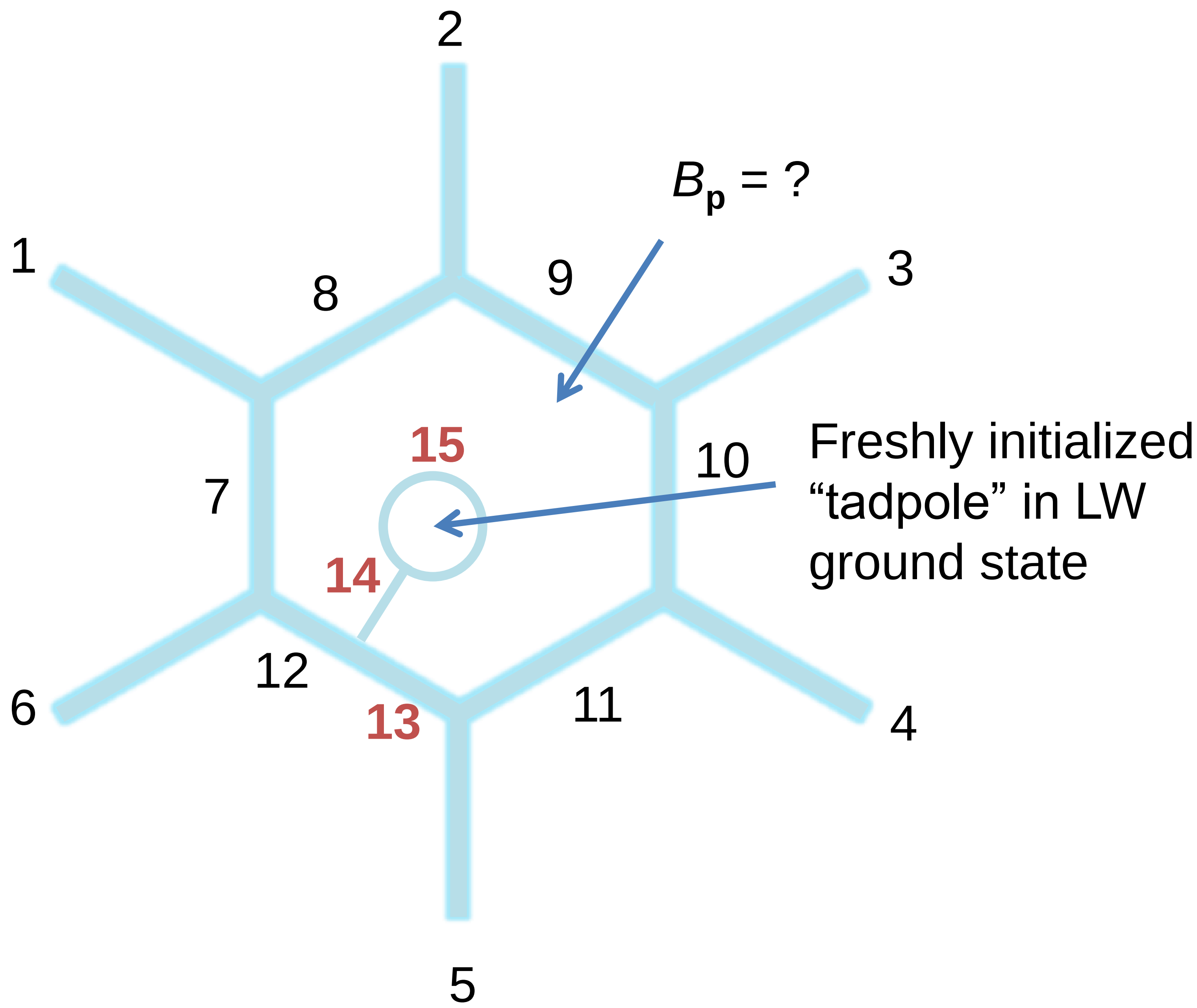
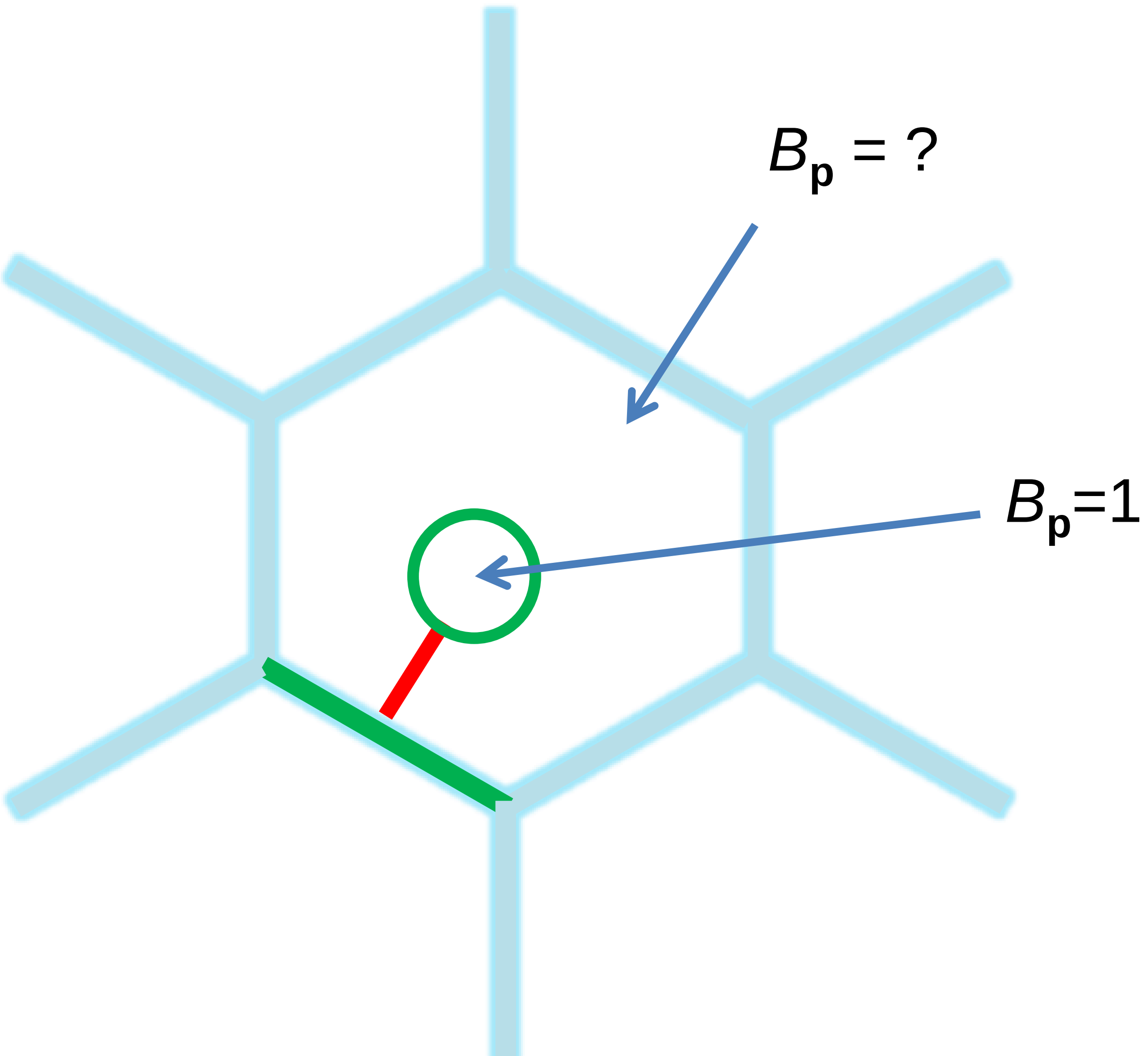
Plaquette Swapping



Plaquette Swapping



Plaquette Swapping



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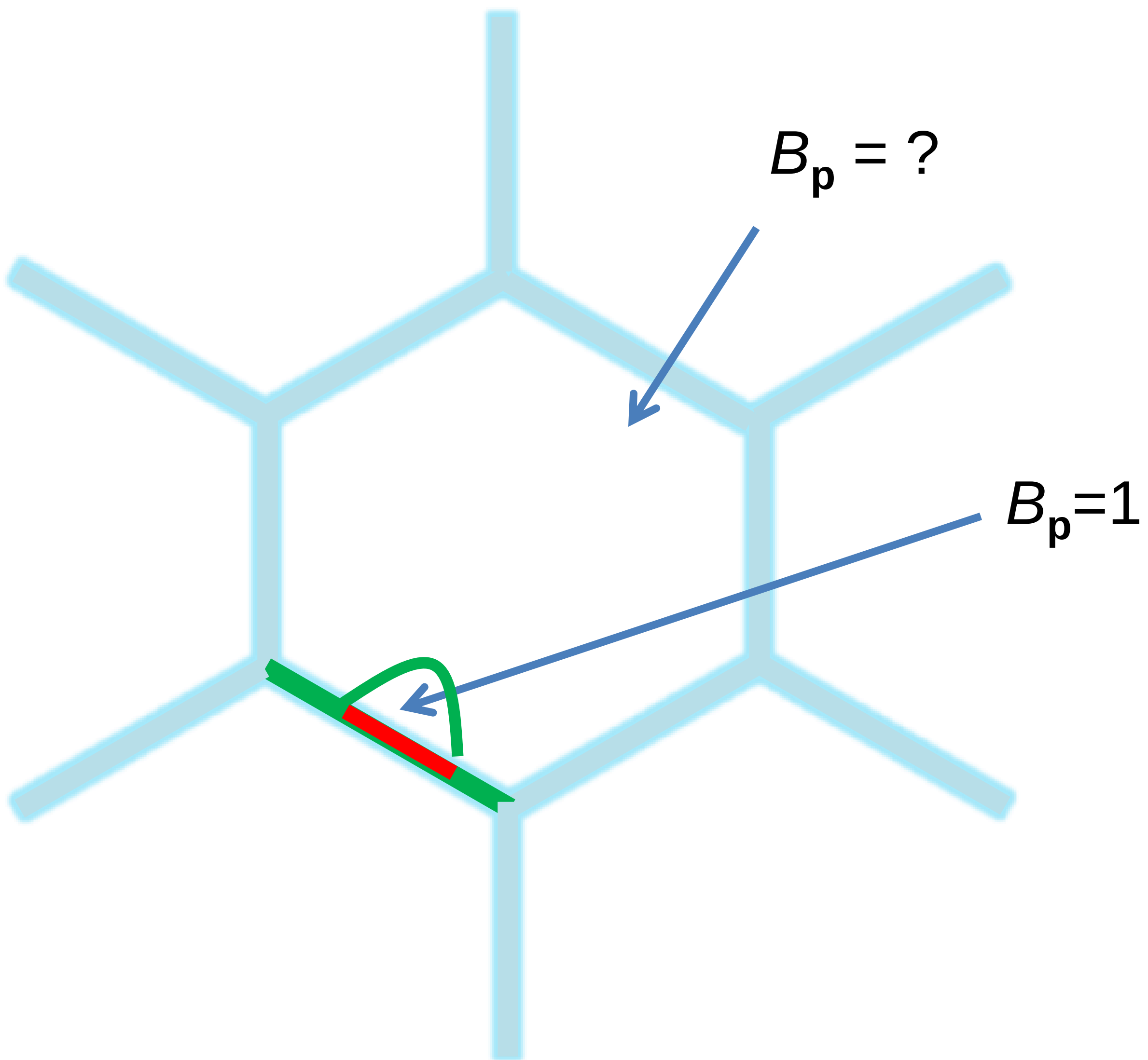

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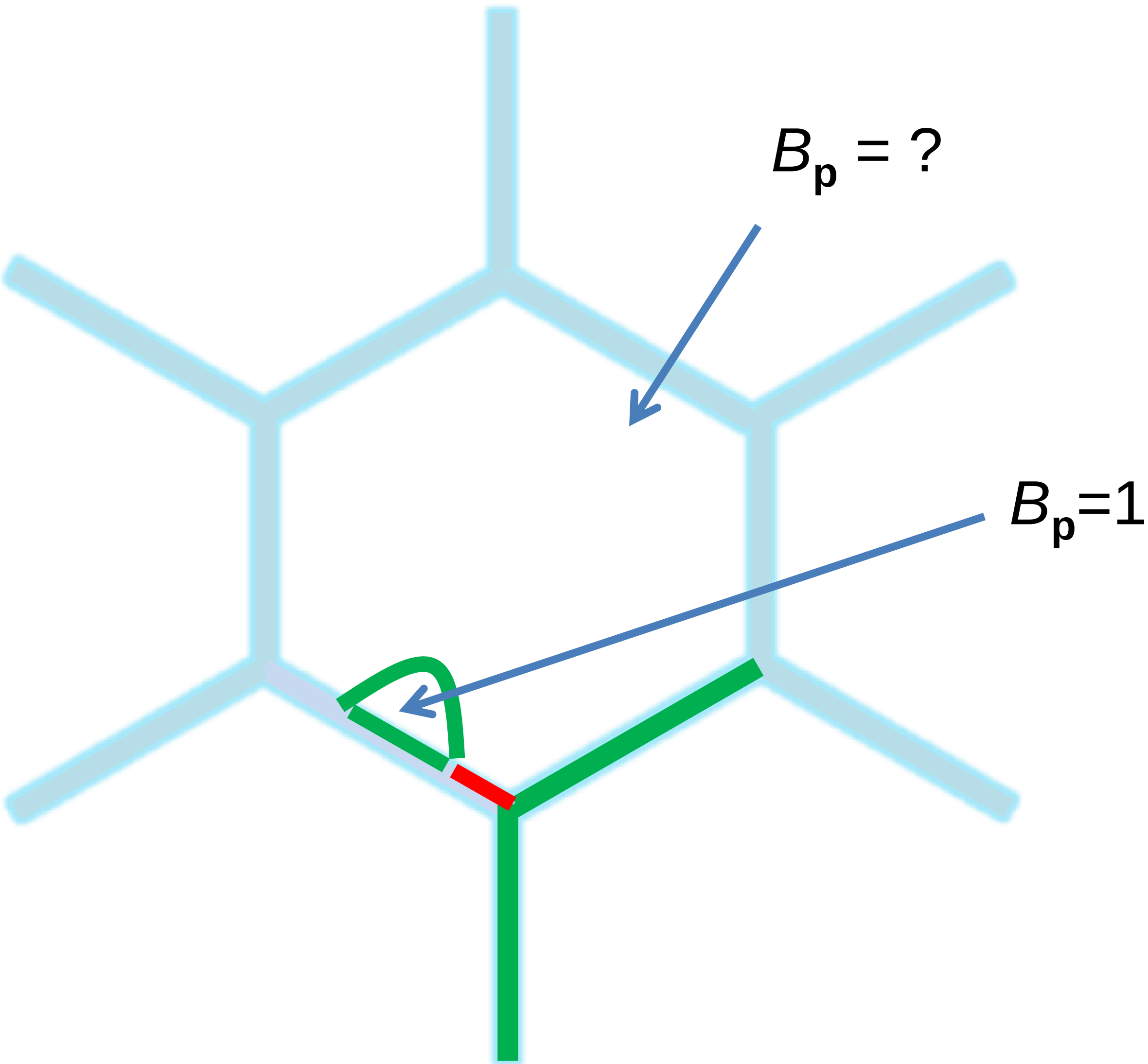

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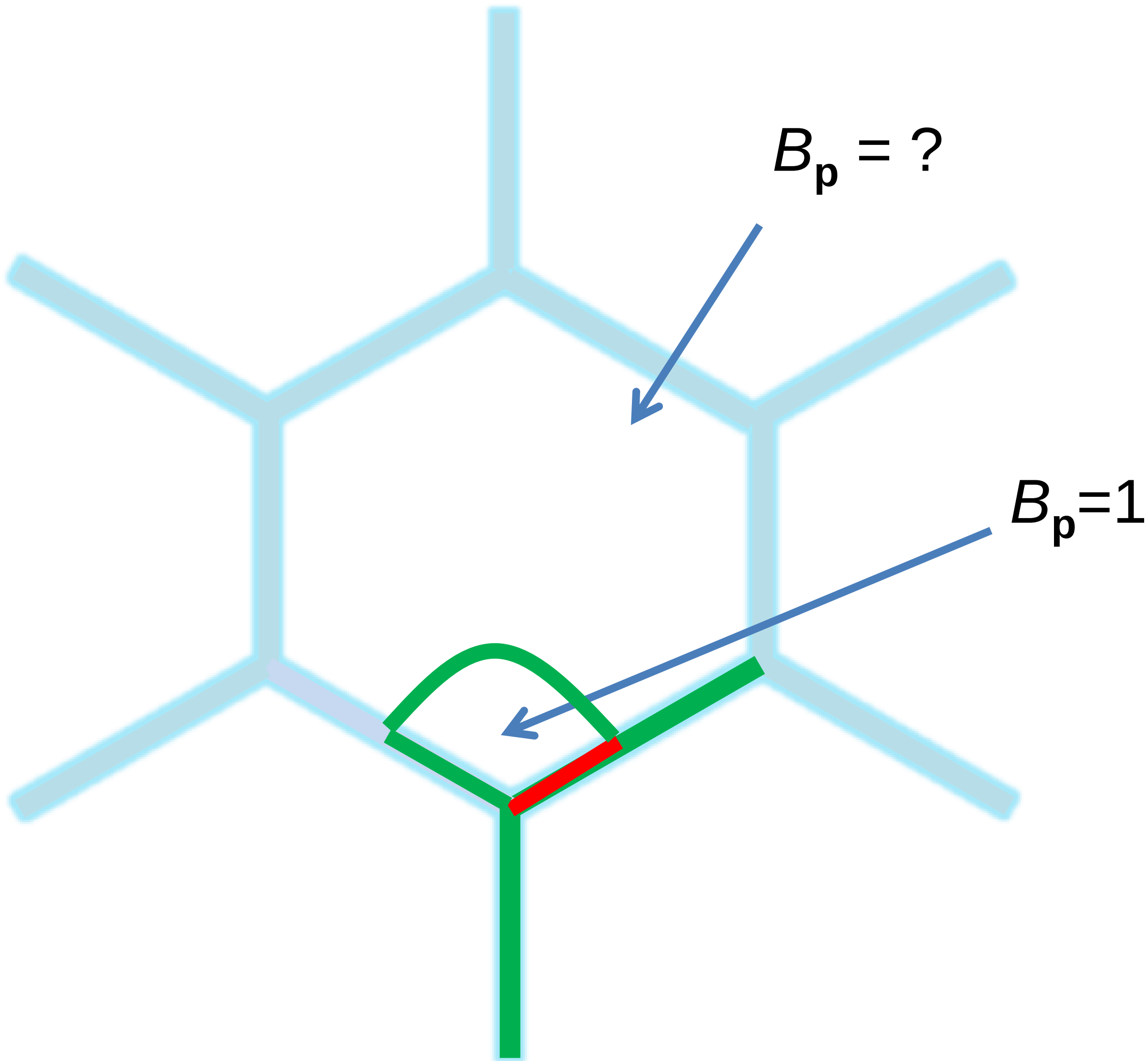

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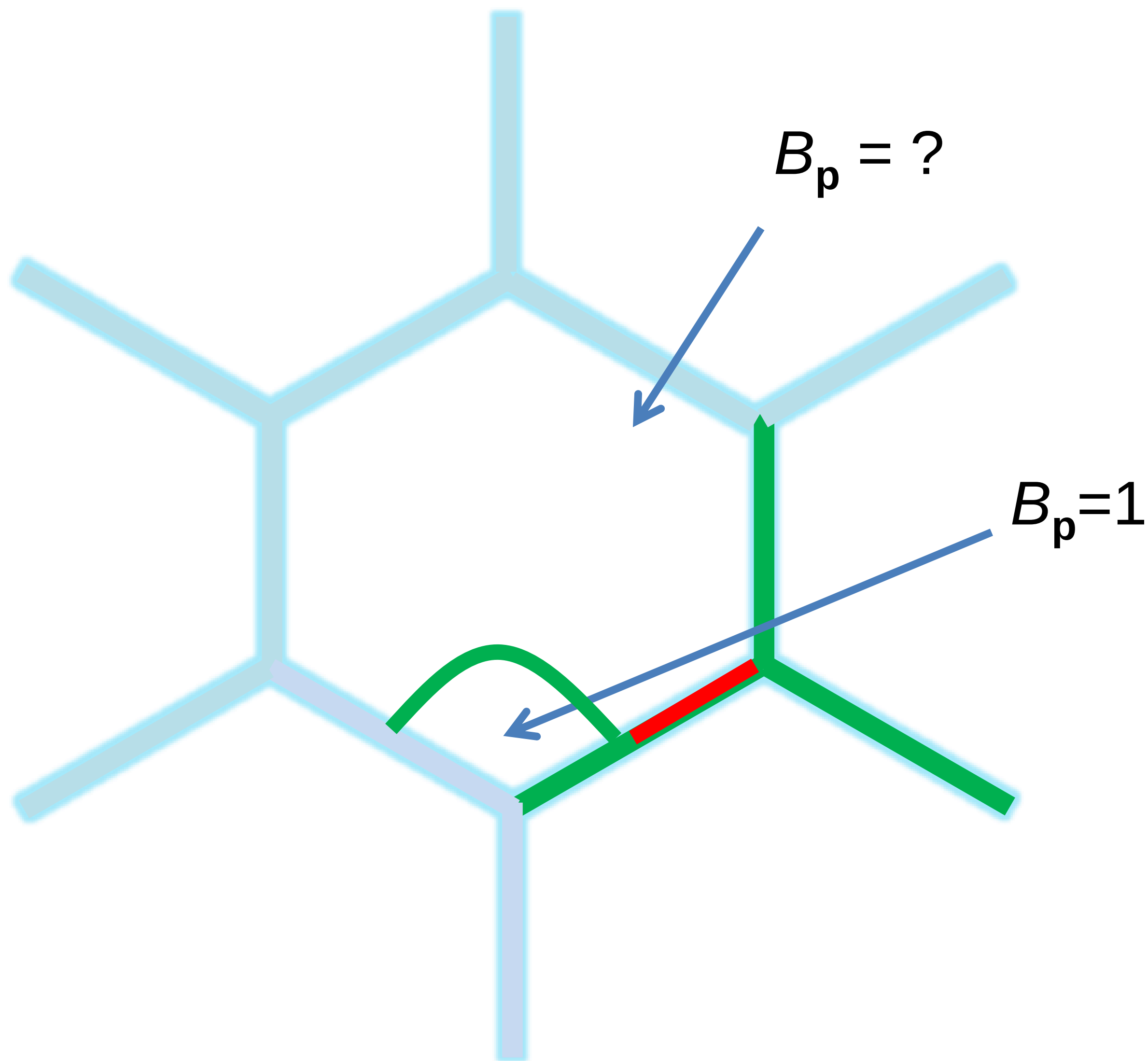

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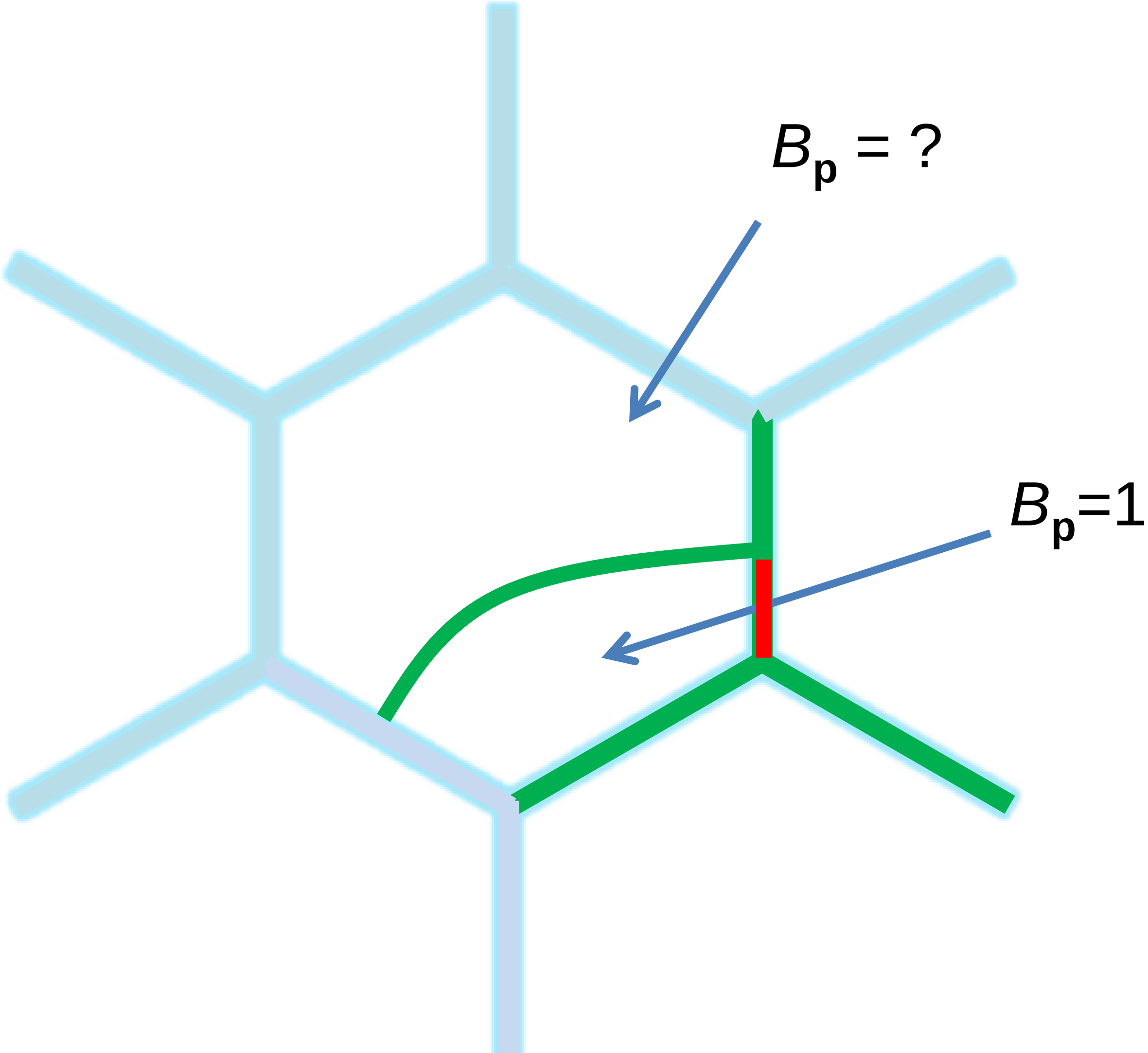

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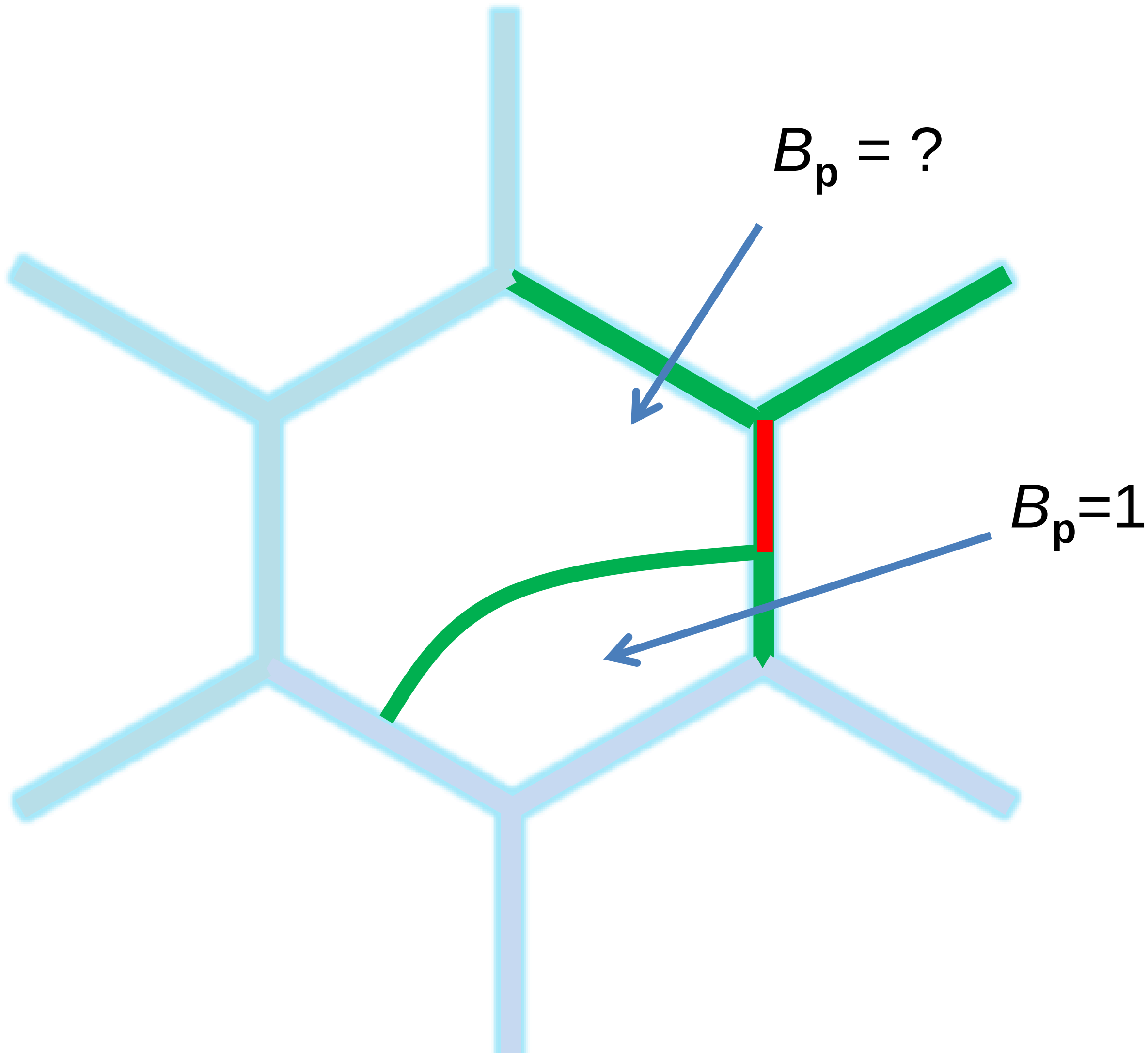

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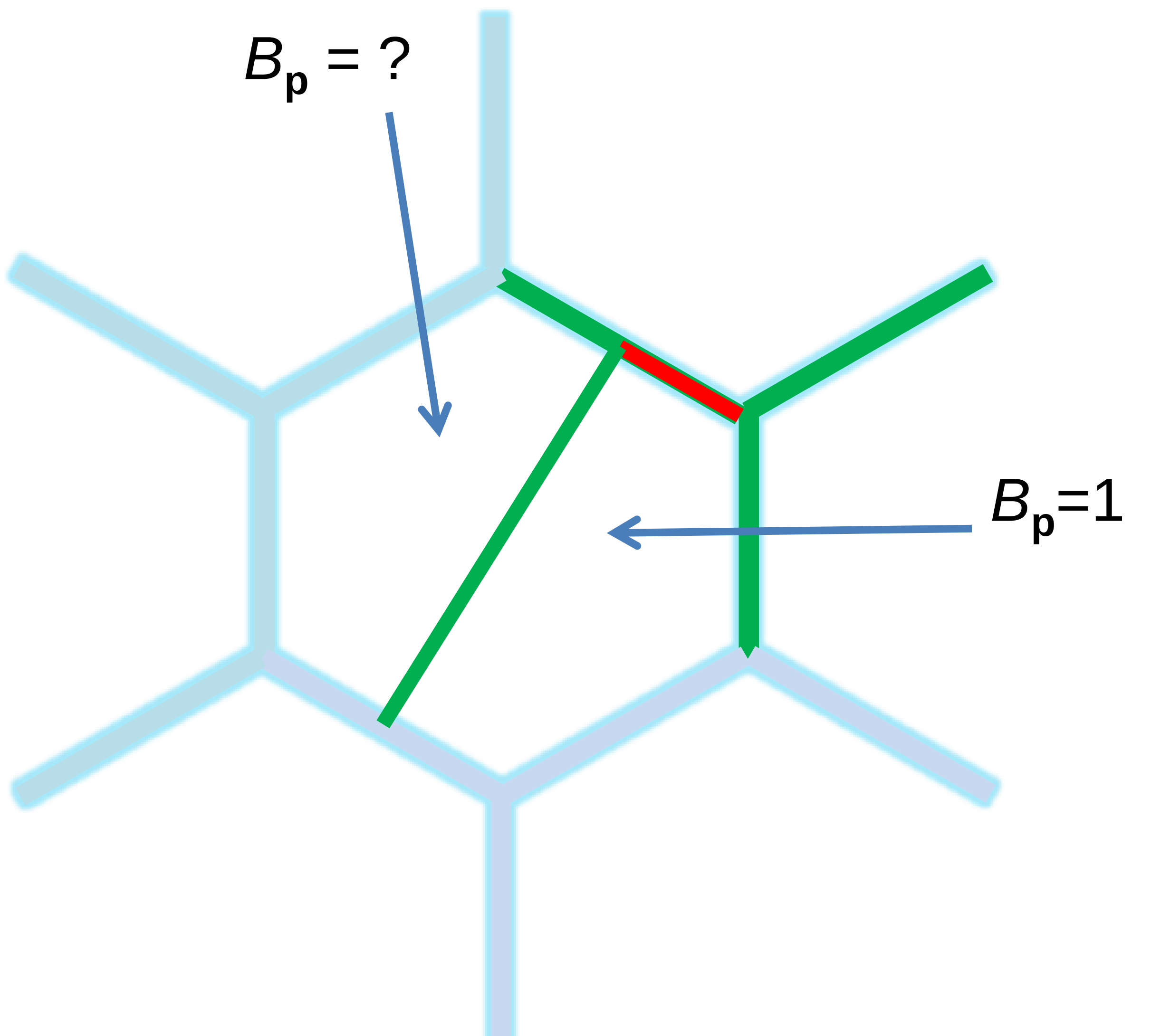

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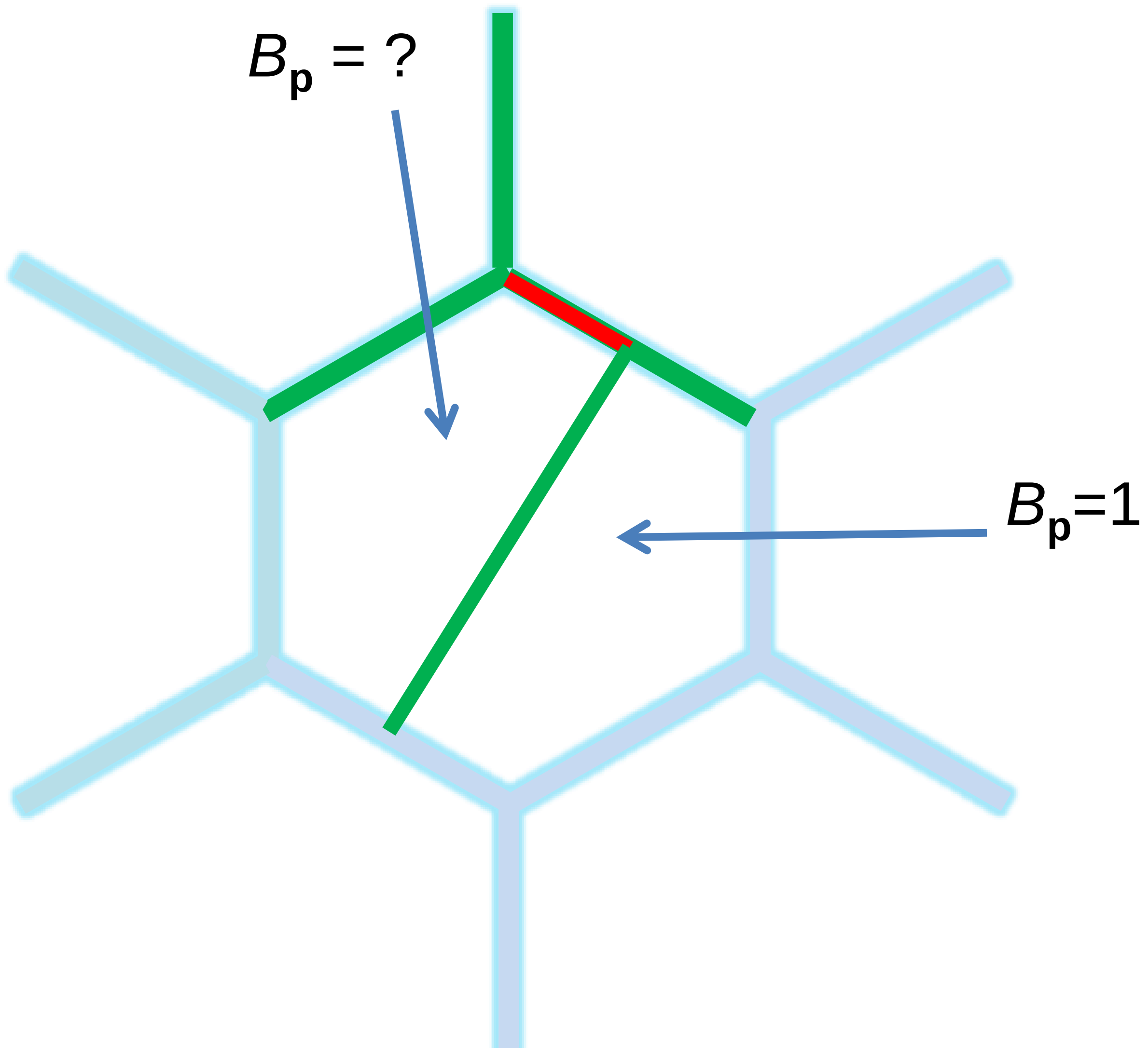

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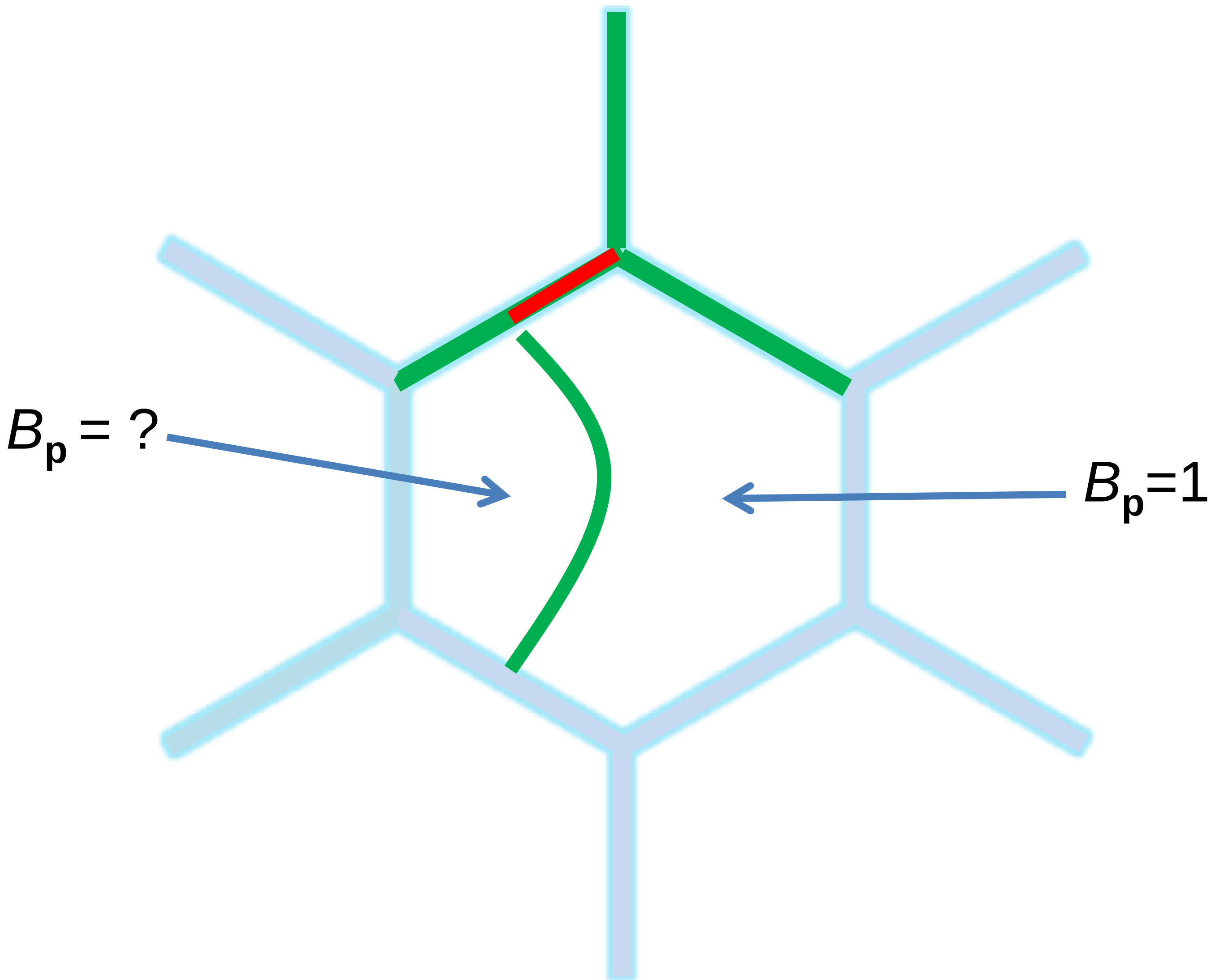

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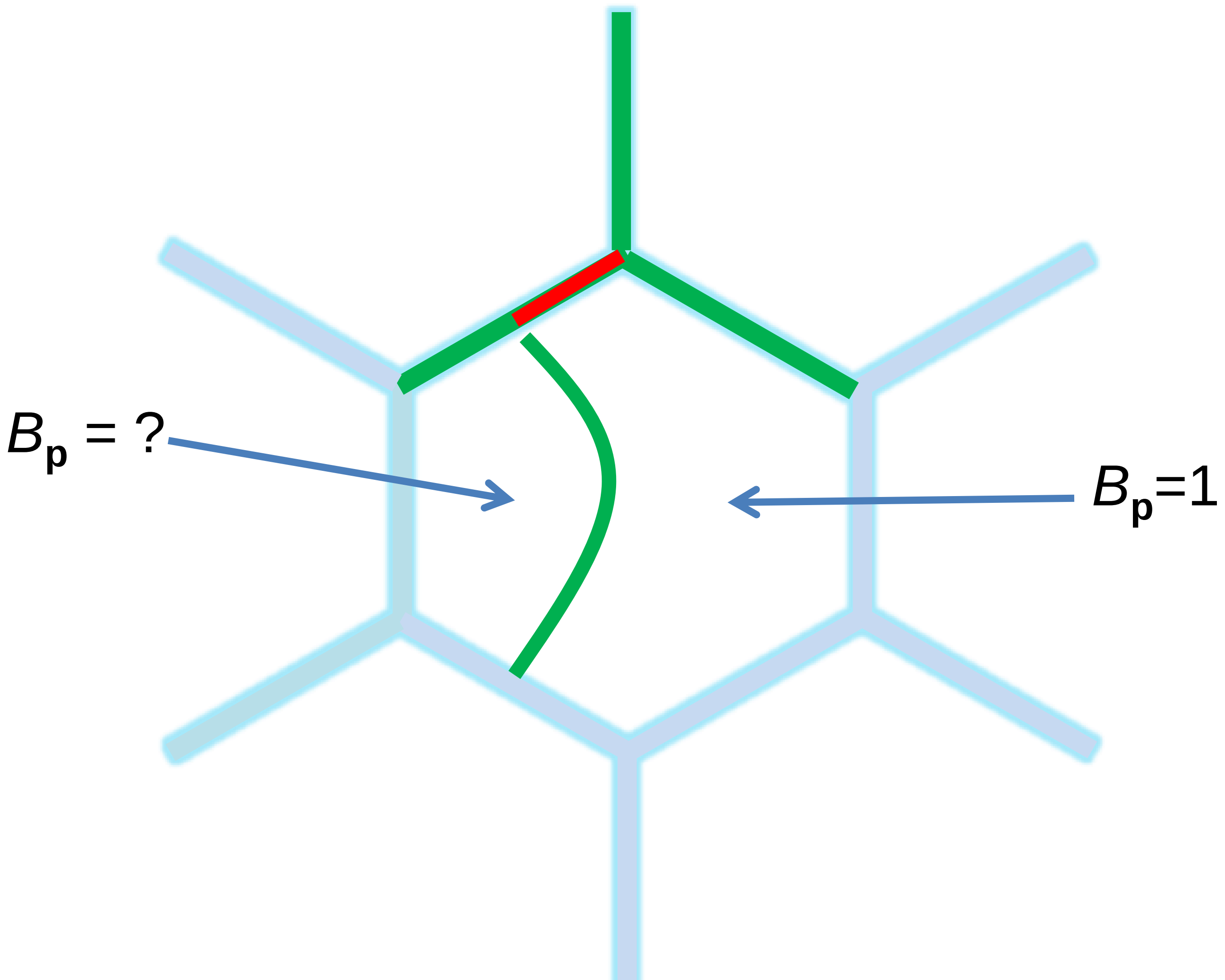

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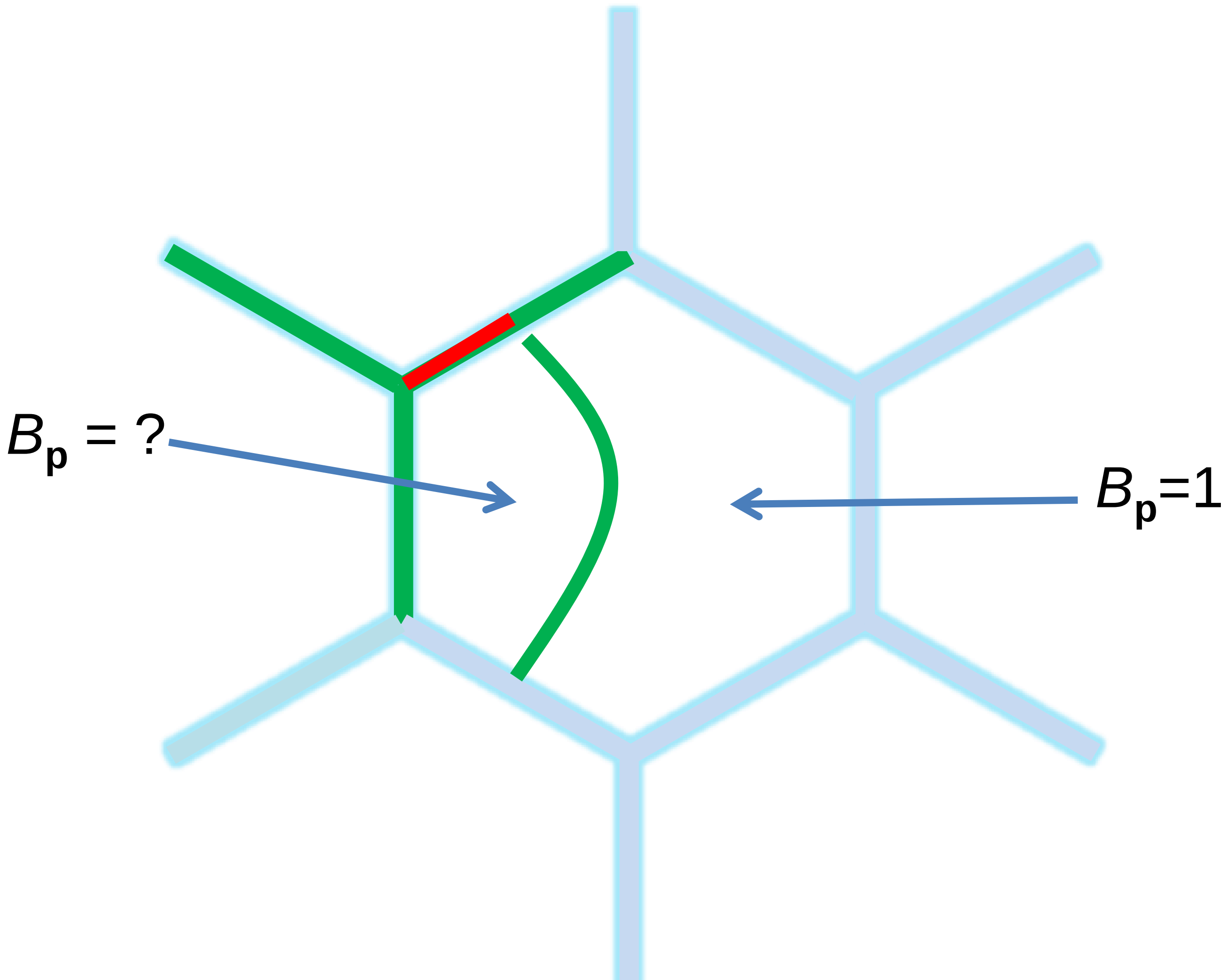

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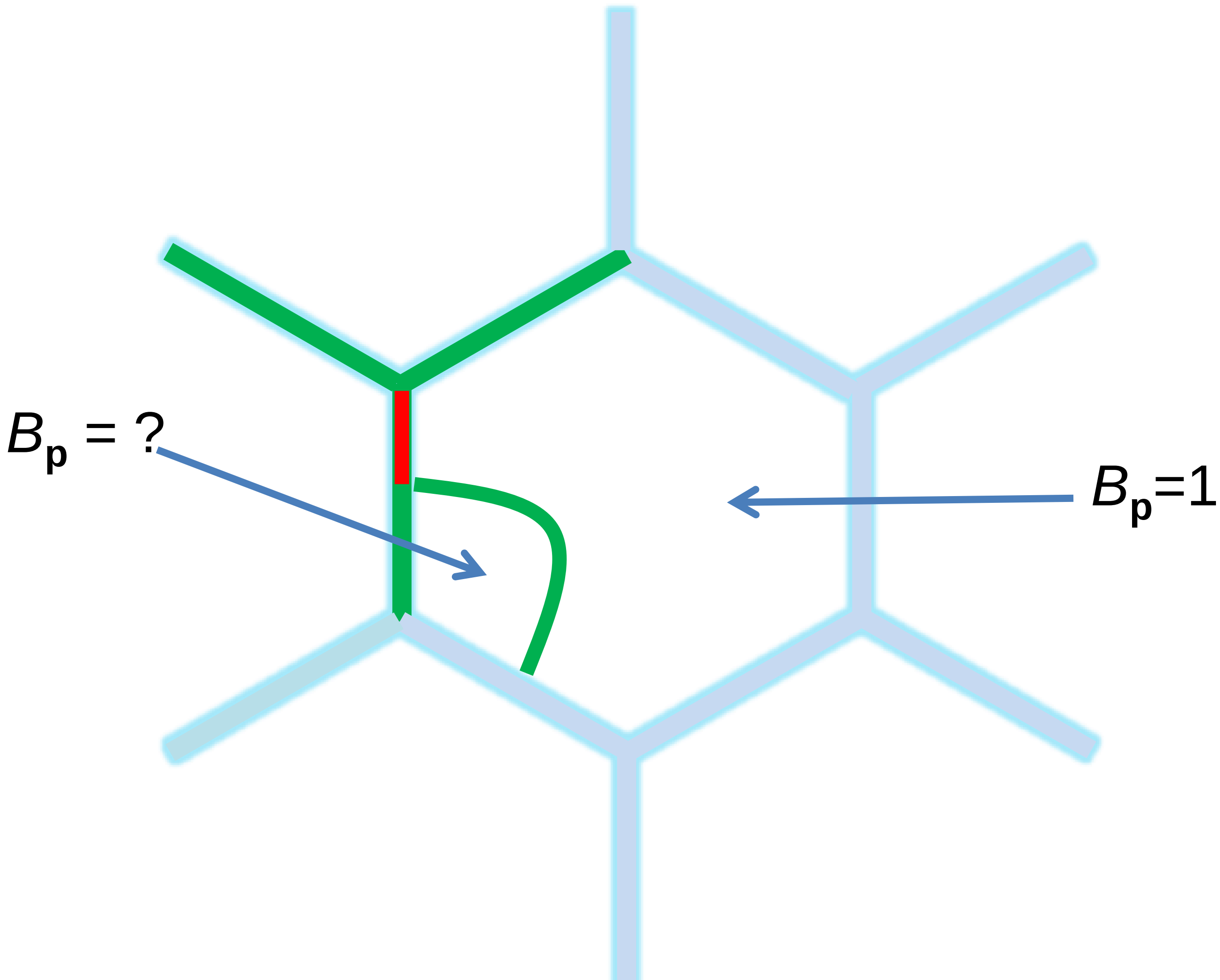

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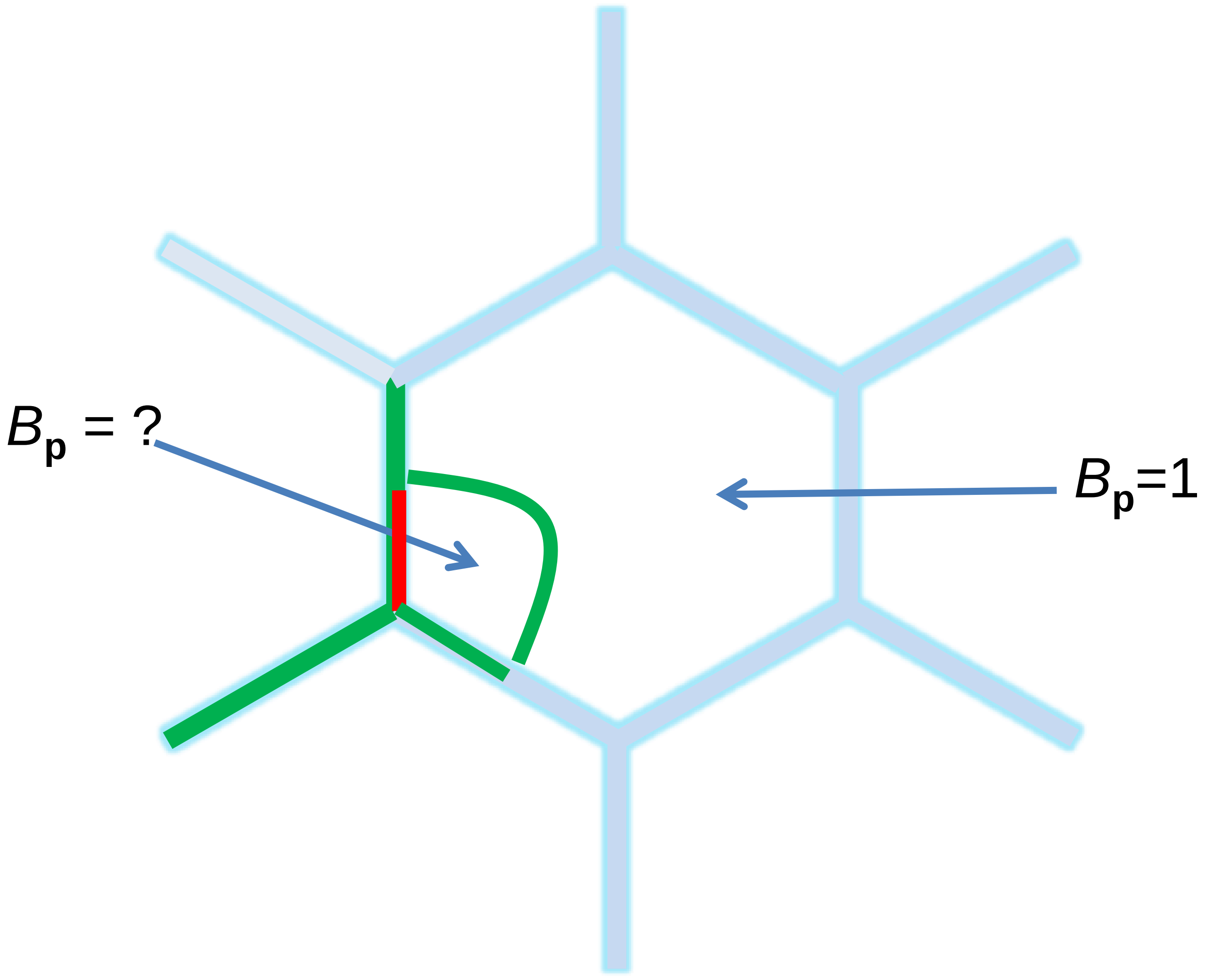

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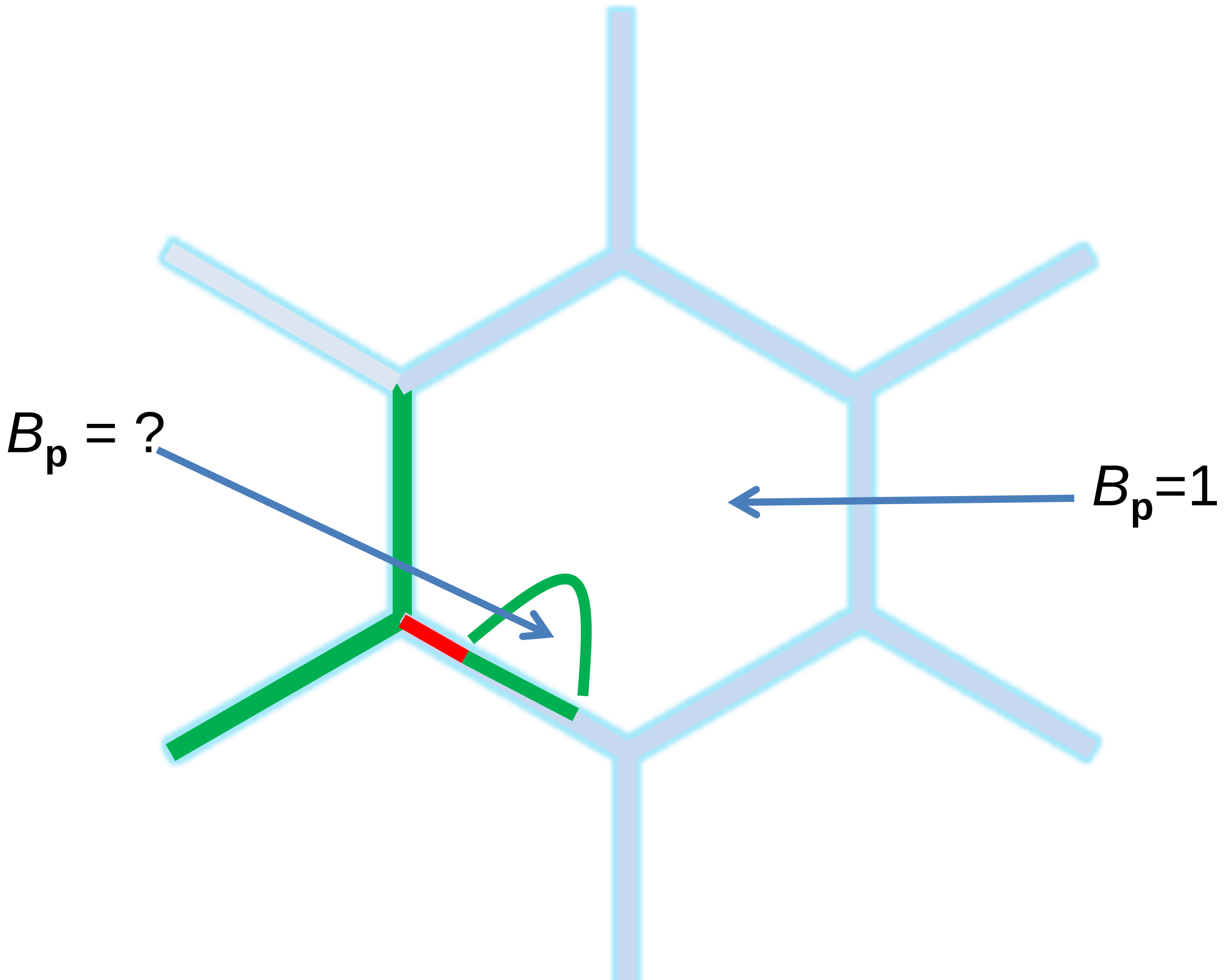

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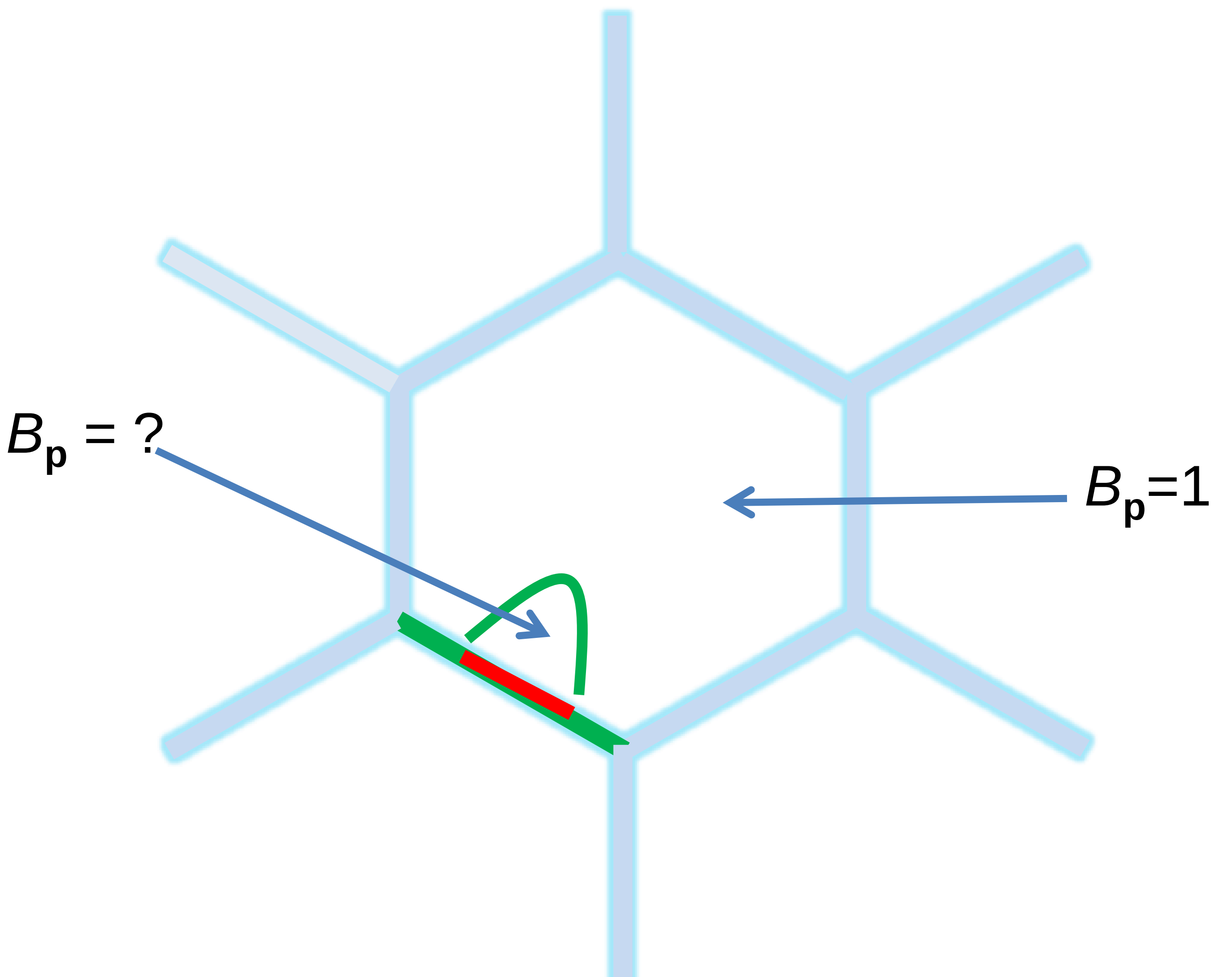

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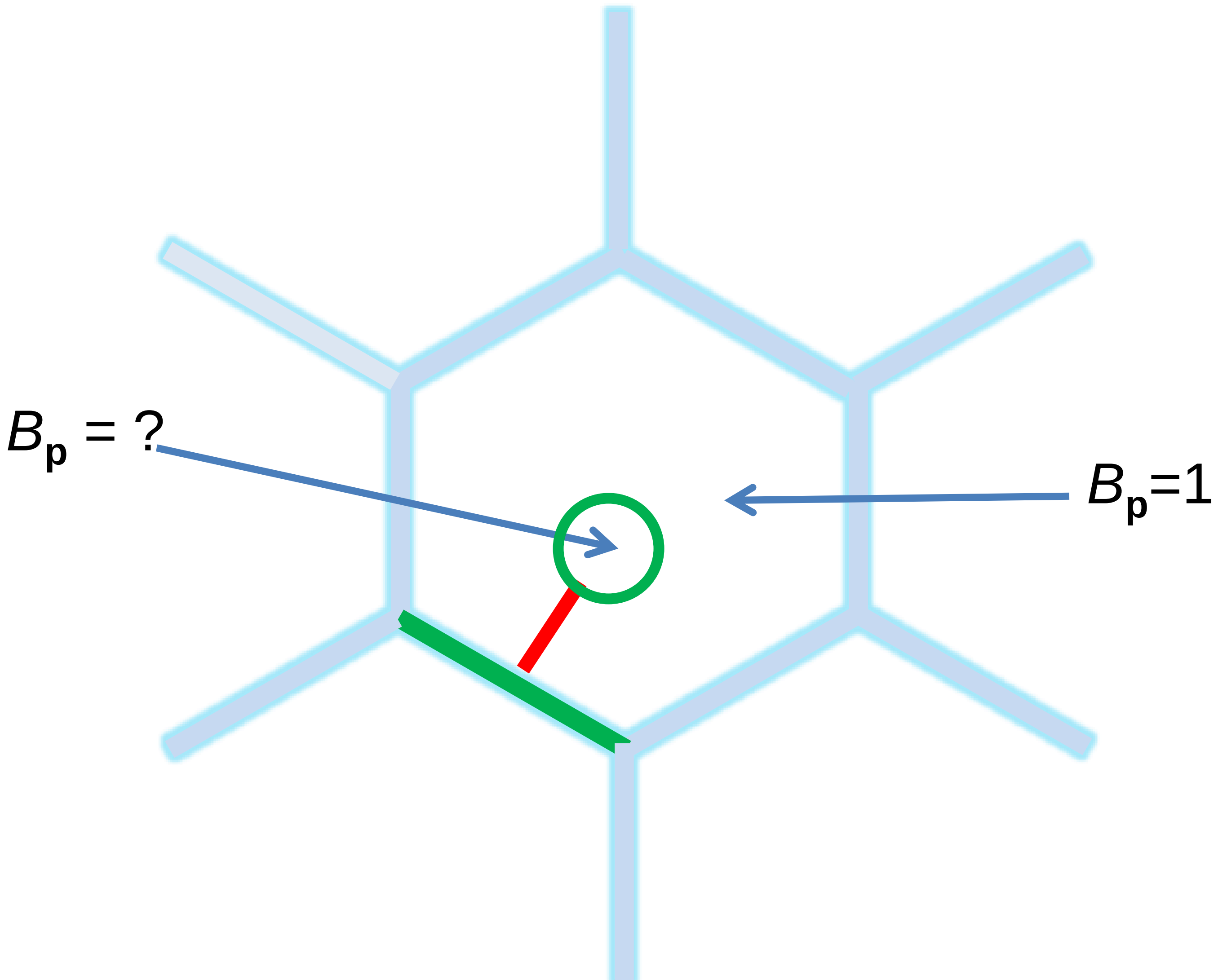

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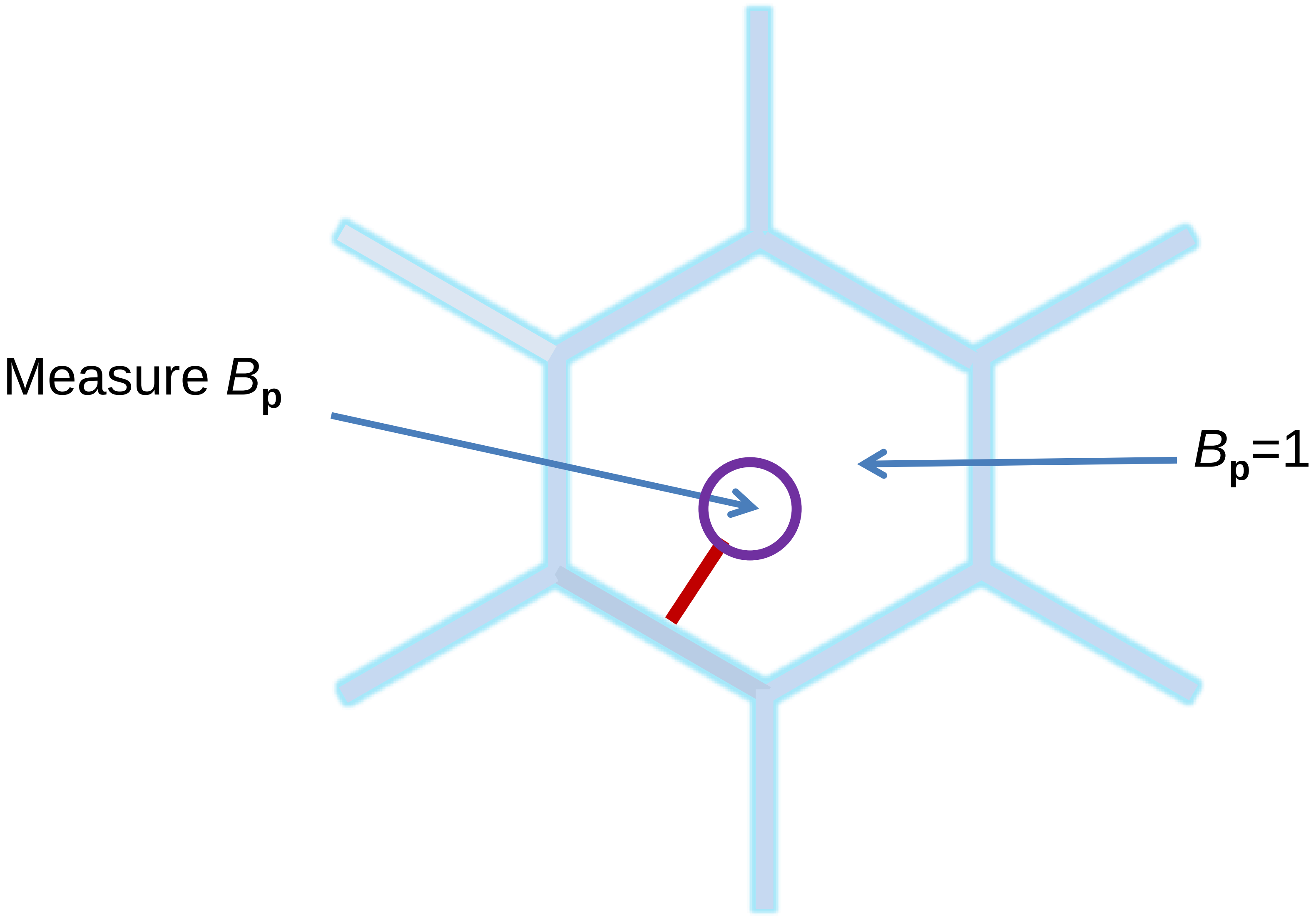
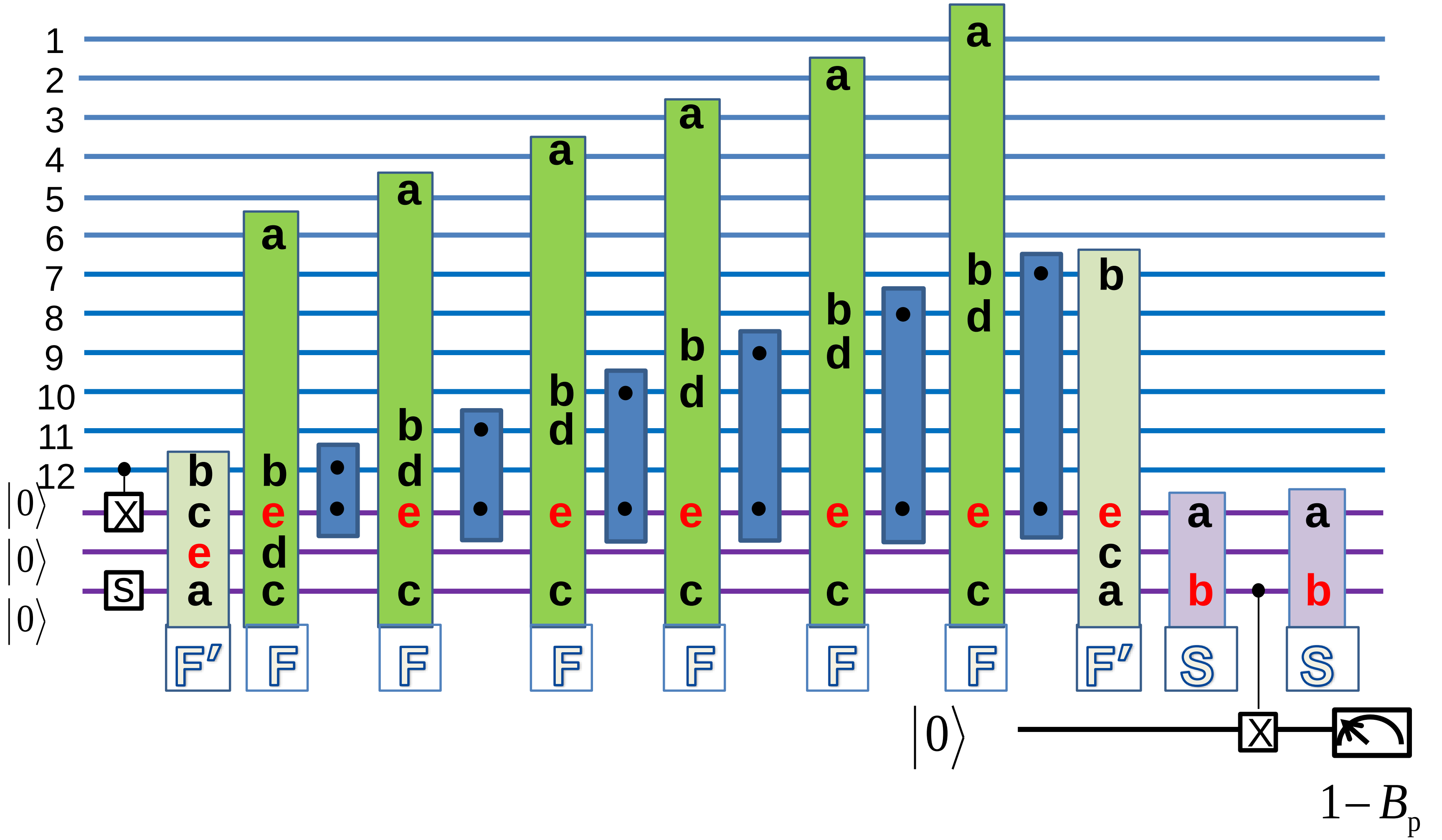
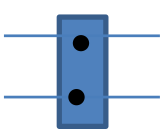


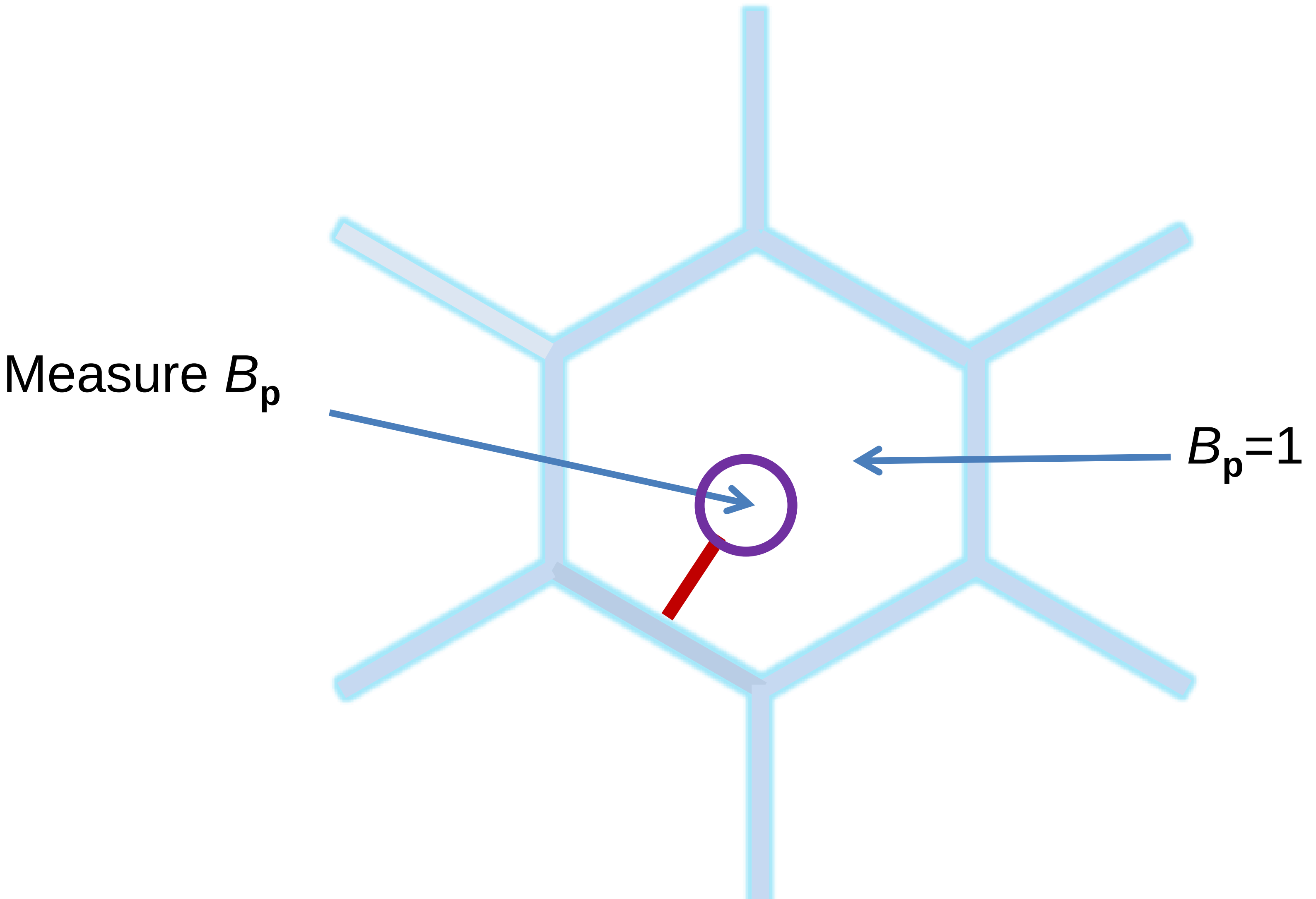
Plaquette Swapping Circuit



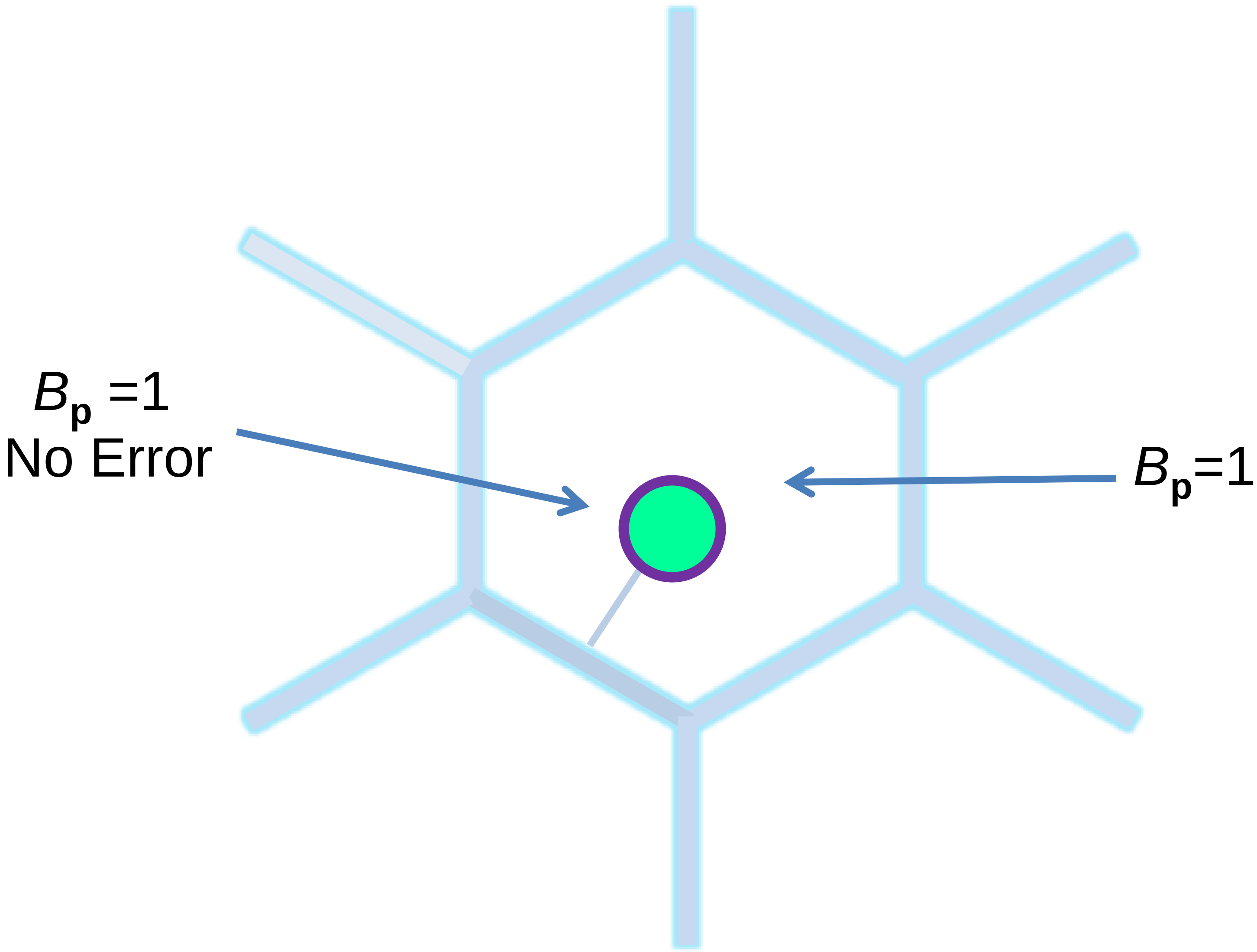


W. Feng, NEB, D.P. DiVincenzo, In Preparation

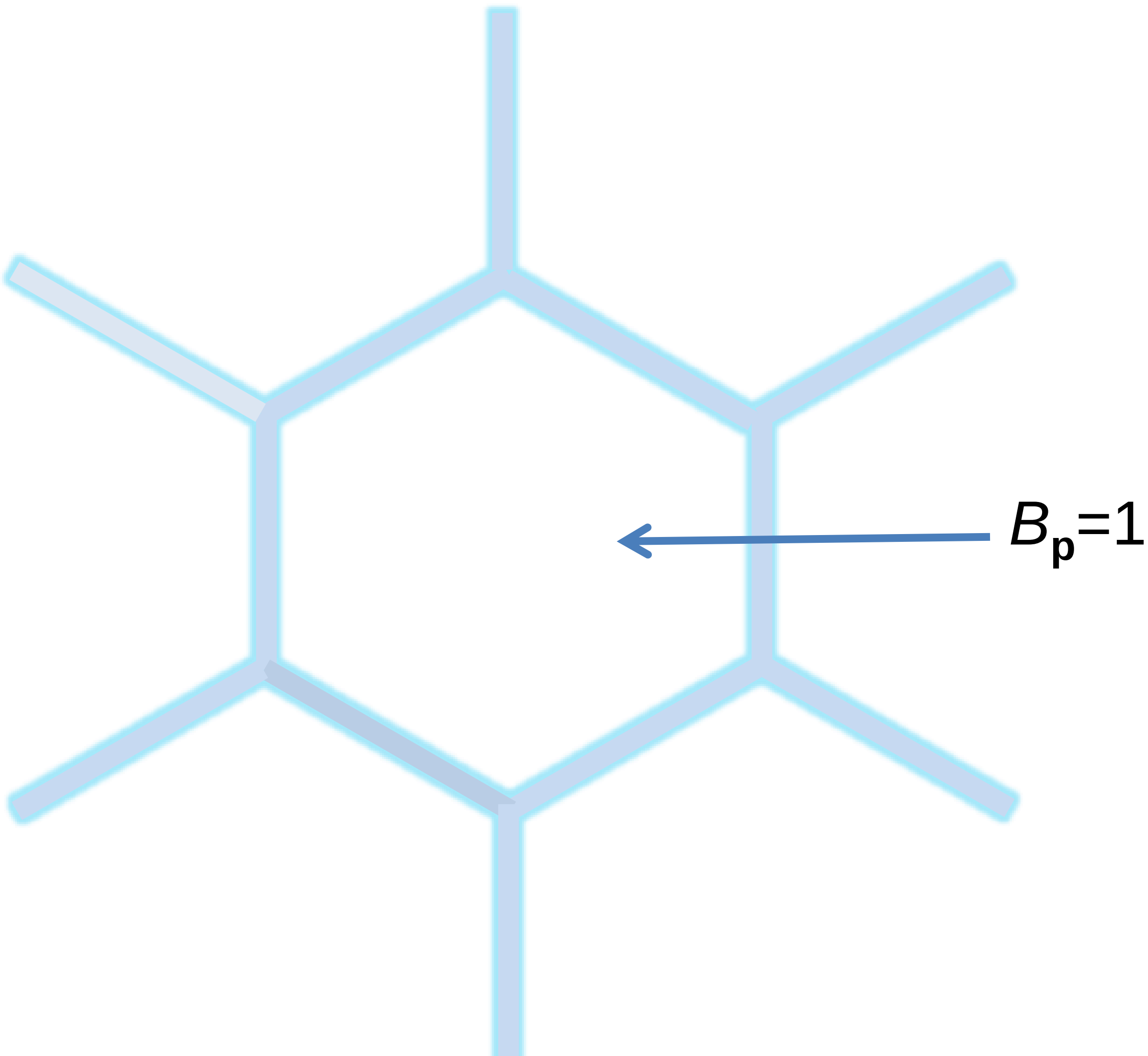
After Swapping



After Swapping

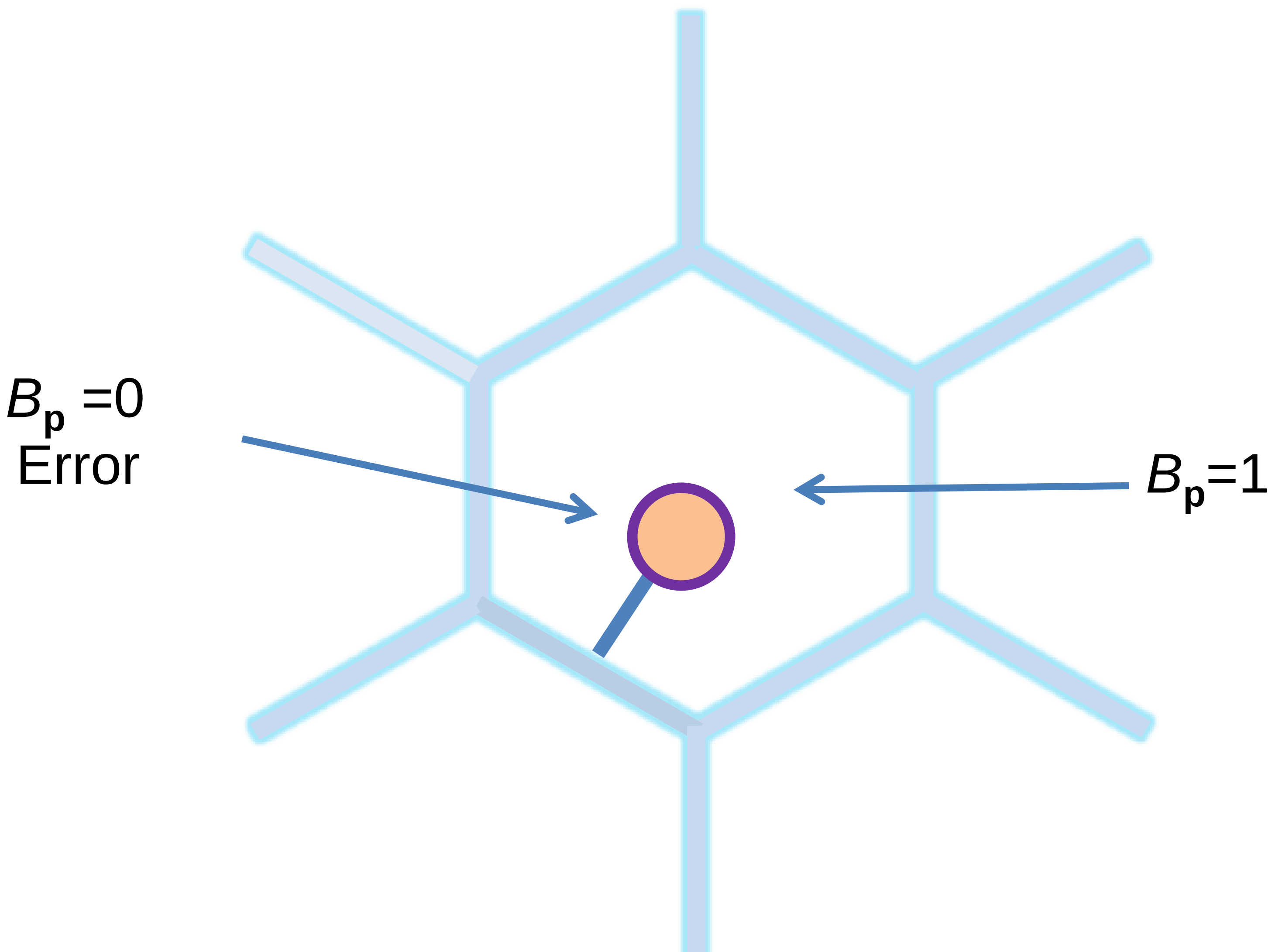


After Swapping

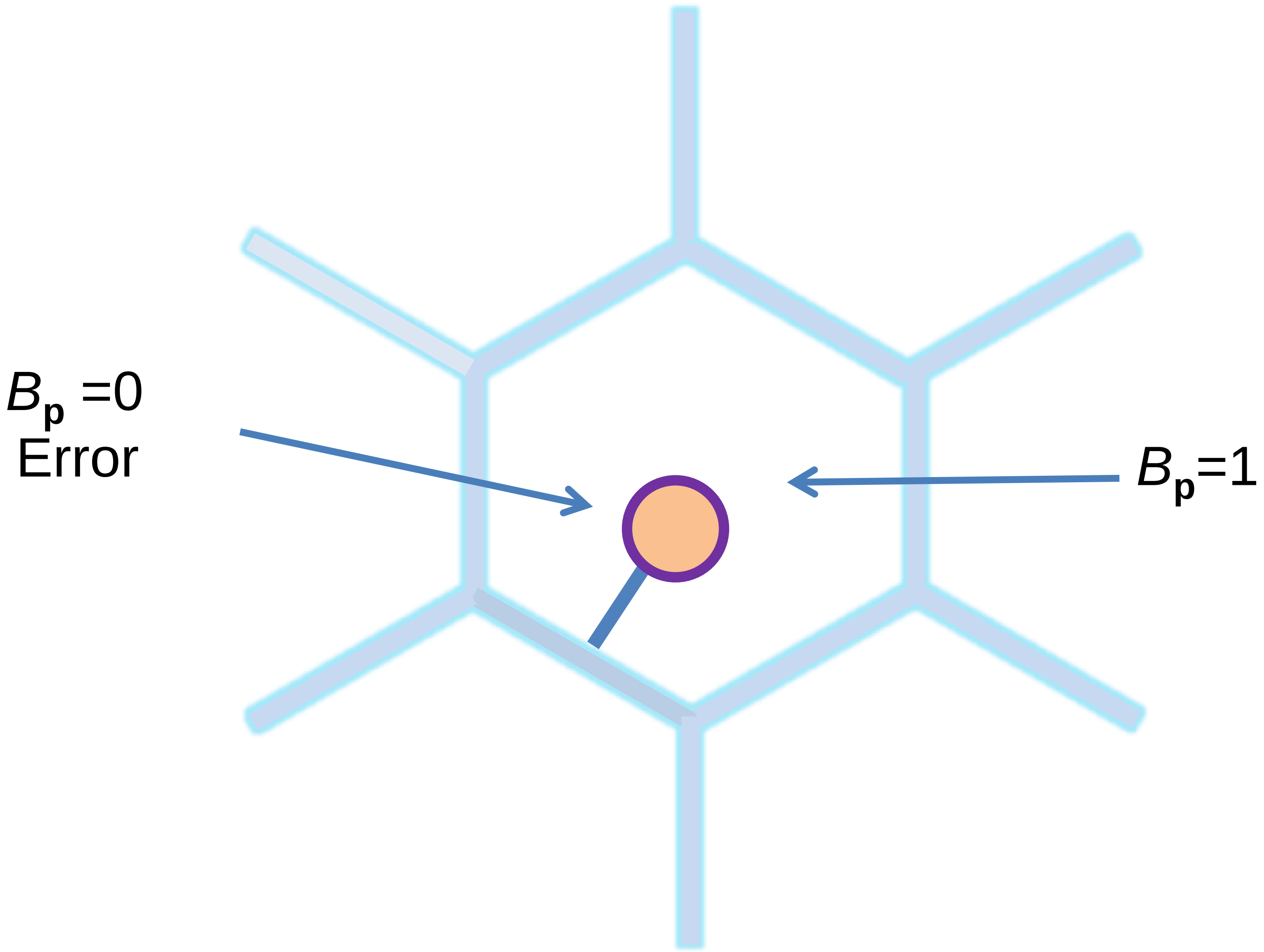


Simply Remove Swapped Plaquette

After Swapping

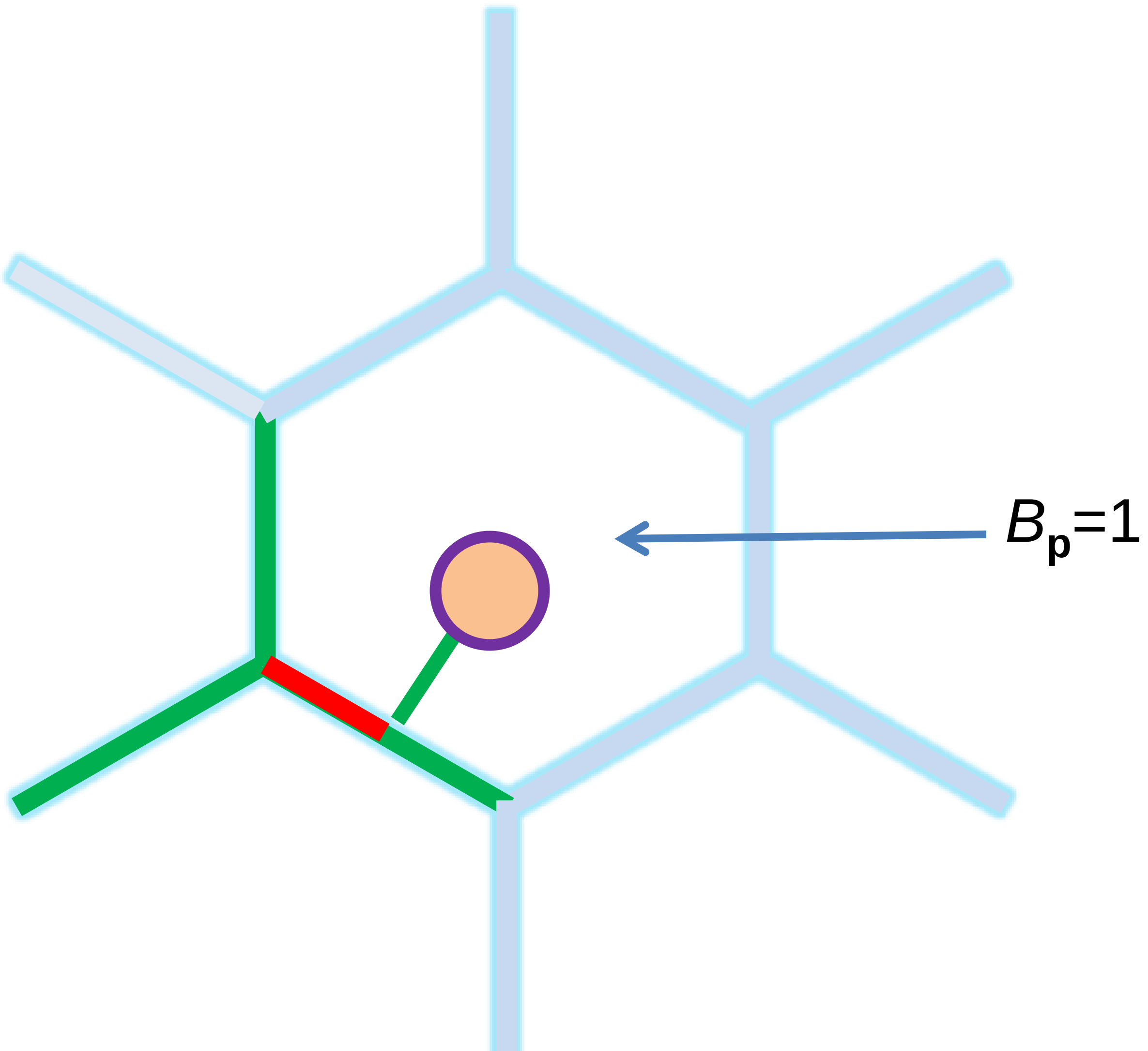


After Swapping



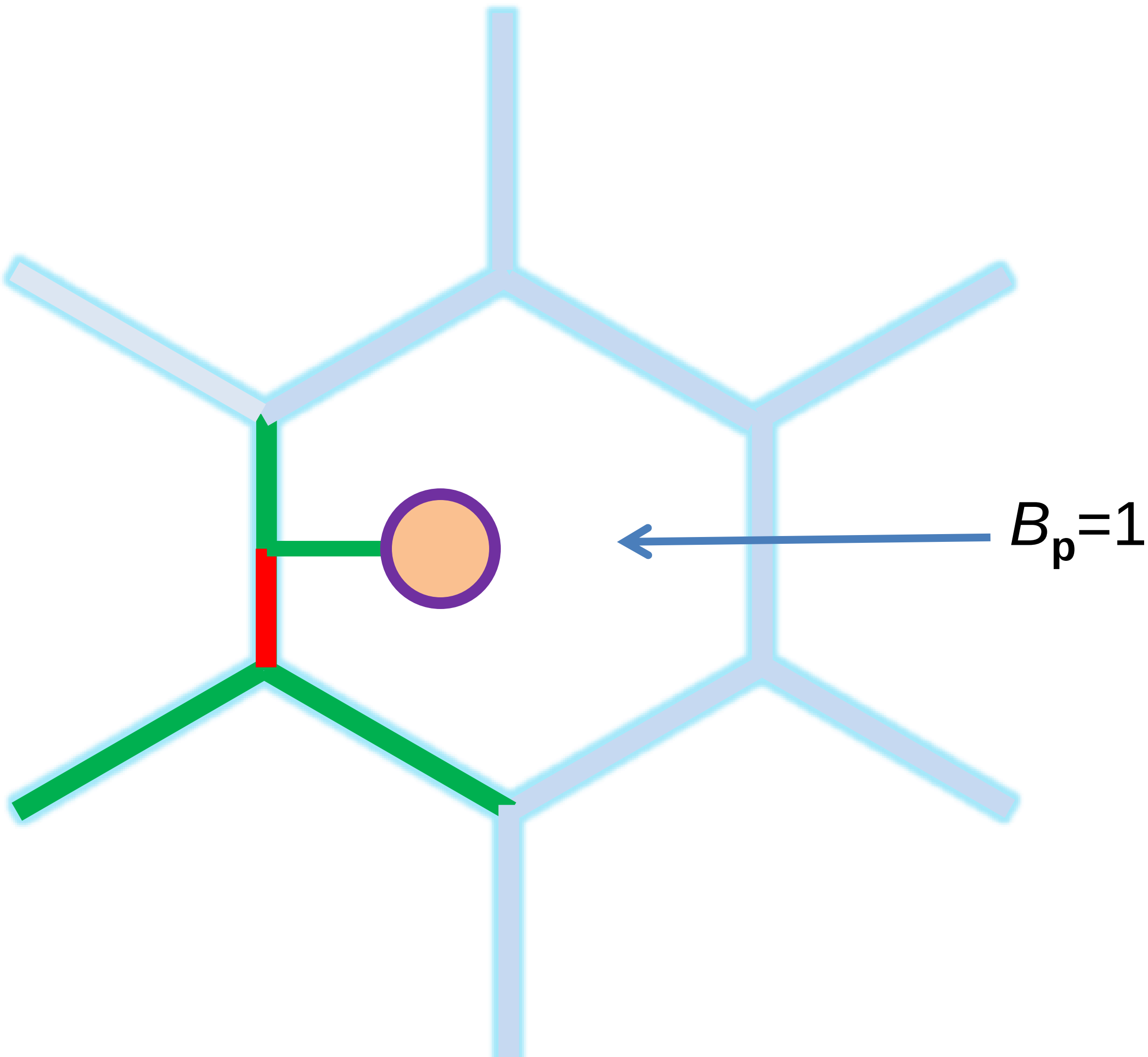
Move “Packaged” Error Using F-Moves

After Swapping



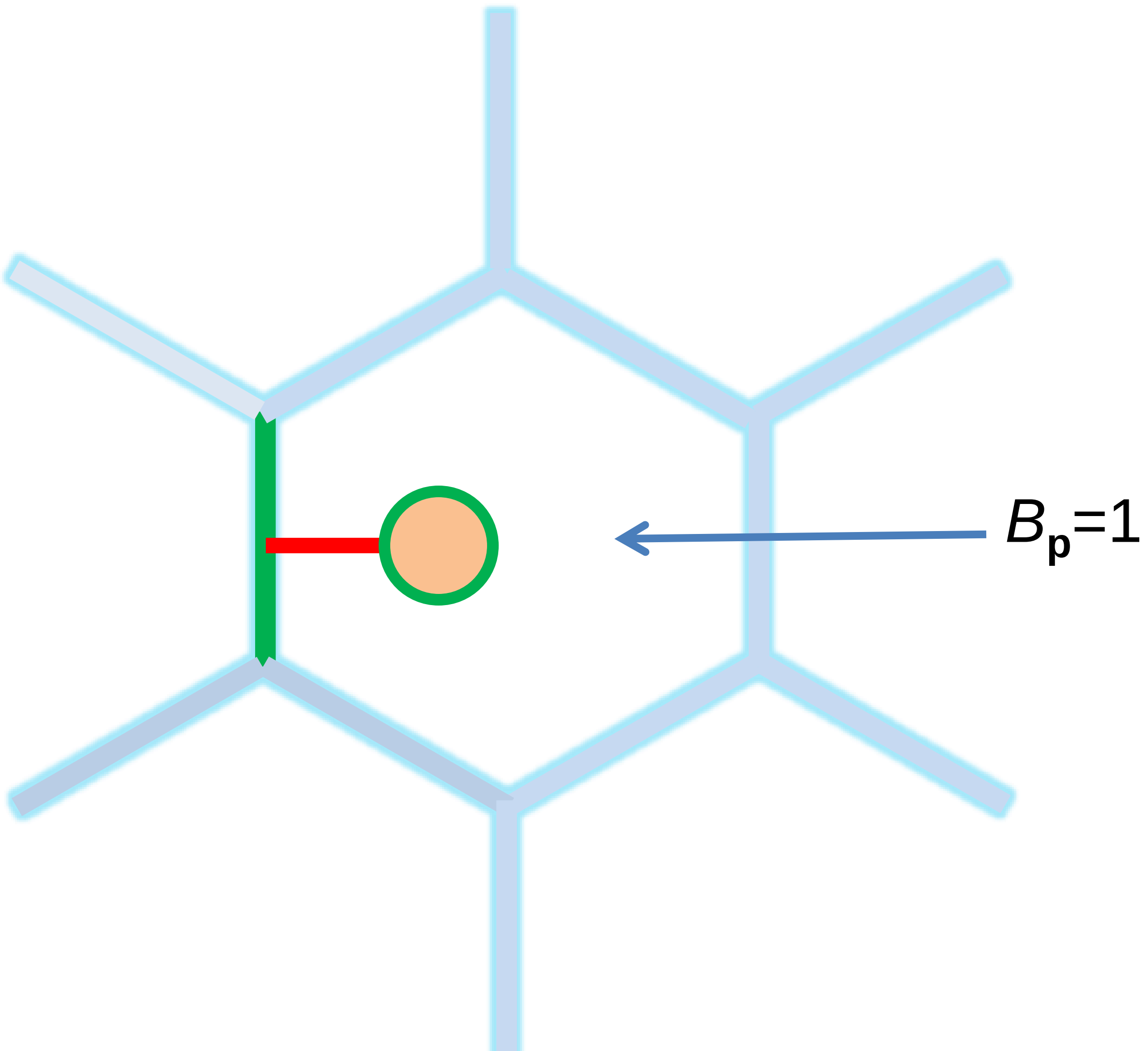
Move "Packaged" Error Using F-Moves

After Swapping



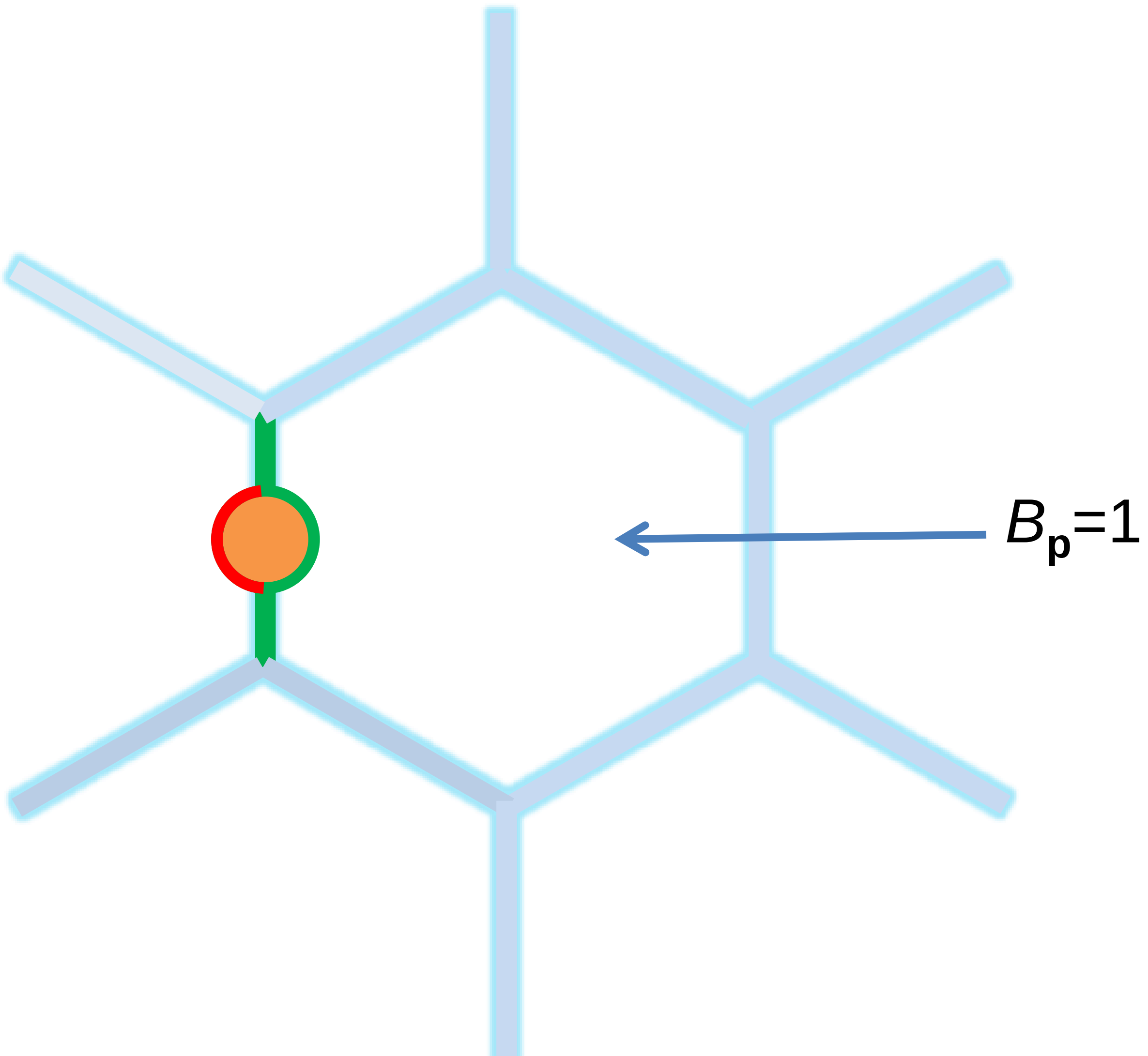
Move "Packaged" Error Using F-Moves

After Swapping



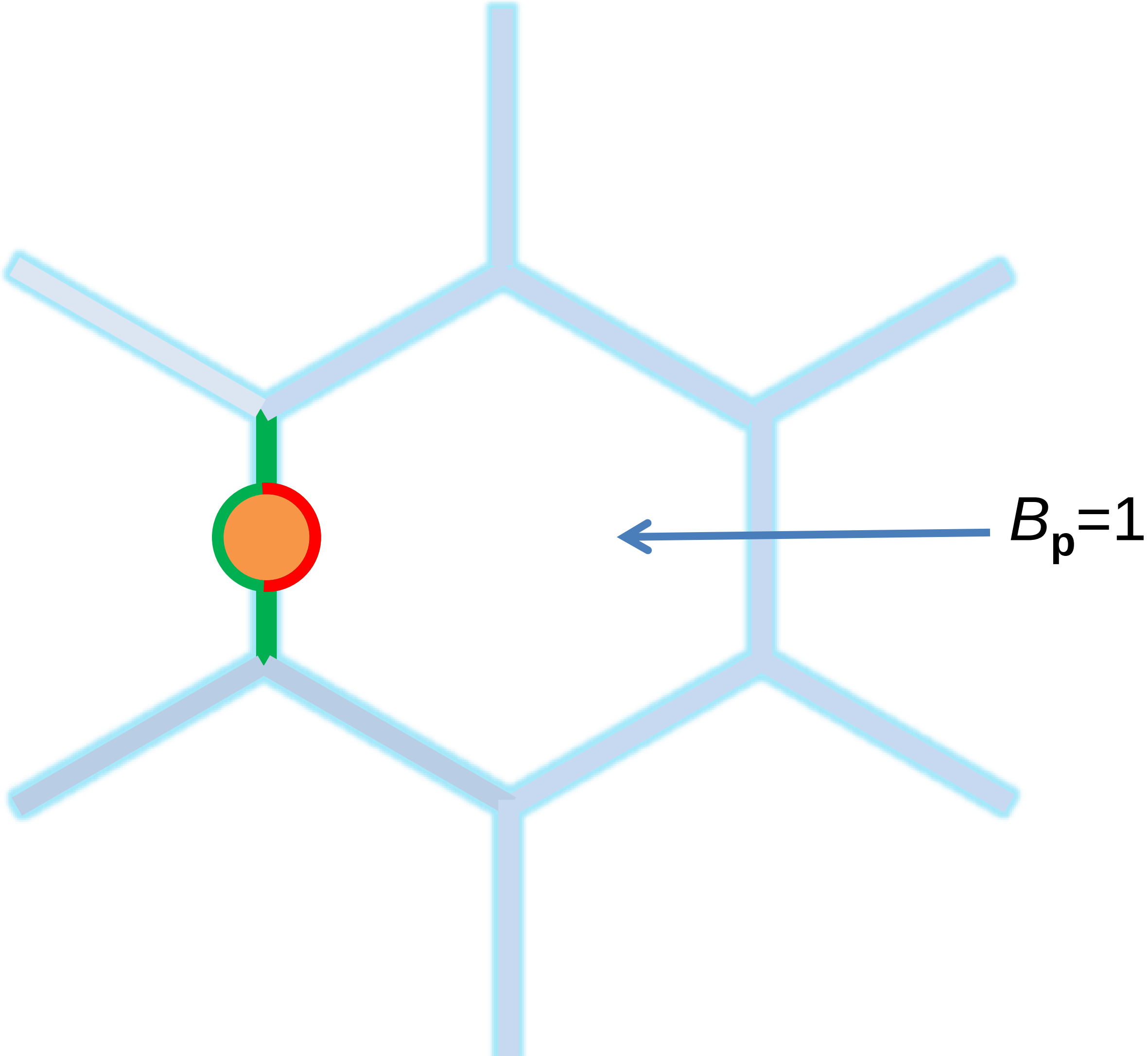
Move "Packaged" Error Using F-Moves

After Swapping



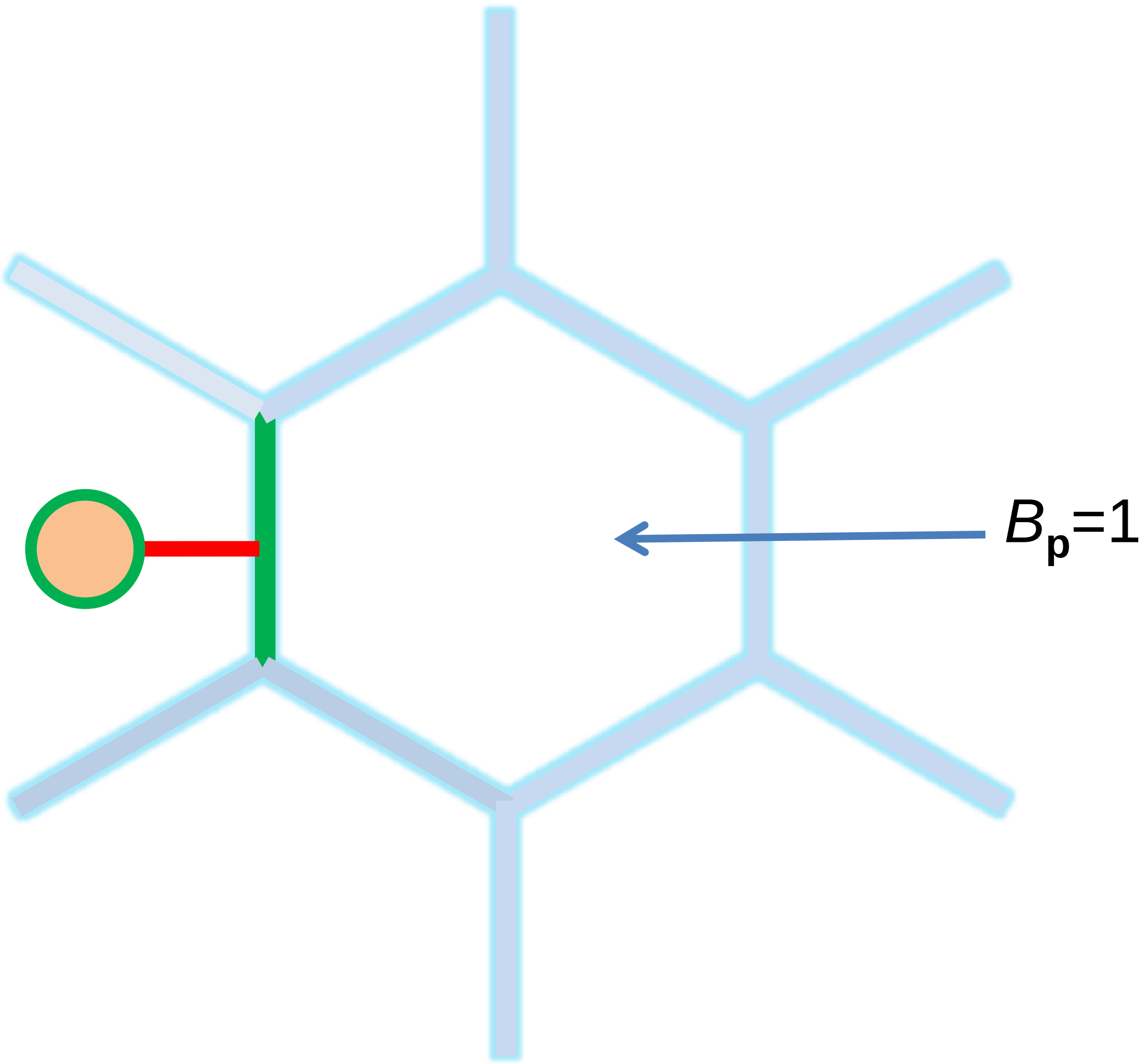
Move "Packaged" Error Using F-Moves

After Swapping



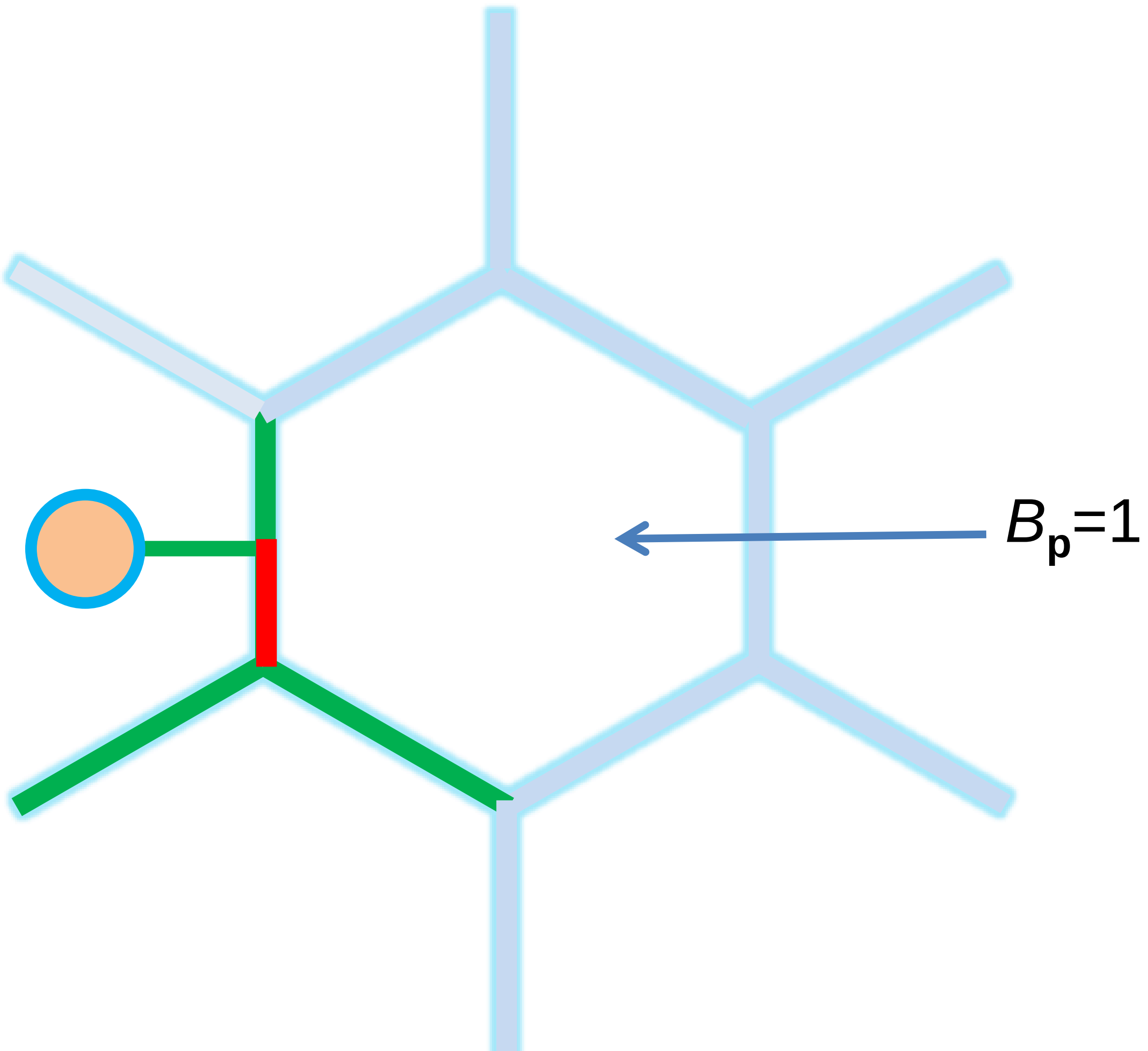
Move “Packaged” Error Using F-Moves

After Swapping



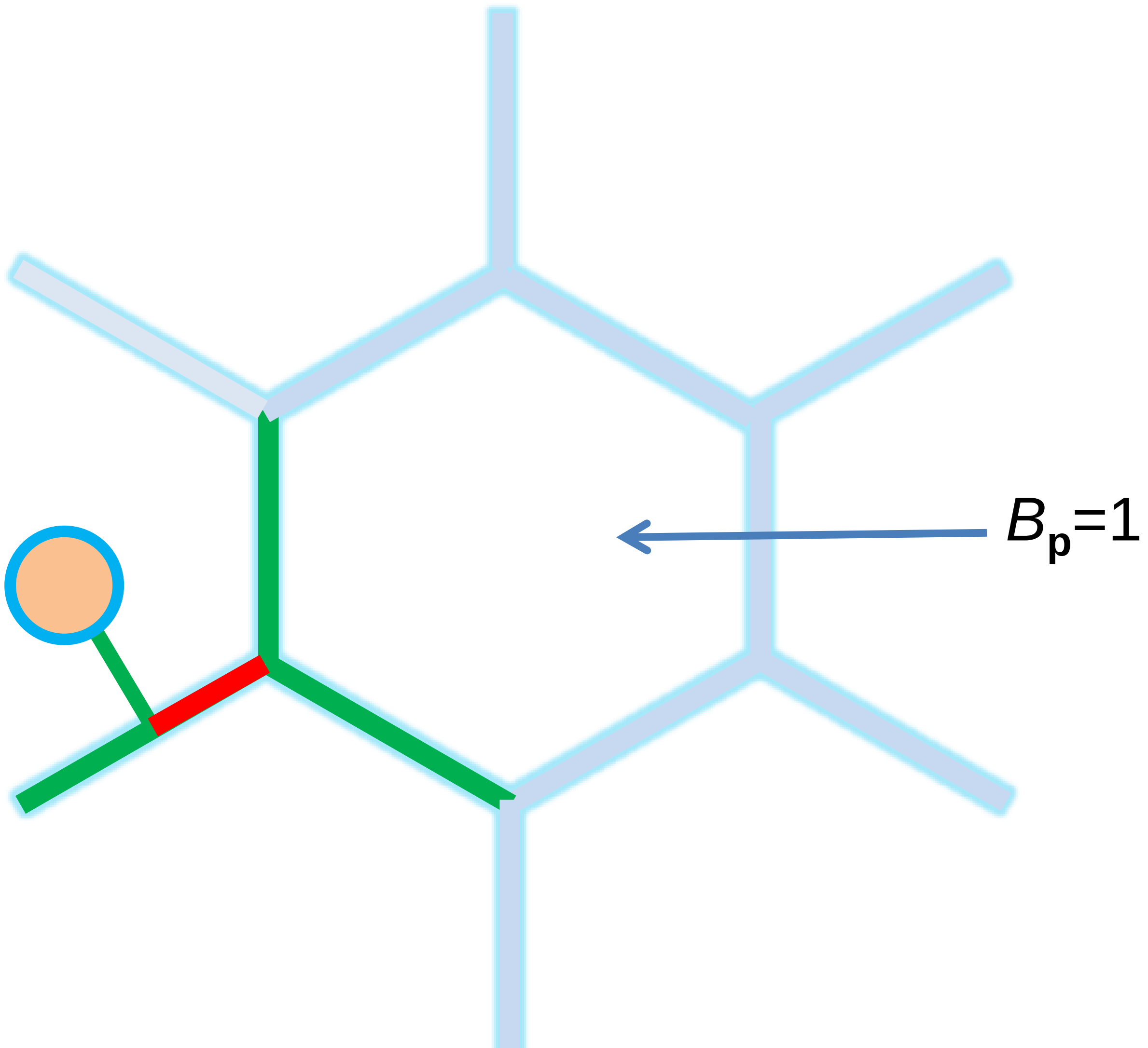
Move “Packaged” Error Using F-Moves

After Swapping



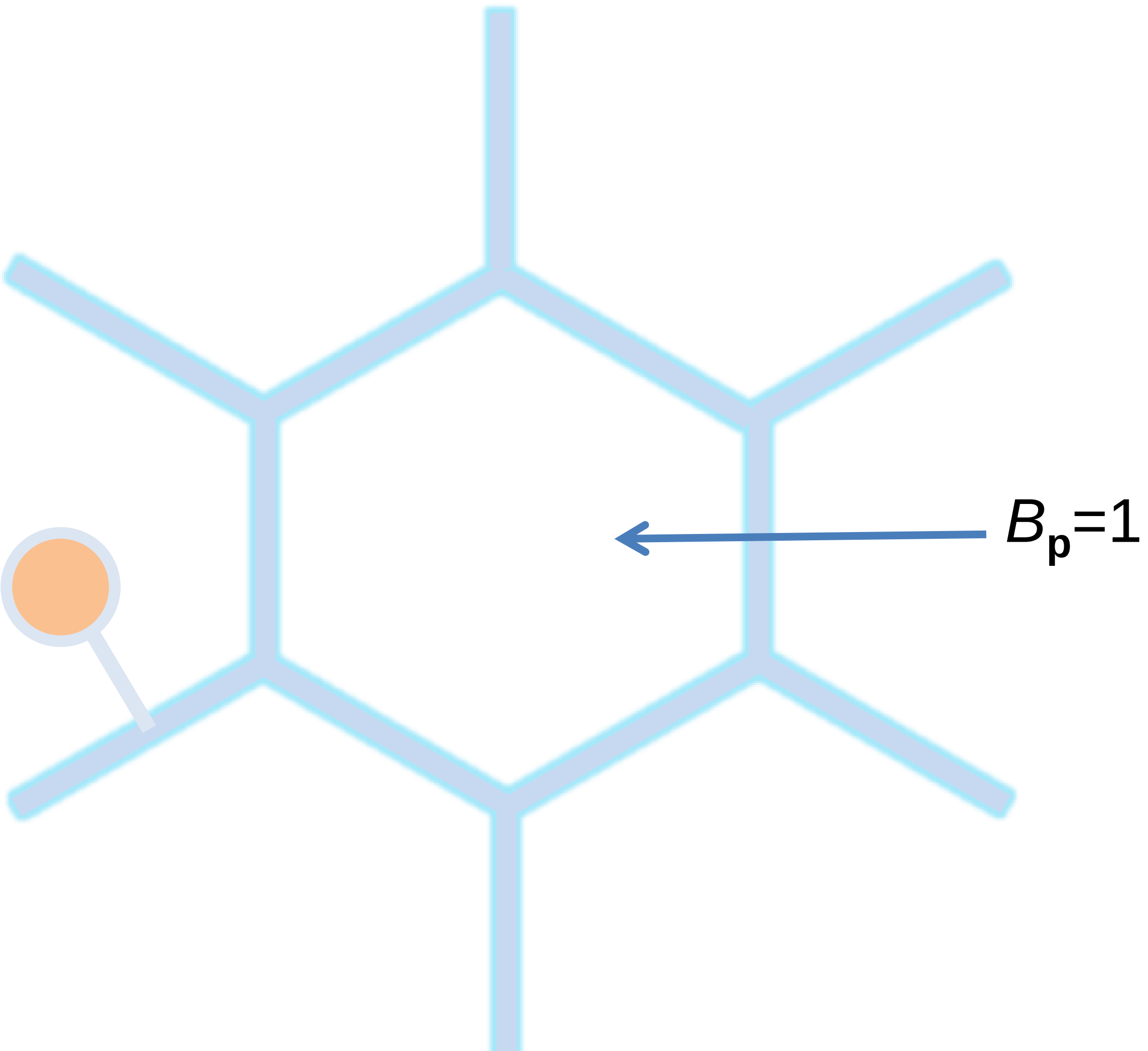
Move "Packaged" Error Using F-Moves

After Swapping



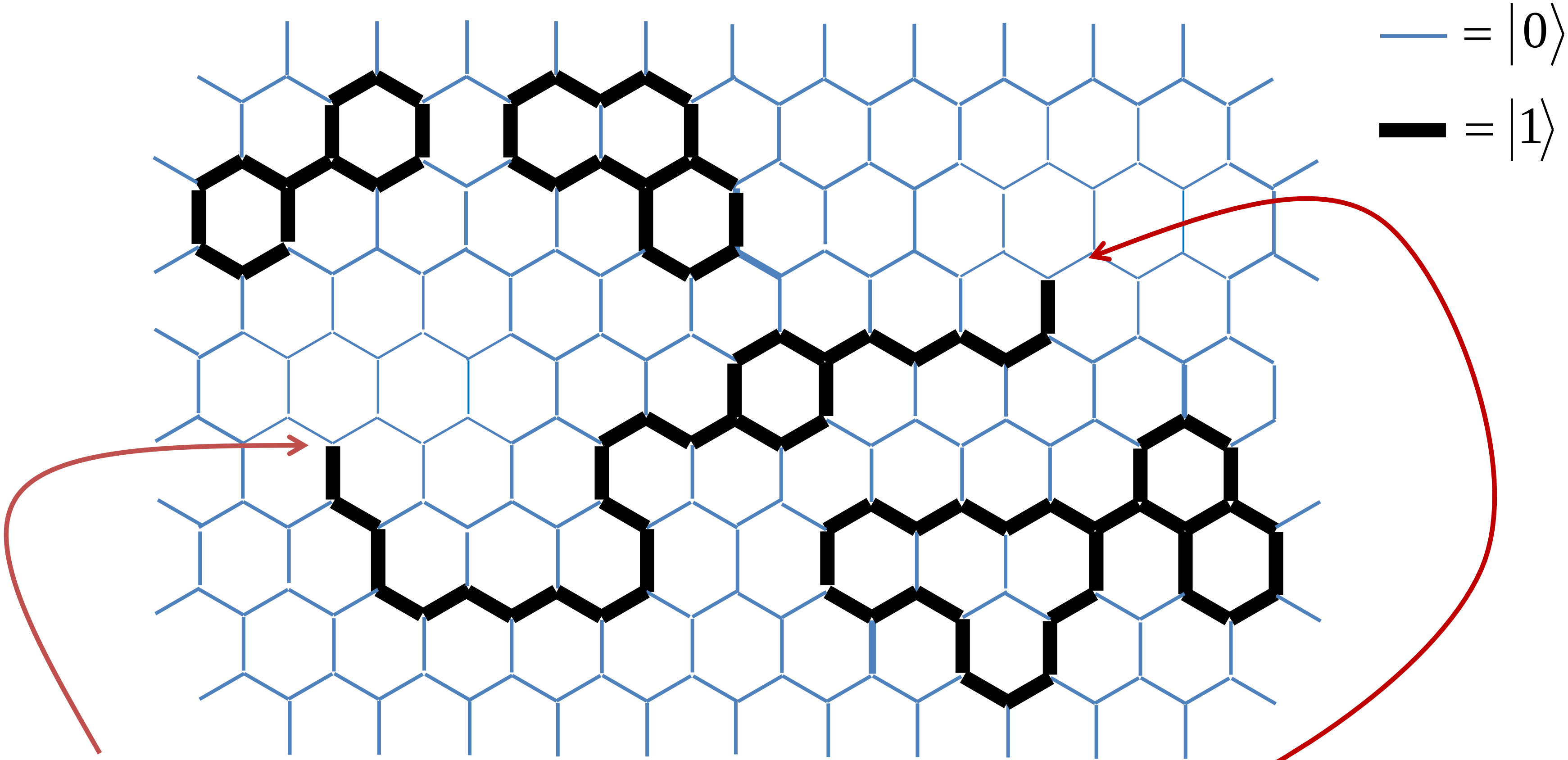
Move "Packaged" Error Using F-Moves

After Swapping



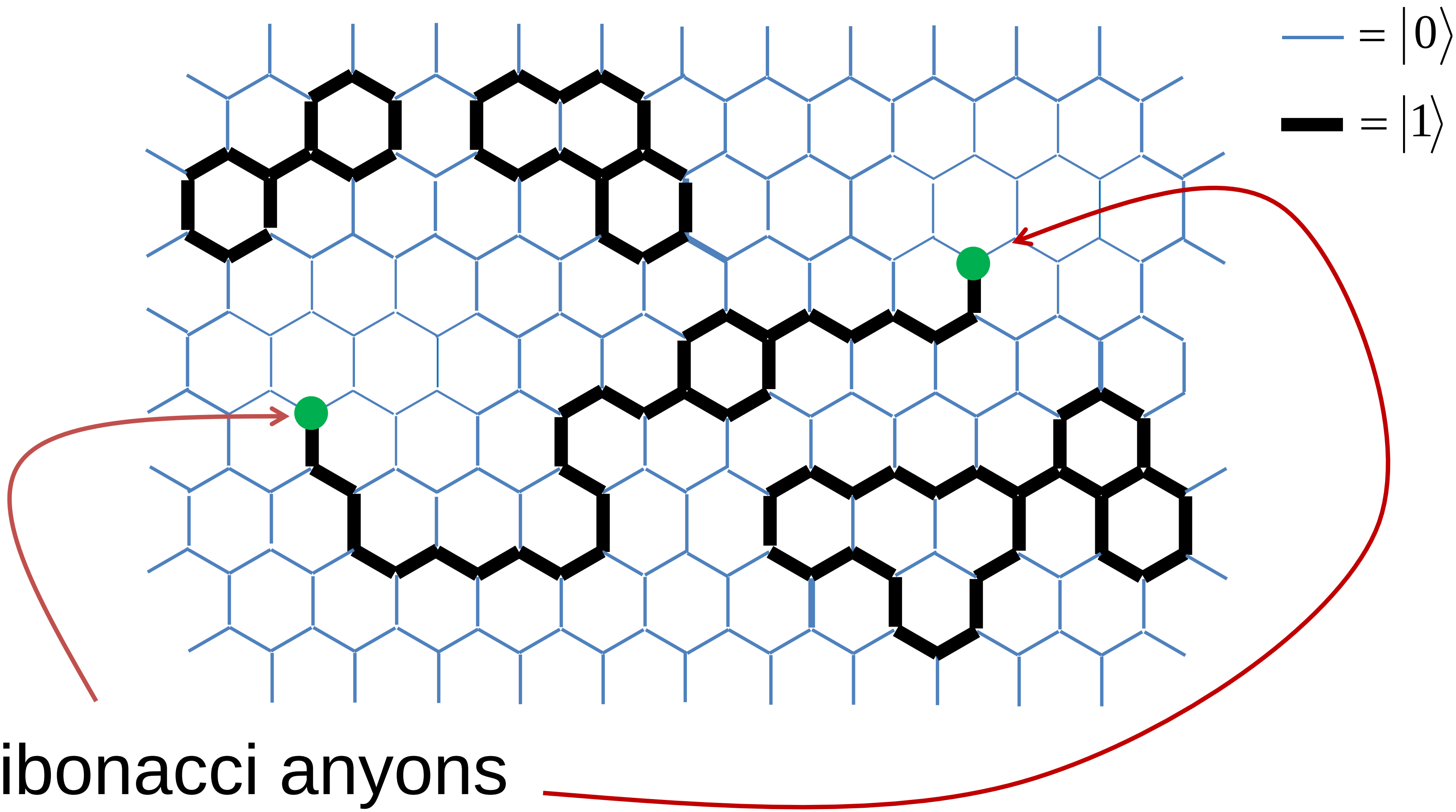
Move “Packaged” Error Using F-Moves

Vertex Errors = String Ends

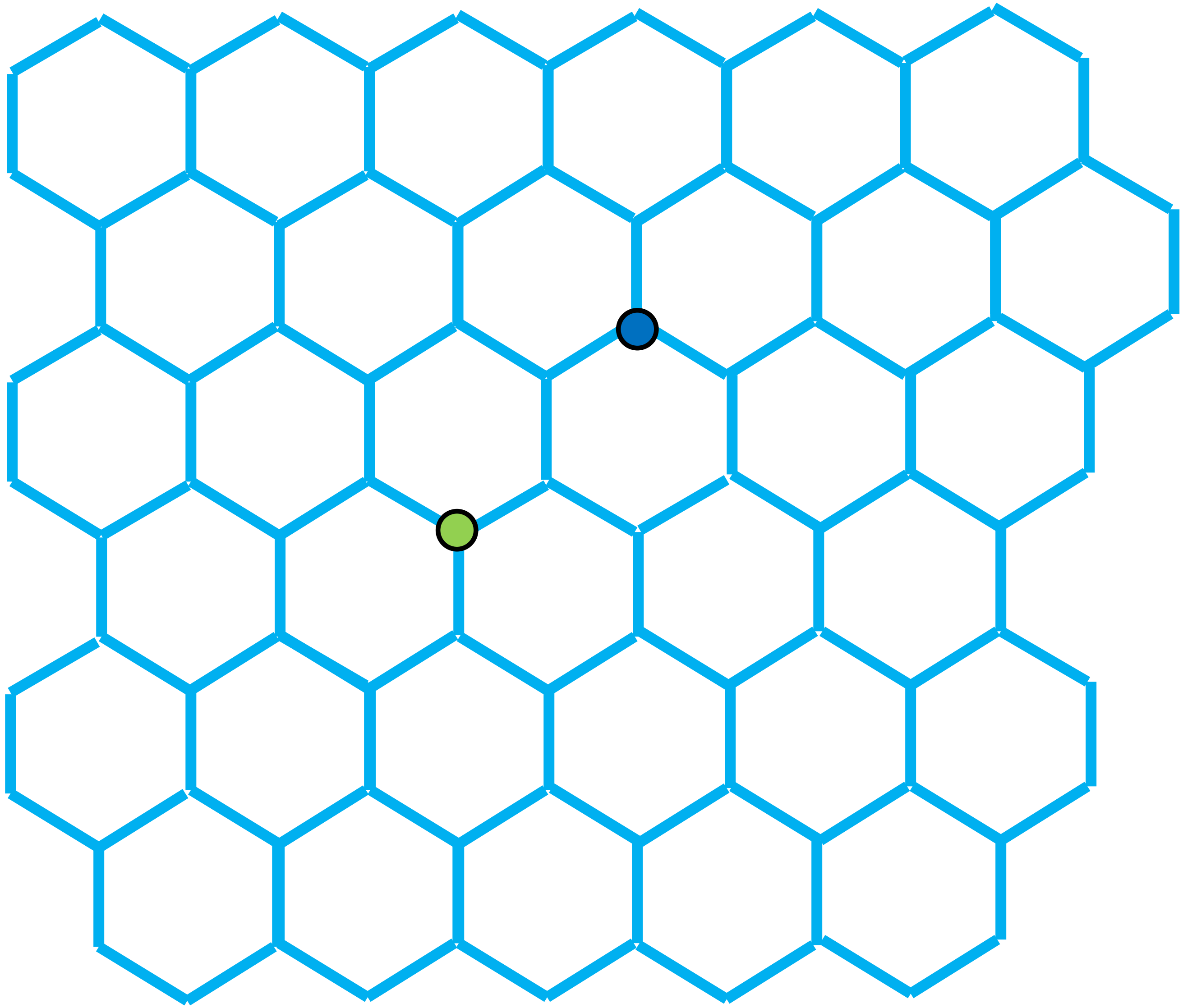


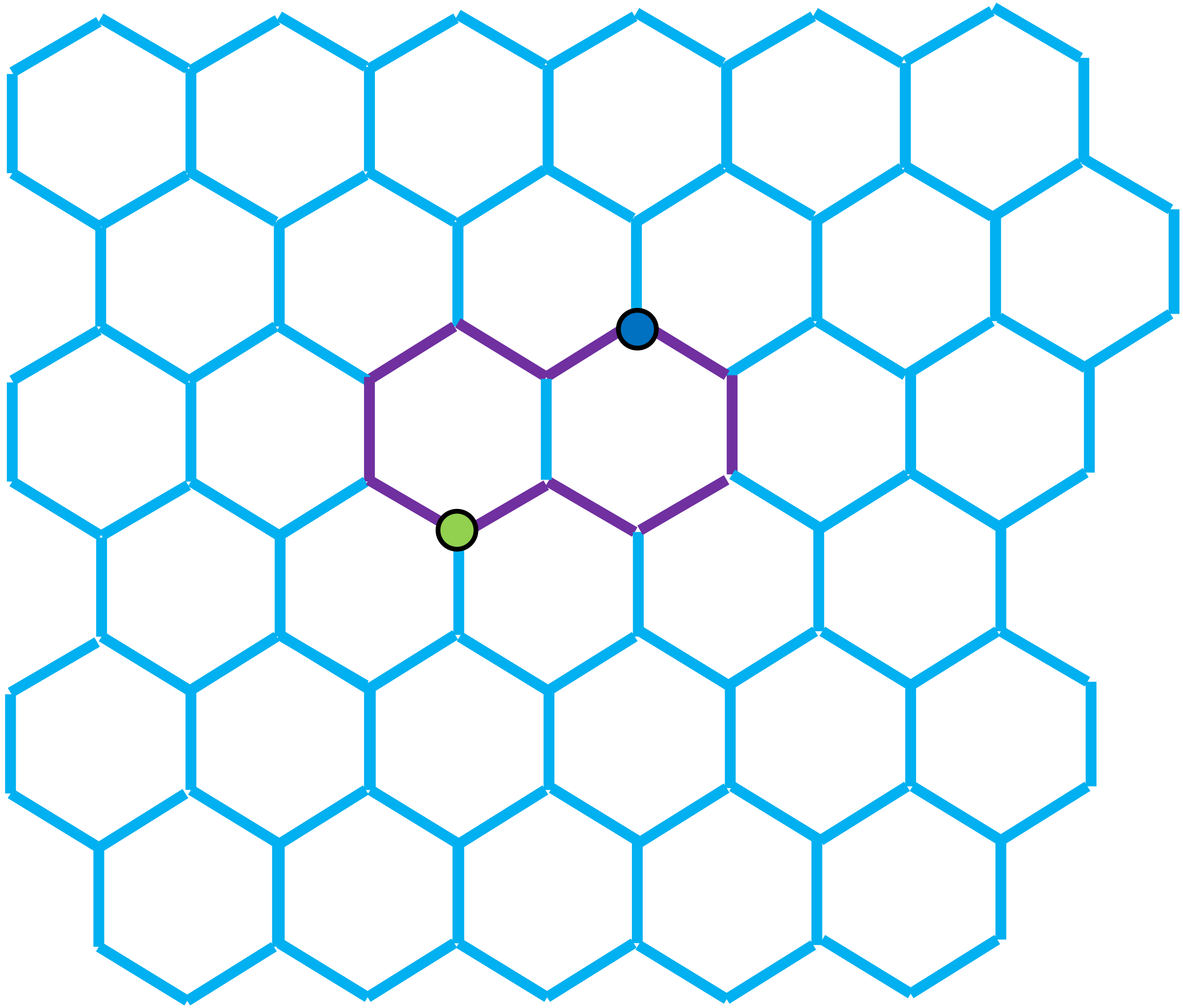
$$Q_{\mathbf{v}} = 0$$

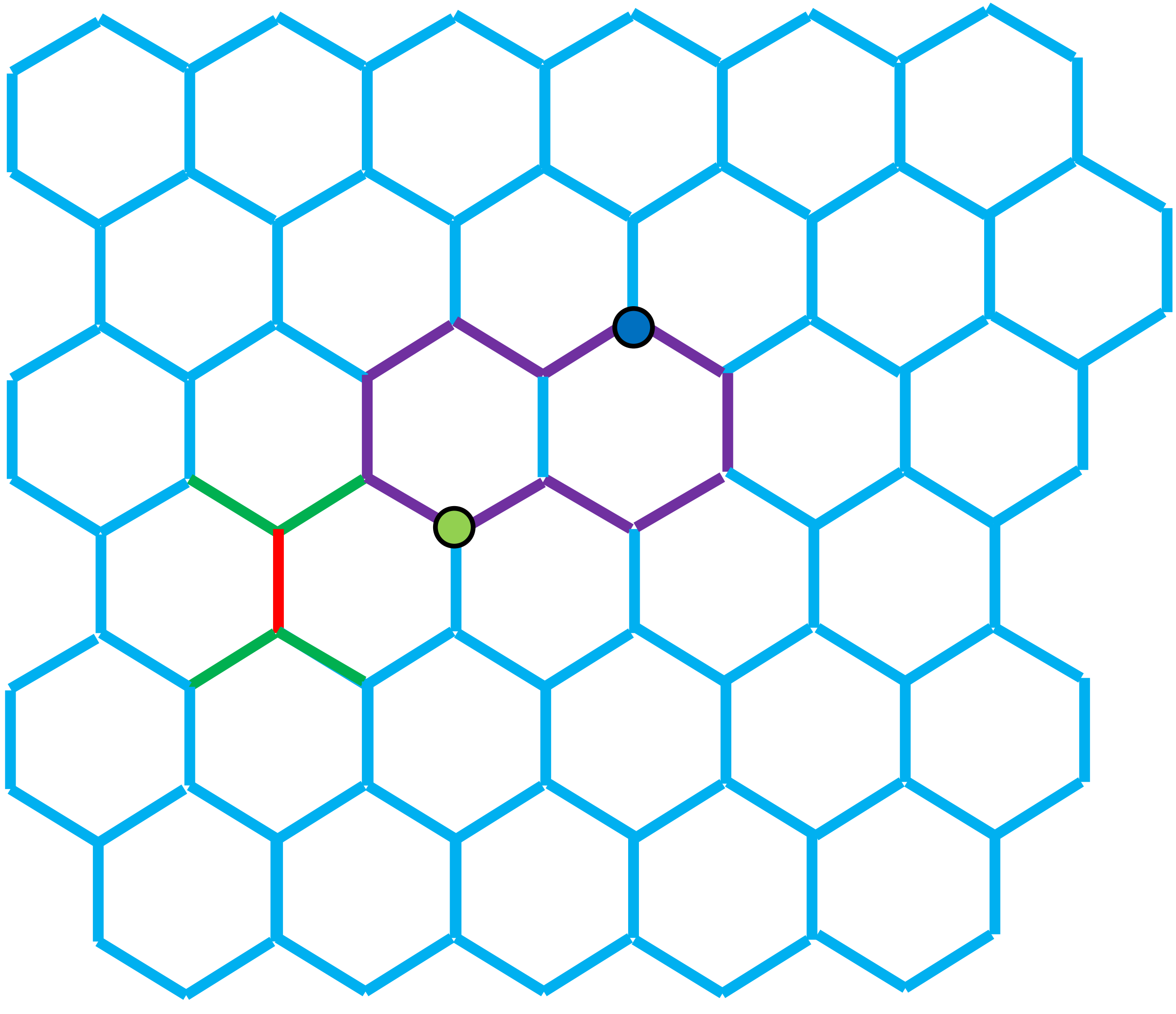
String Ends = Fibonacci Anyons

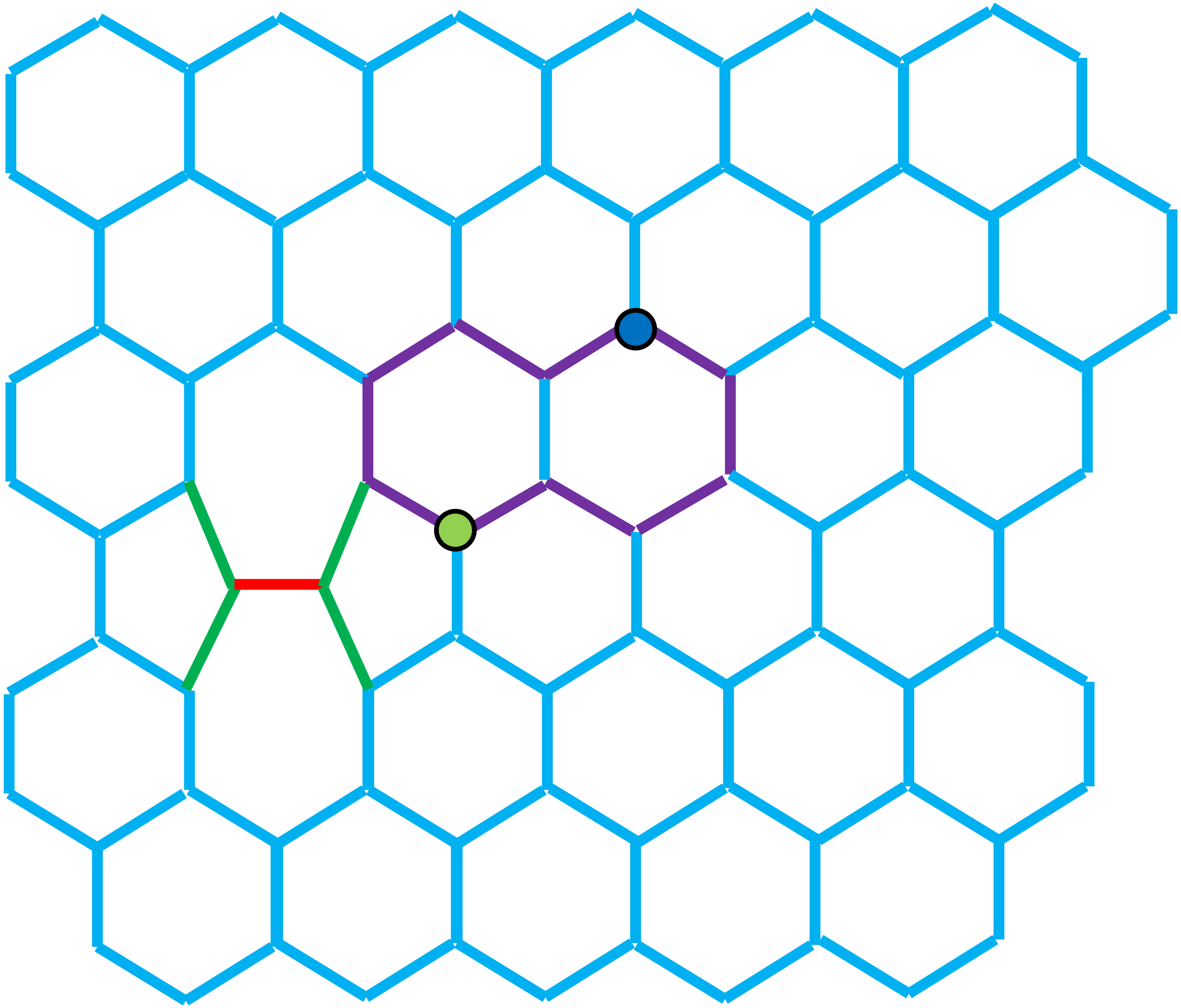


Fibonacci anyons

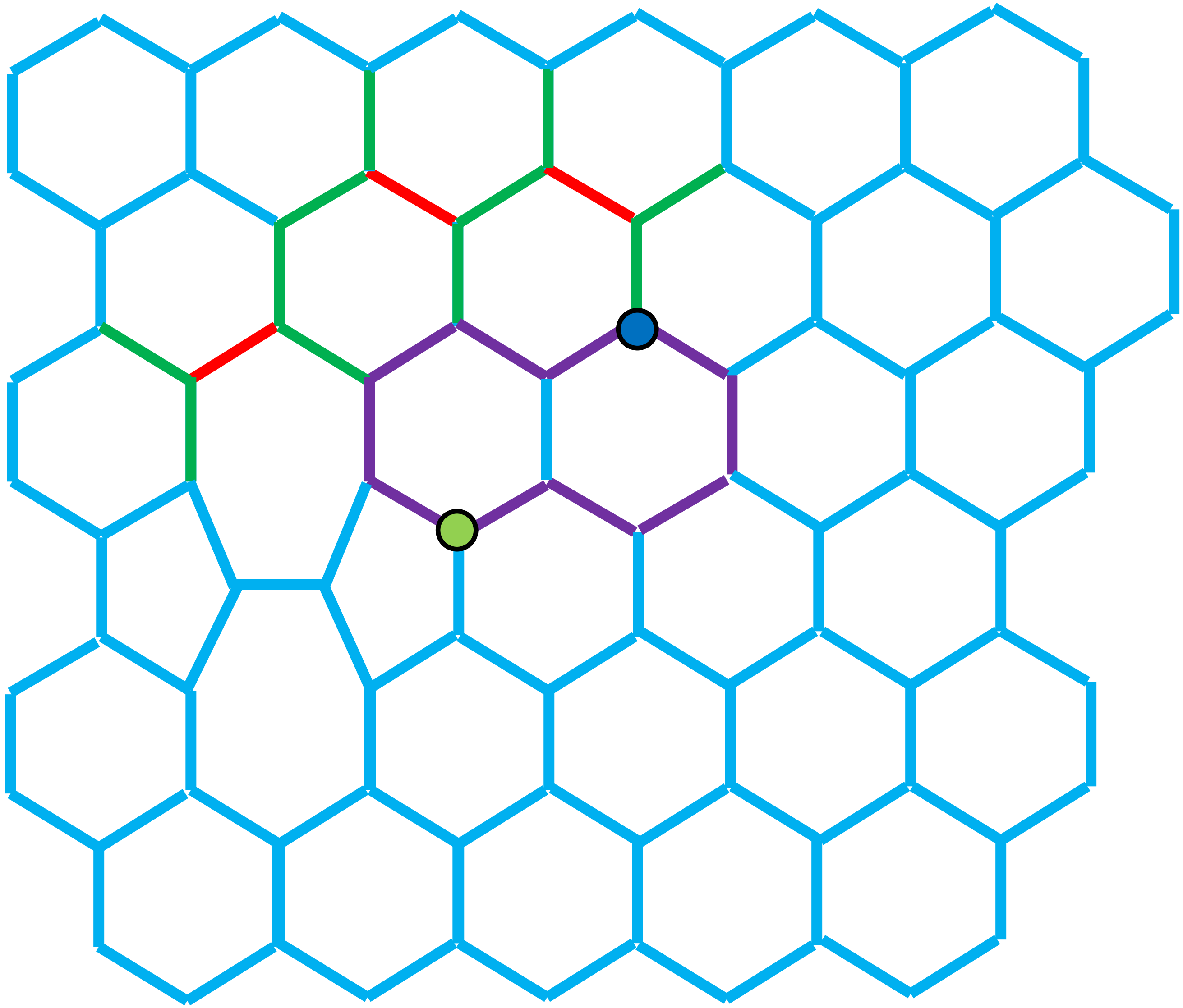




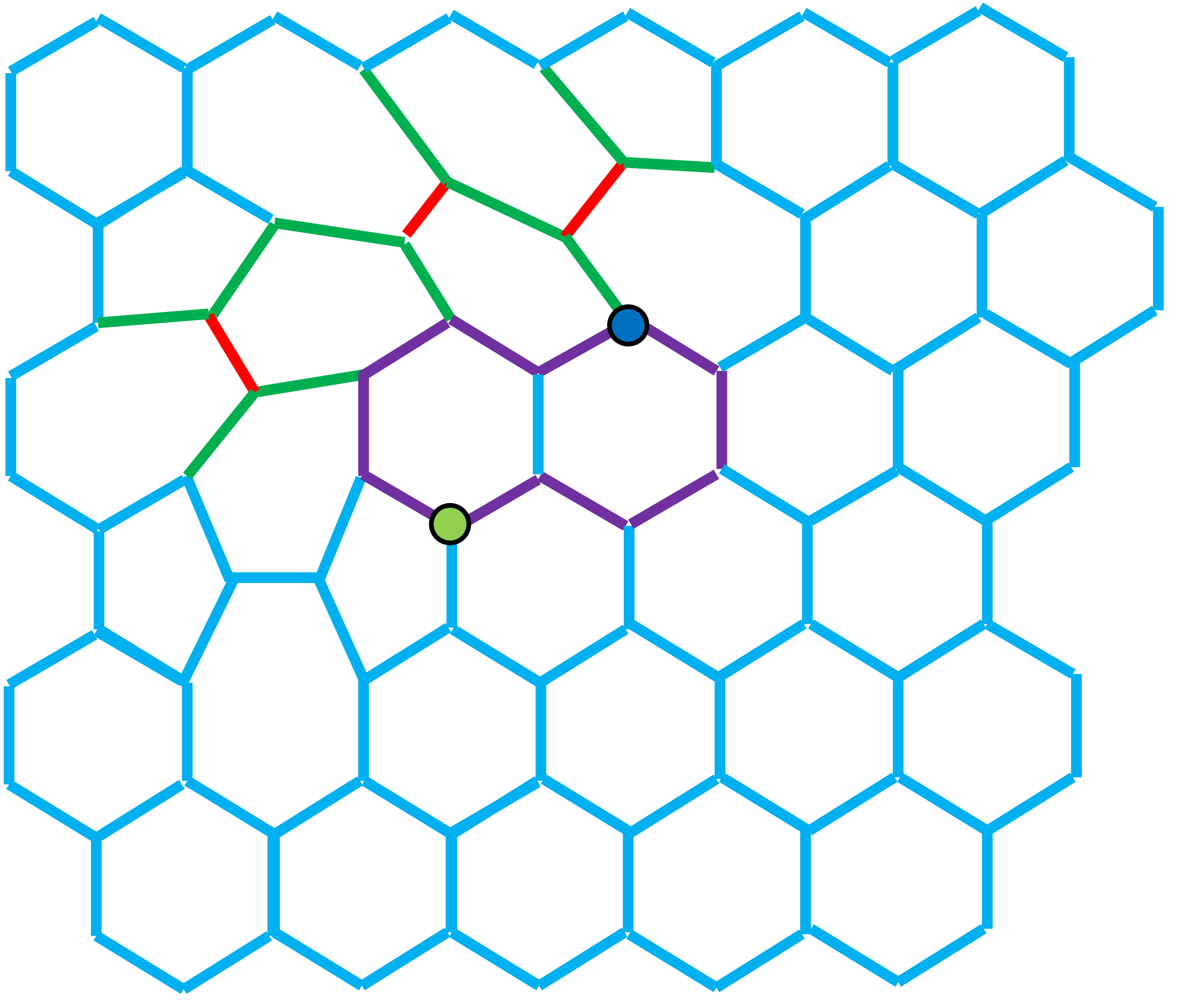




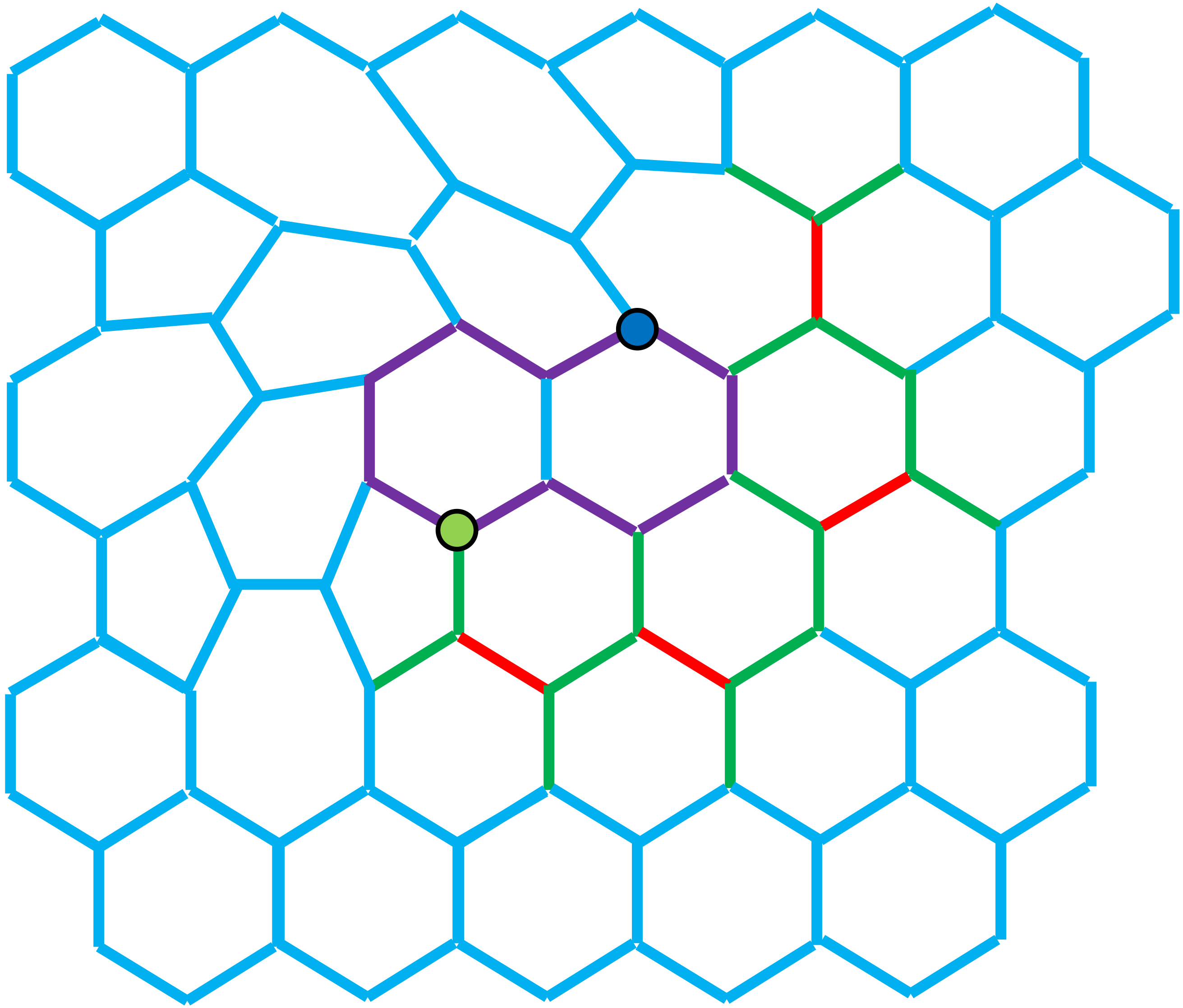
1 F-moves



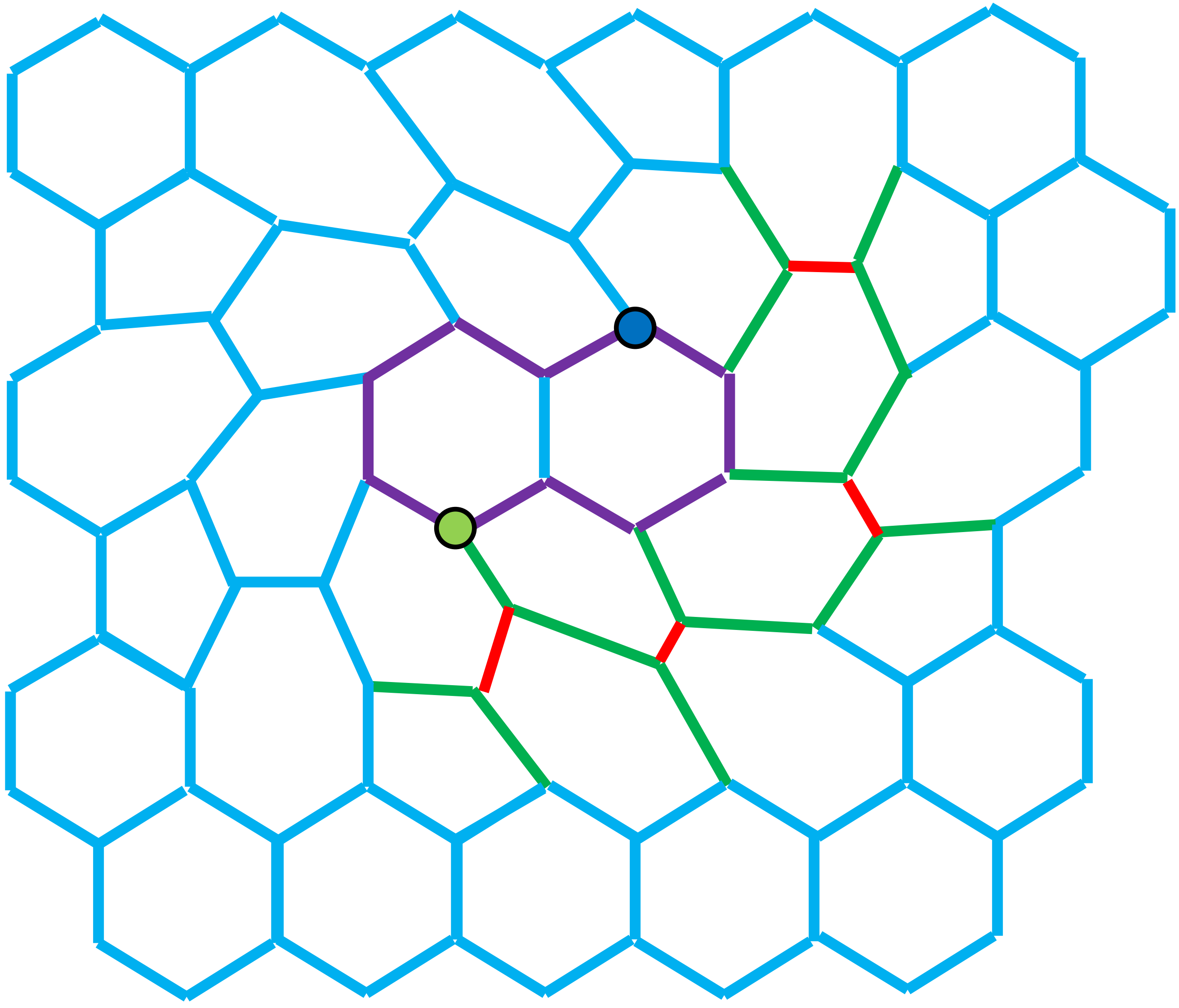
1 F-moves



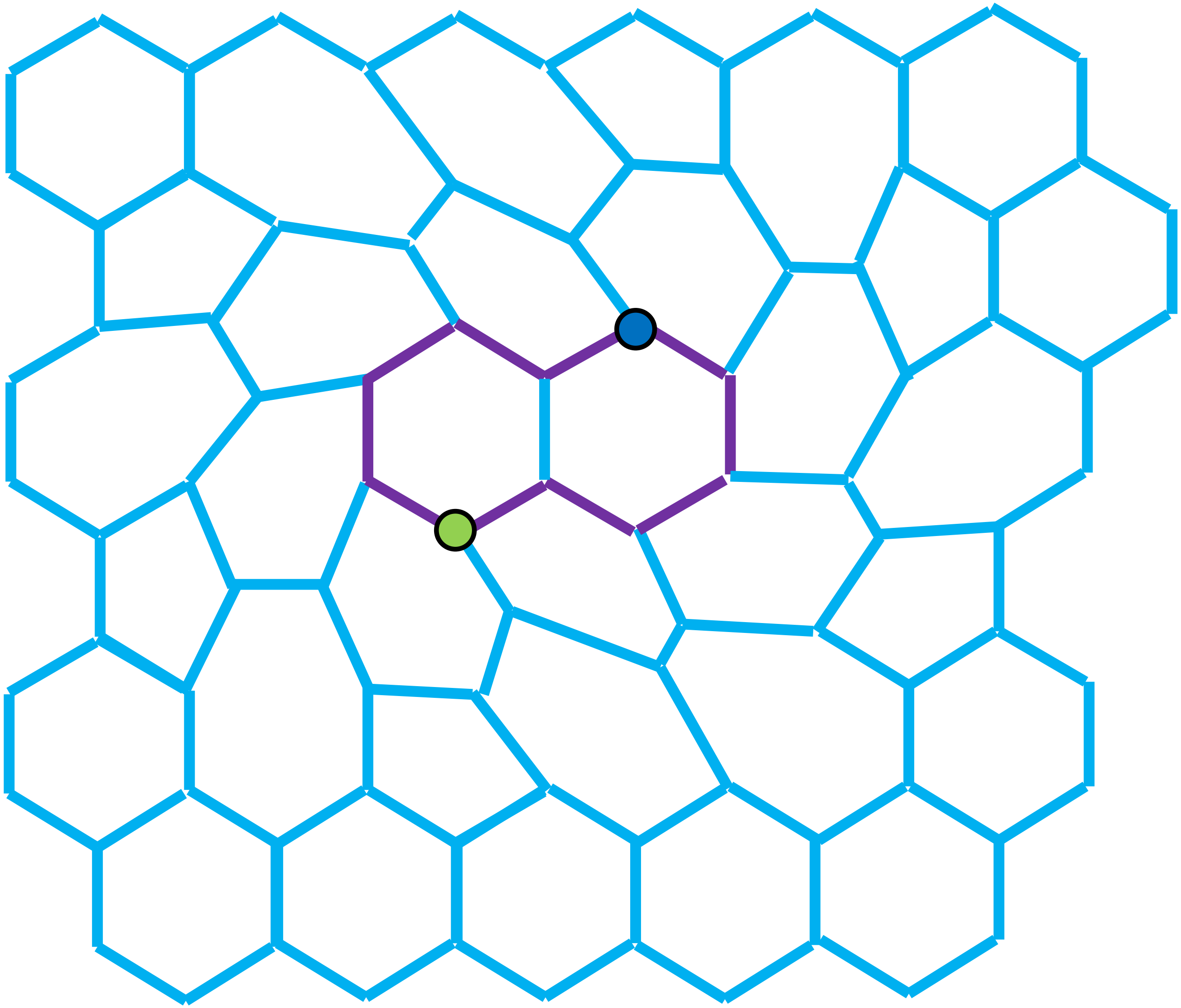
4 F-moves



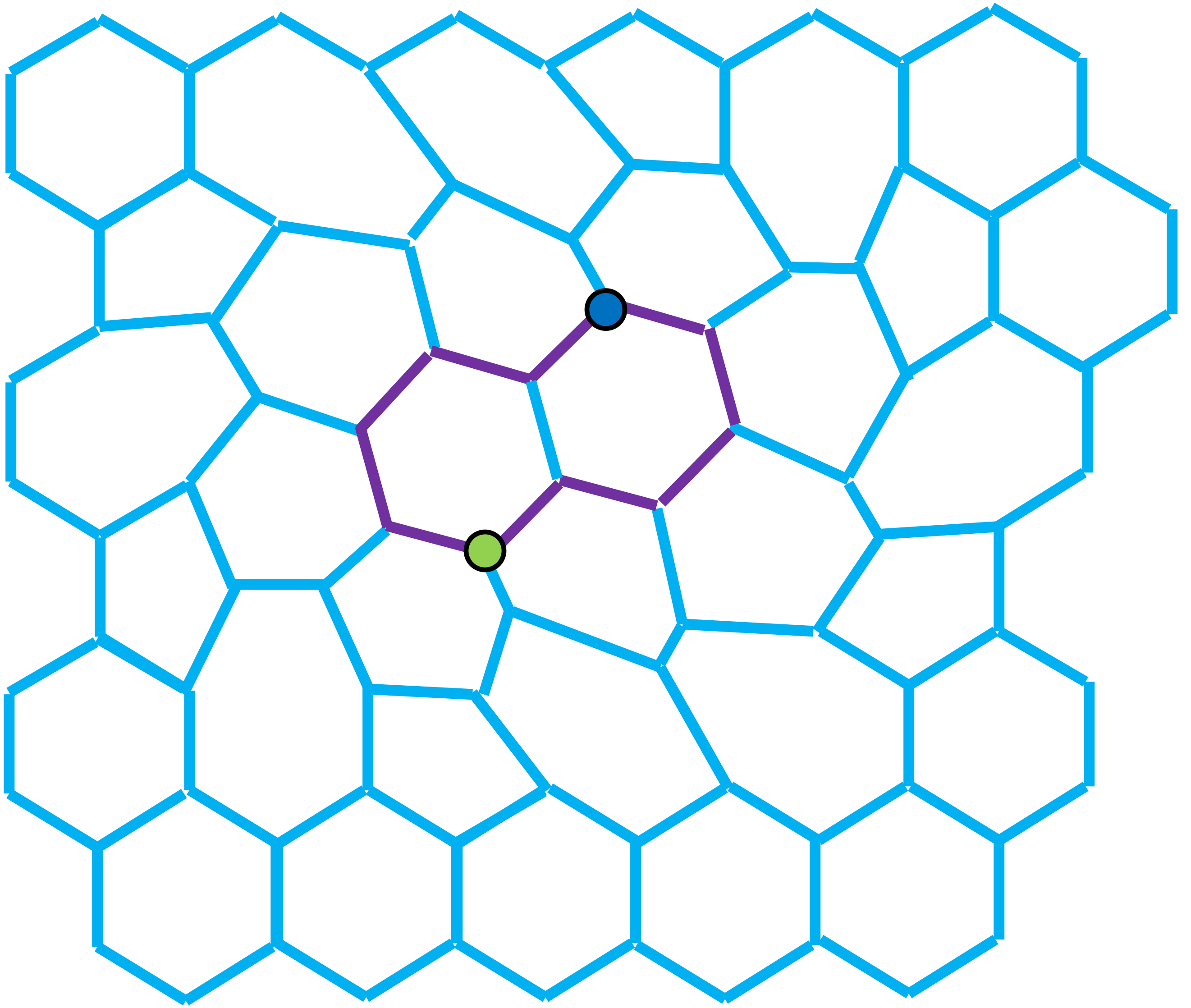
4 F-moves



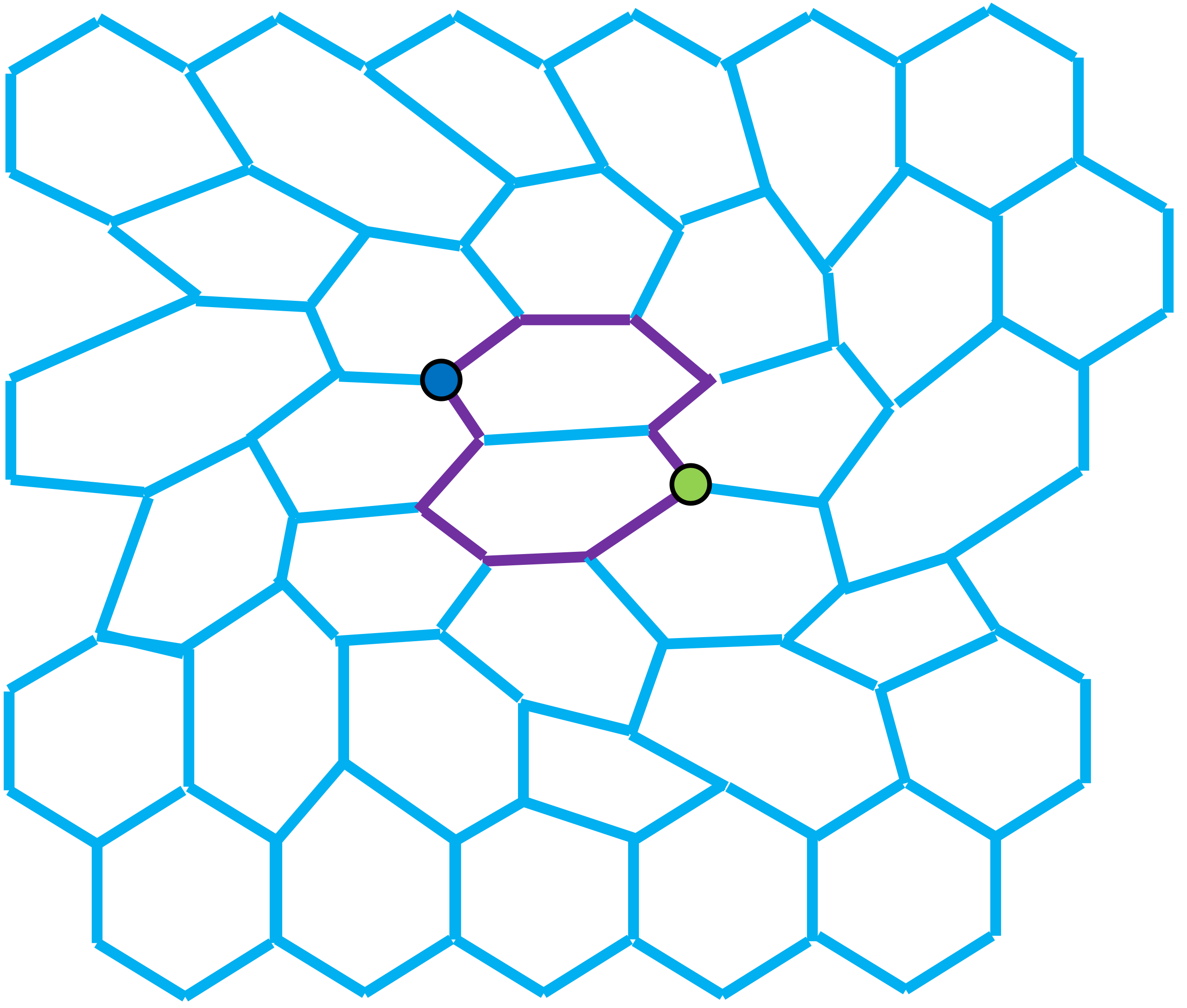
8 F-moves



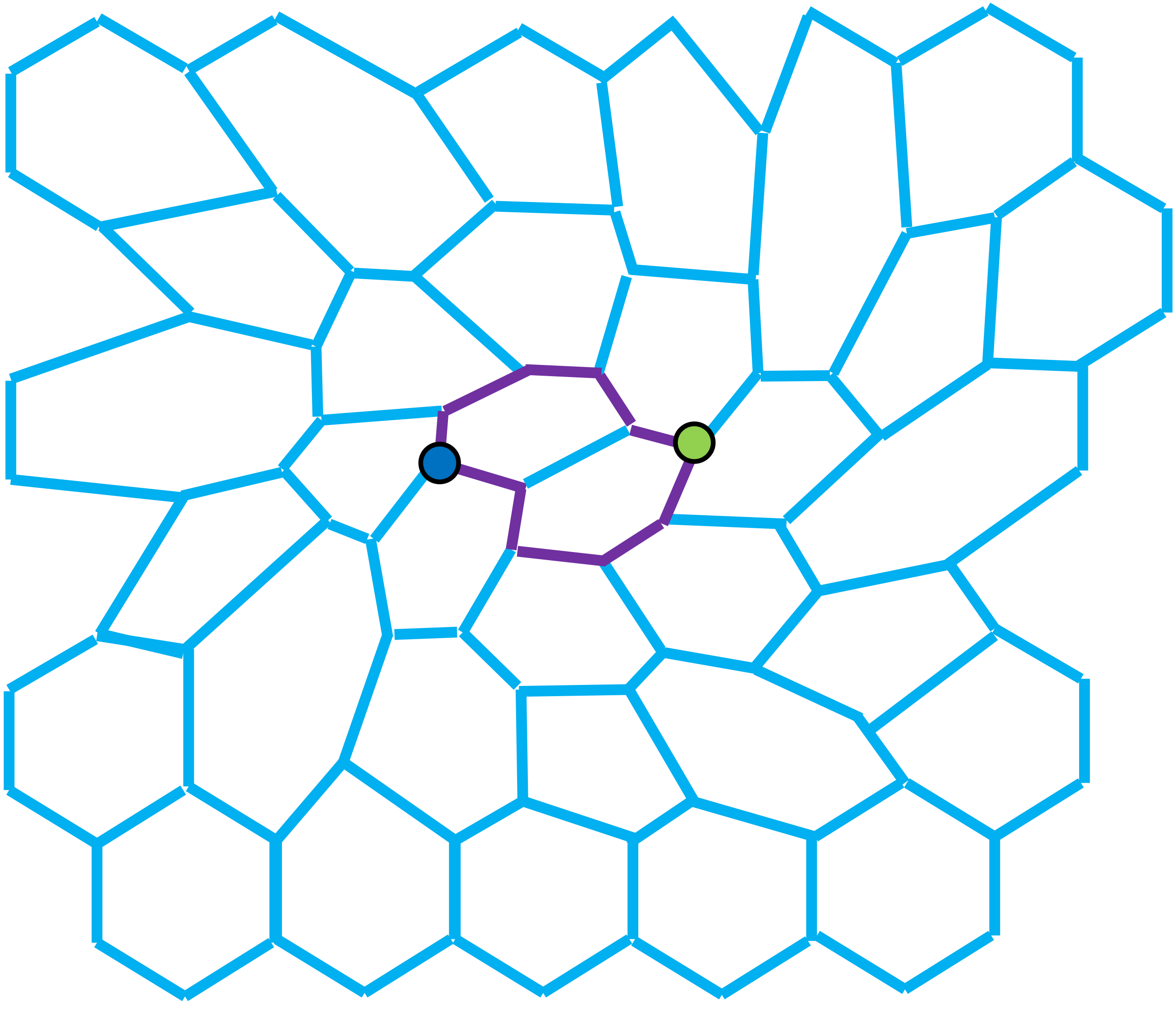
8 F-moves



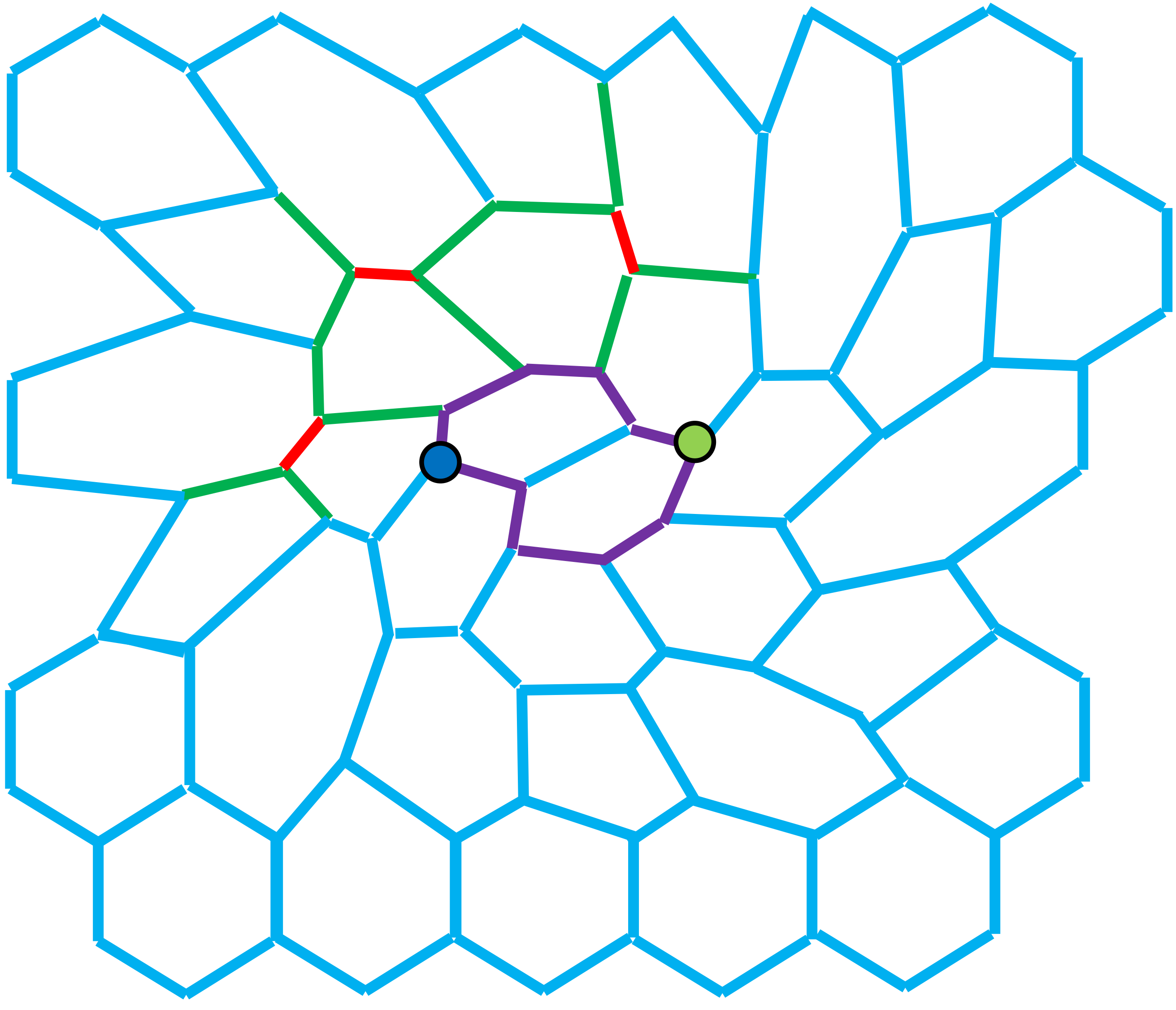
8 F-moves



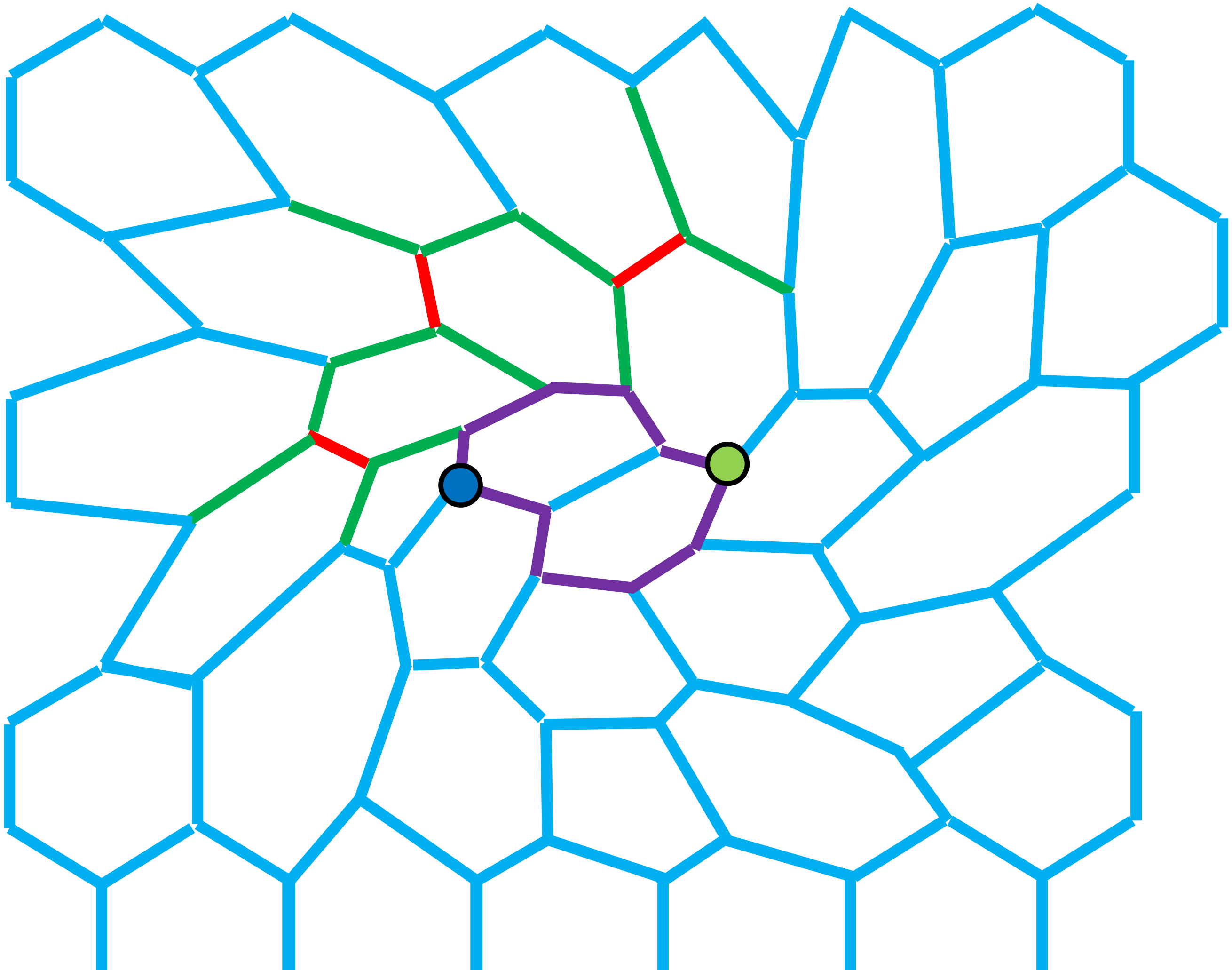
32 F-moves



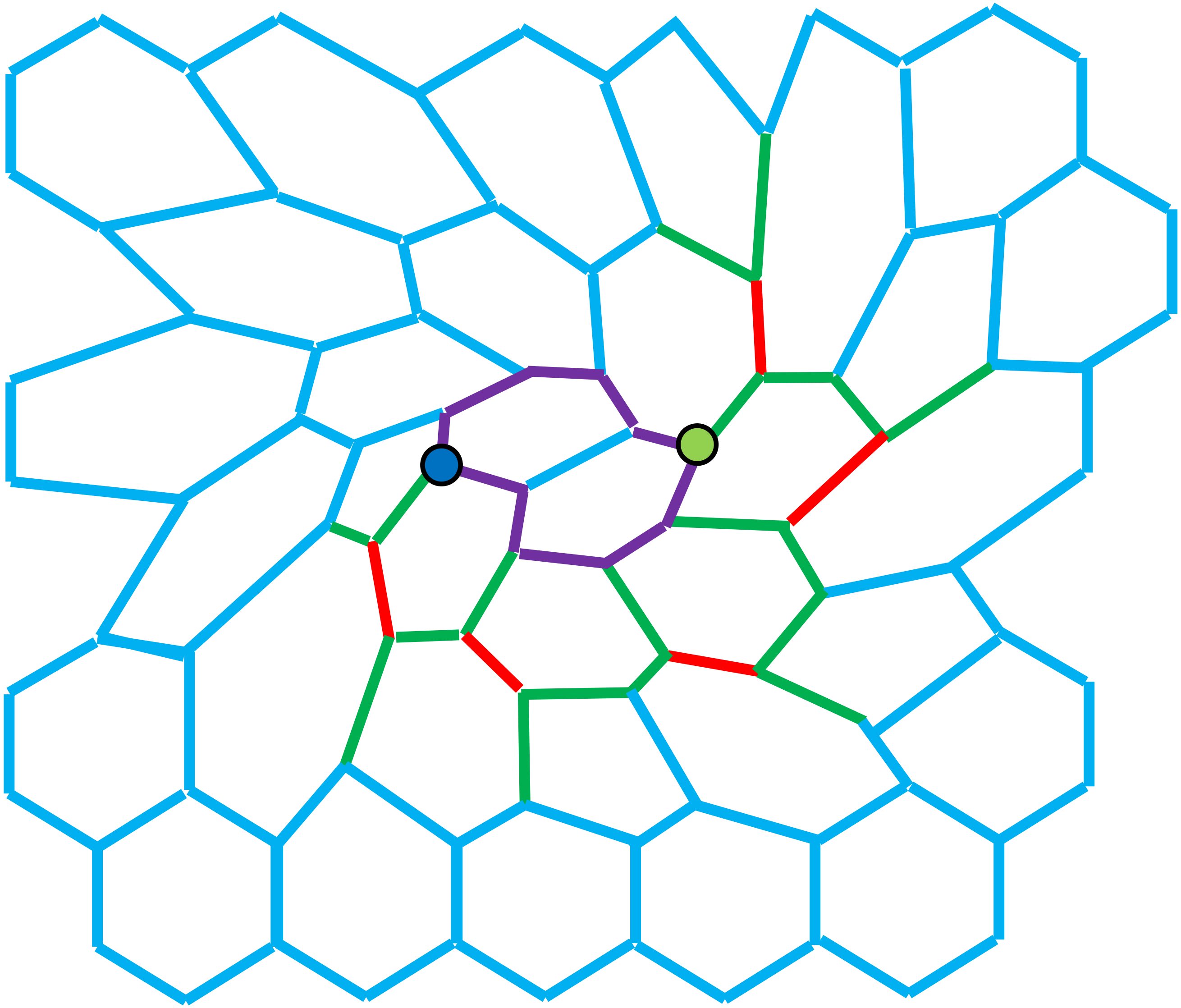
48 F-moves



48 F-moves



51 F-moves



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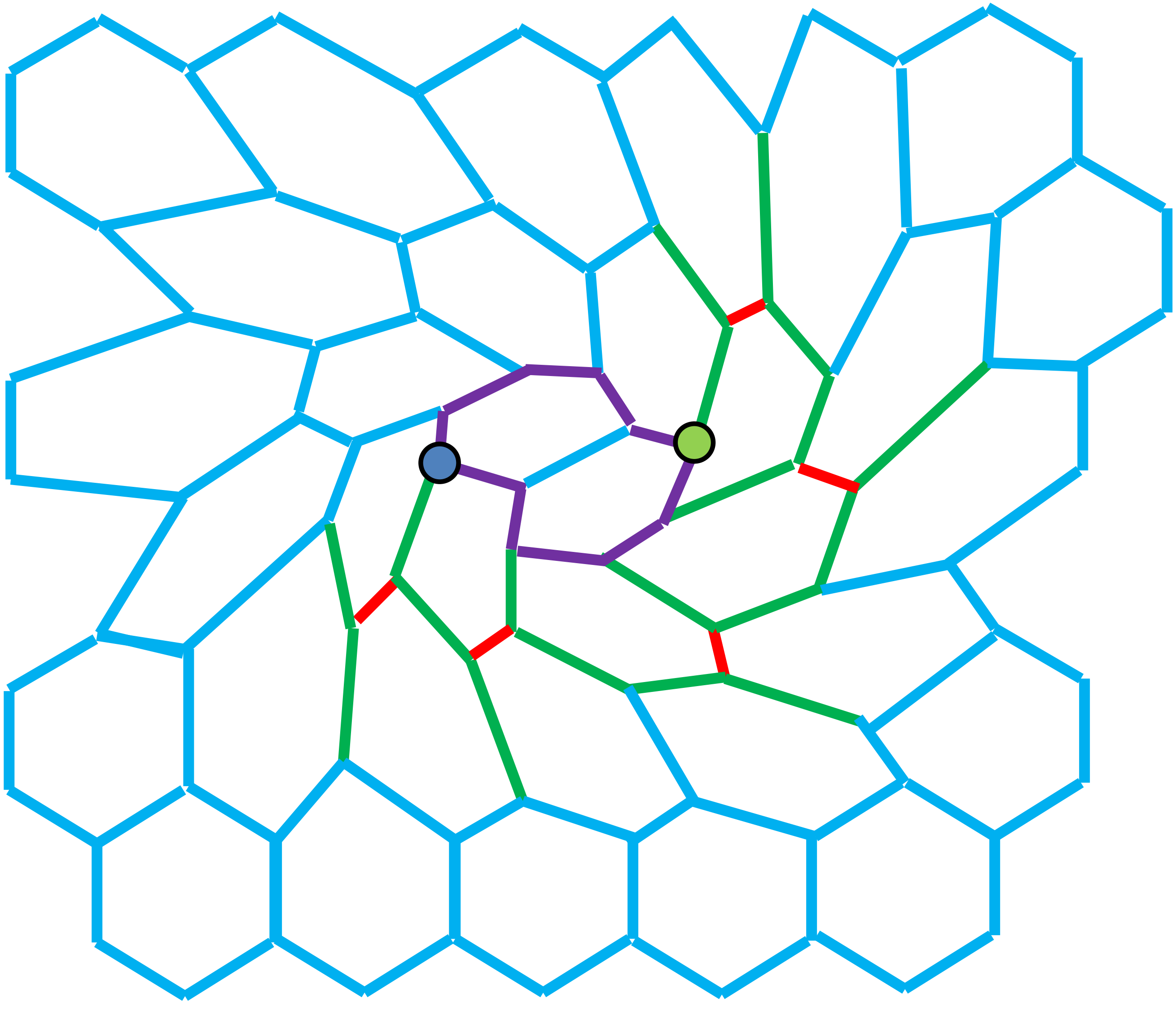
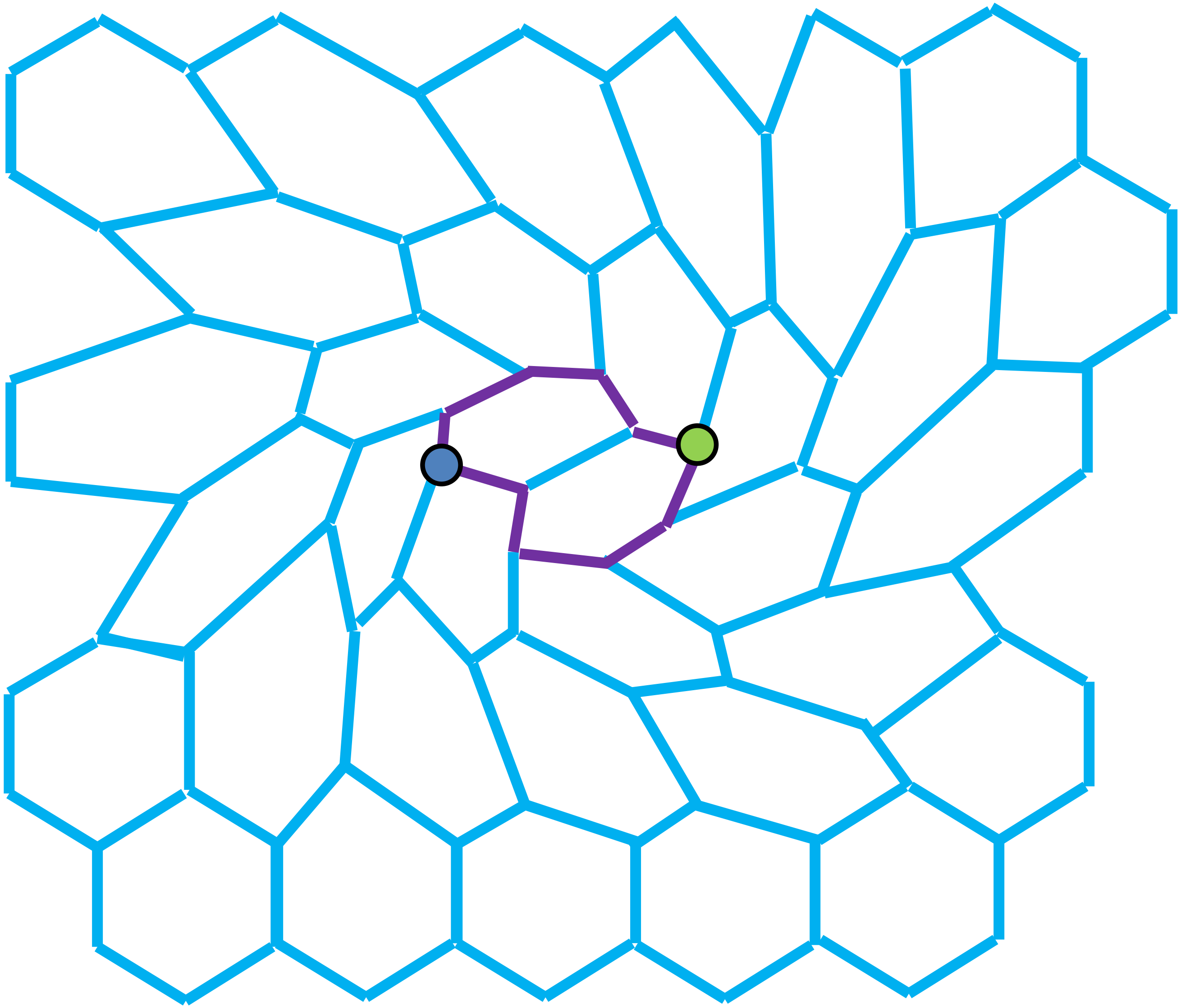
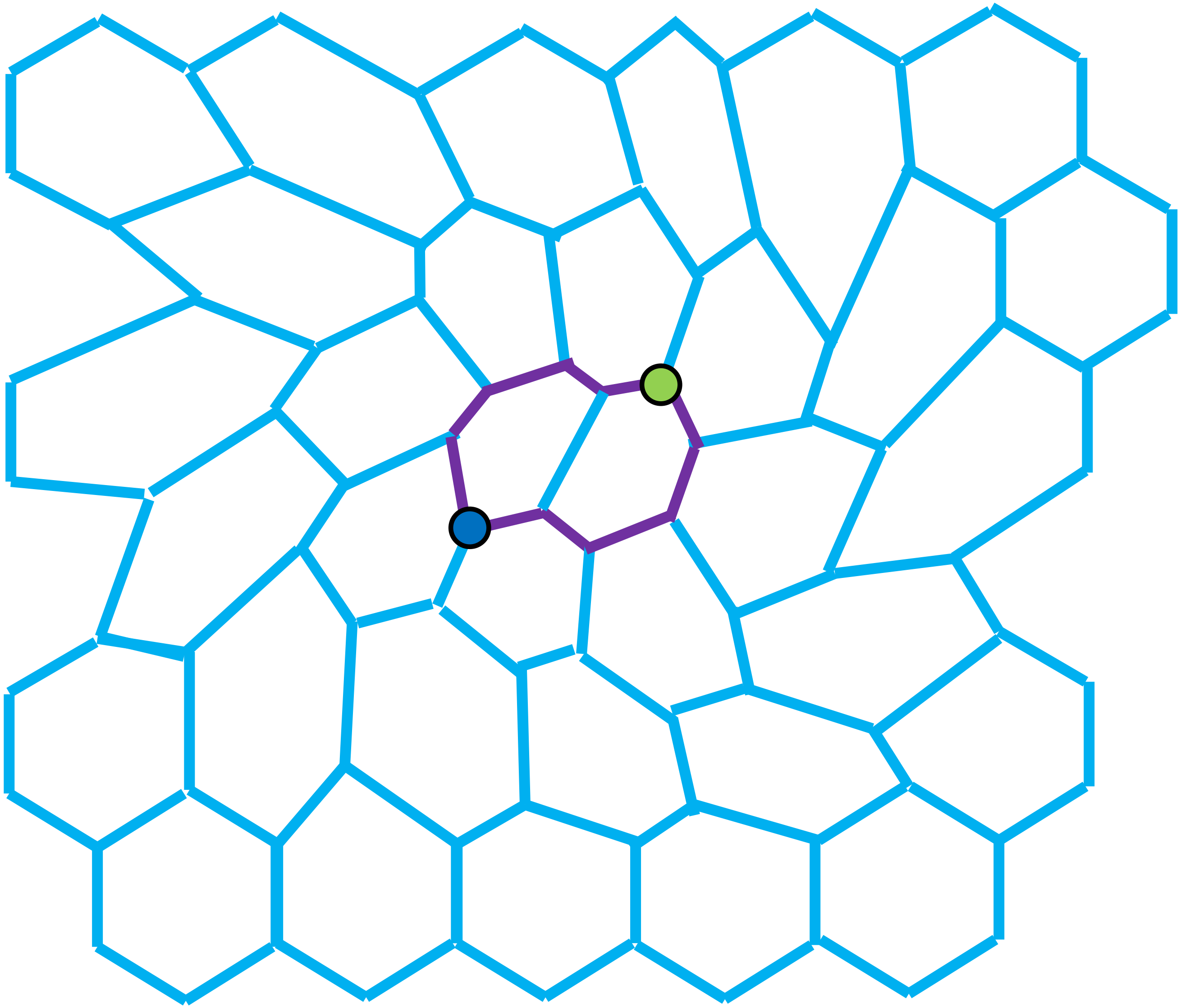


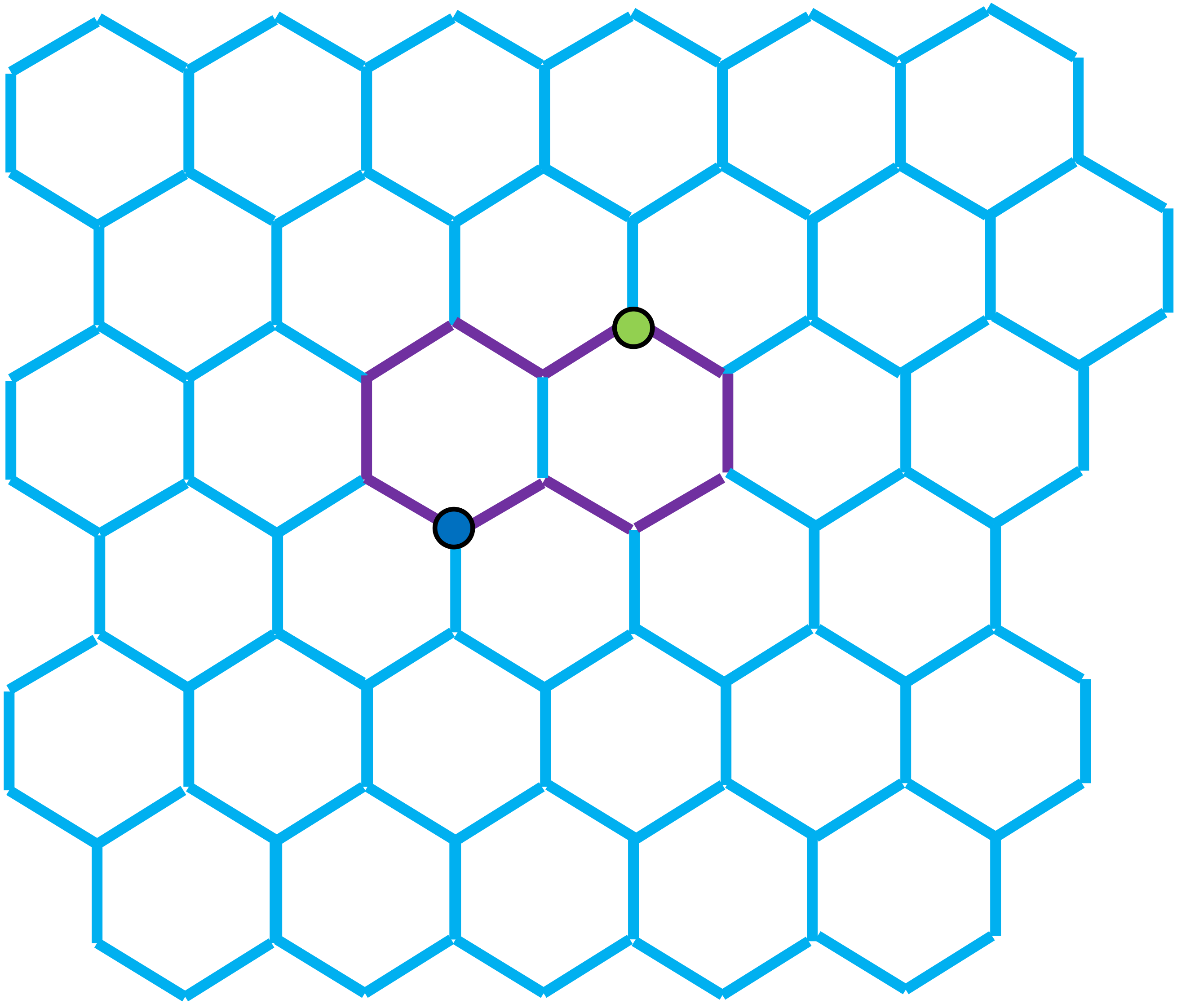
56 F-moves



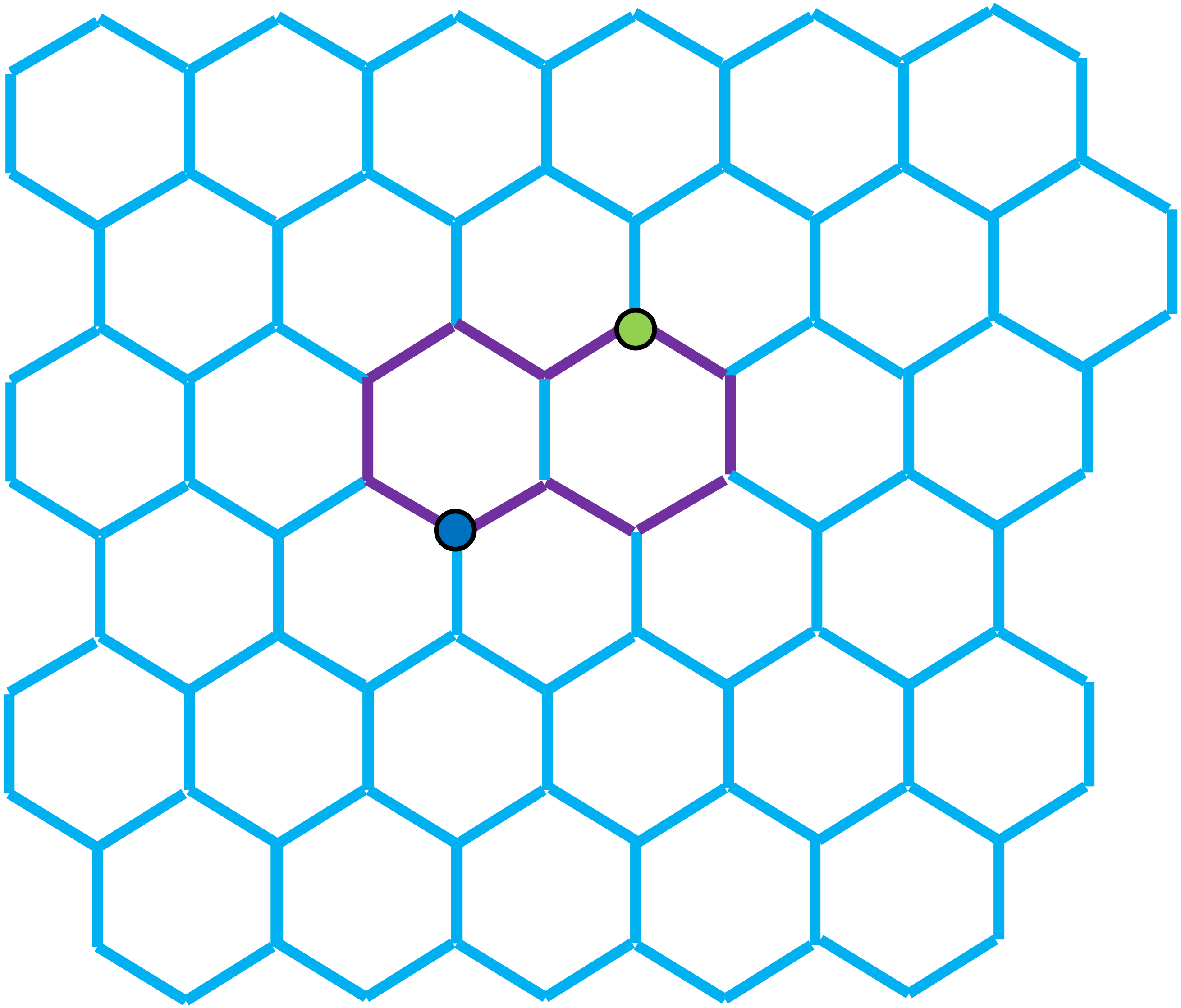
56 F-moves



56 F-moves



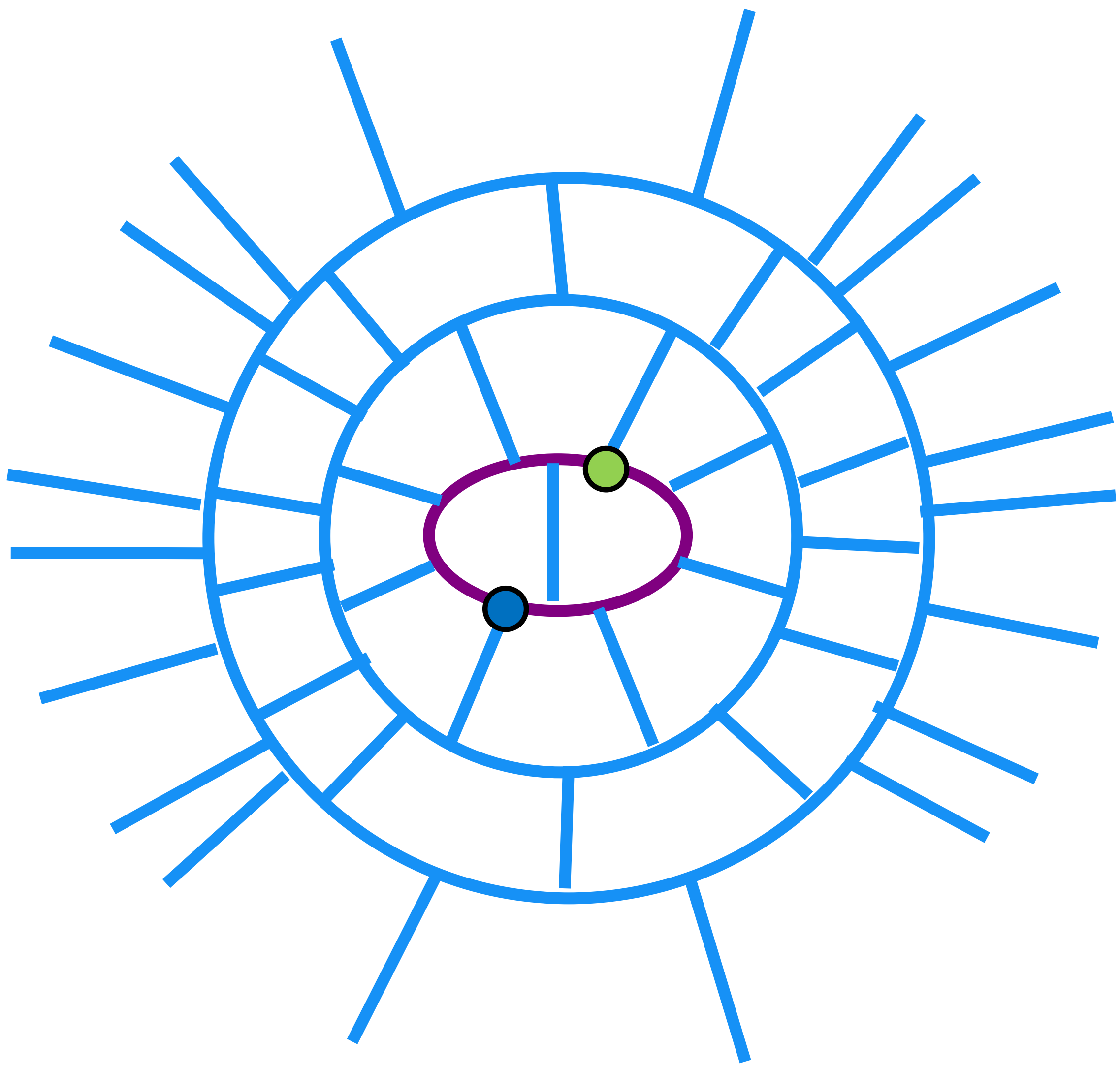
56 F-moves

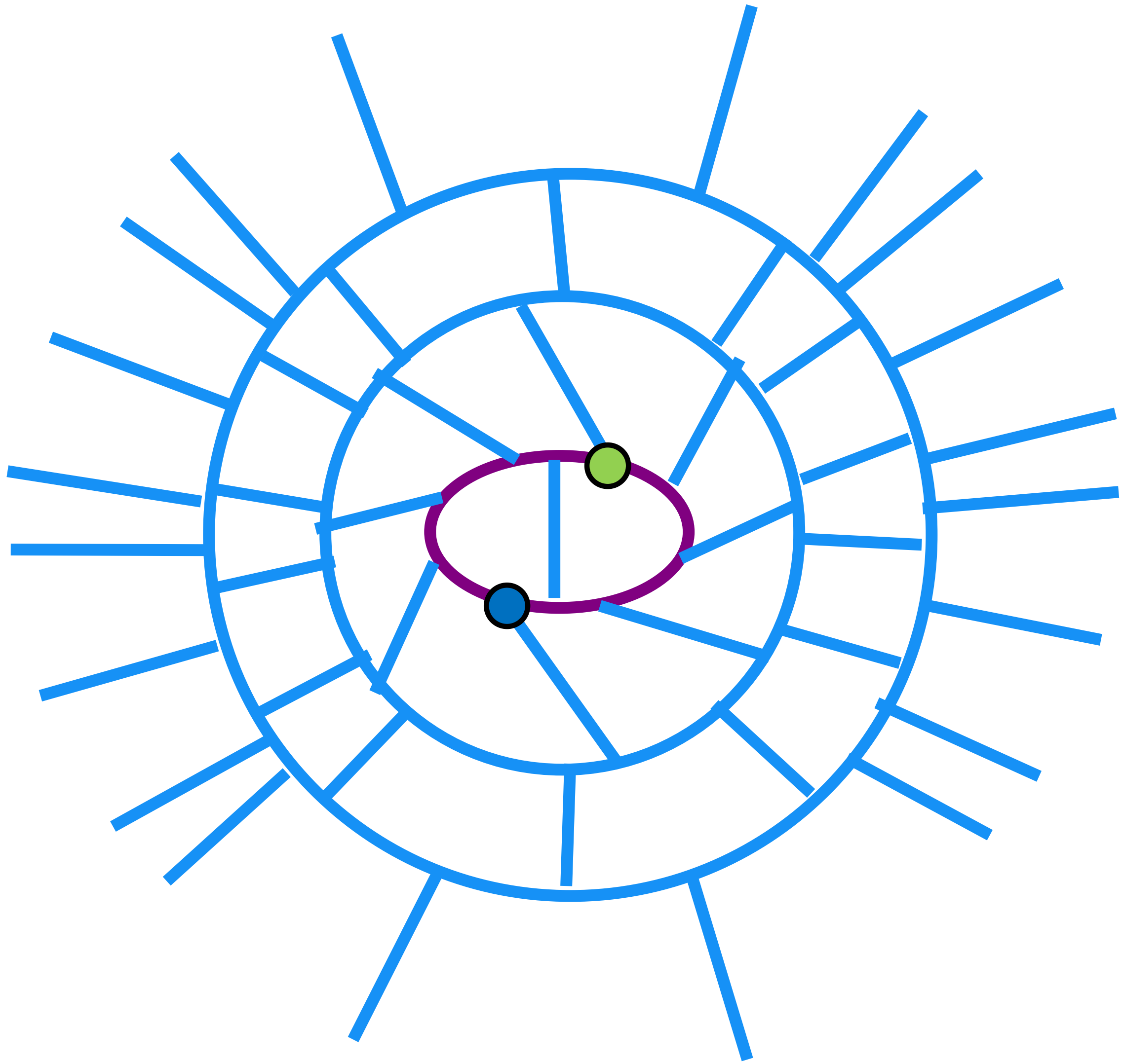


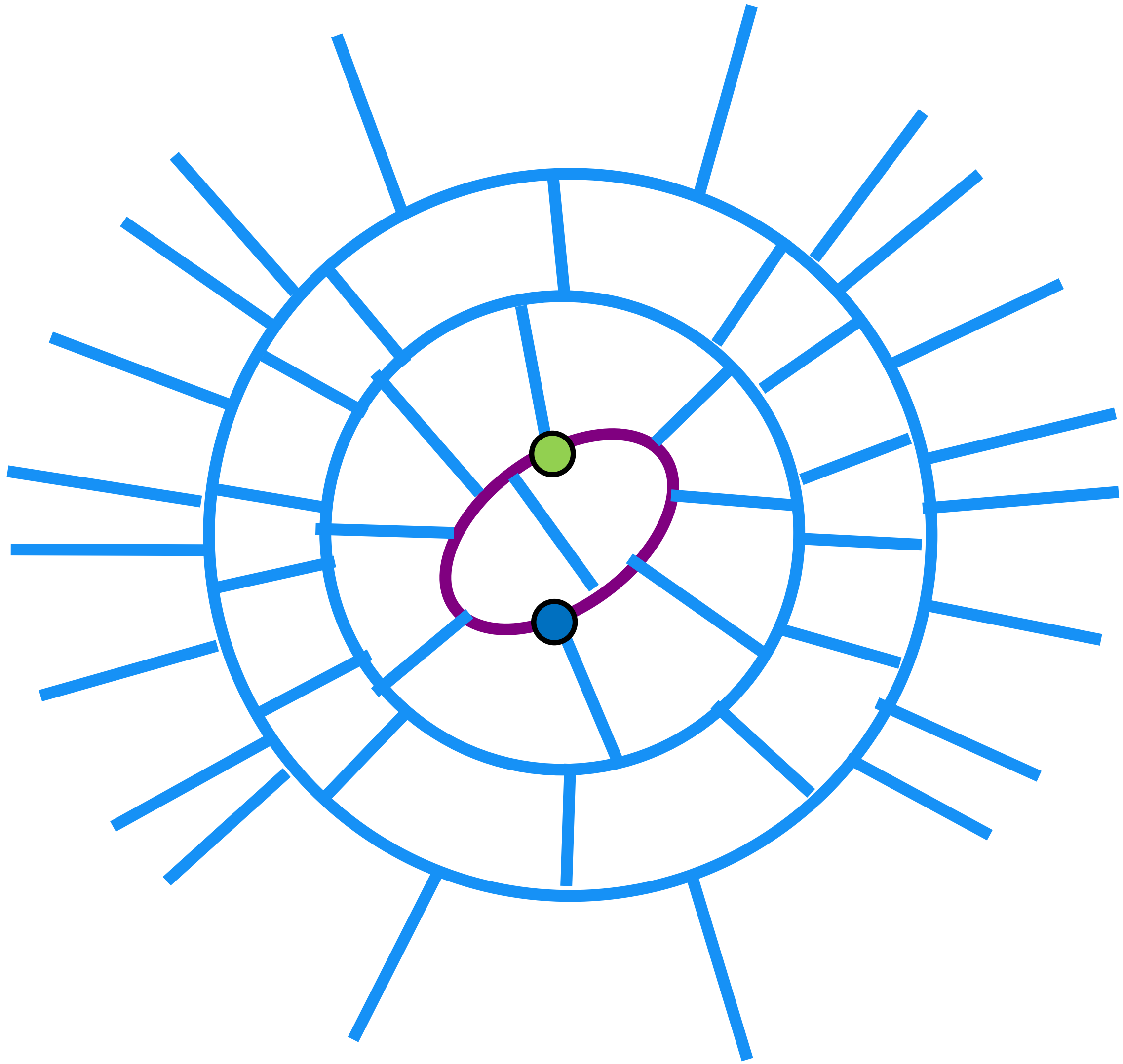
Dehn Twist

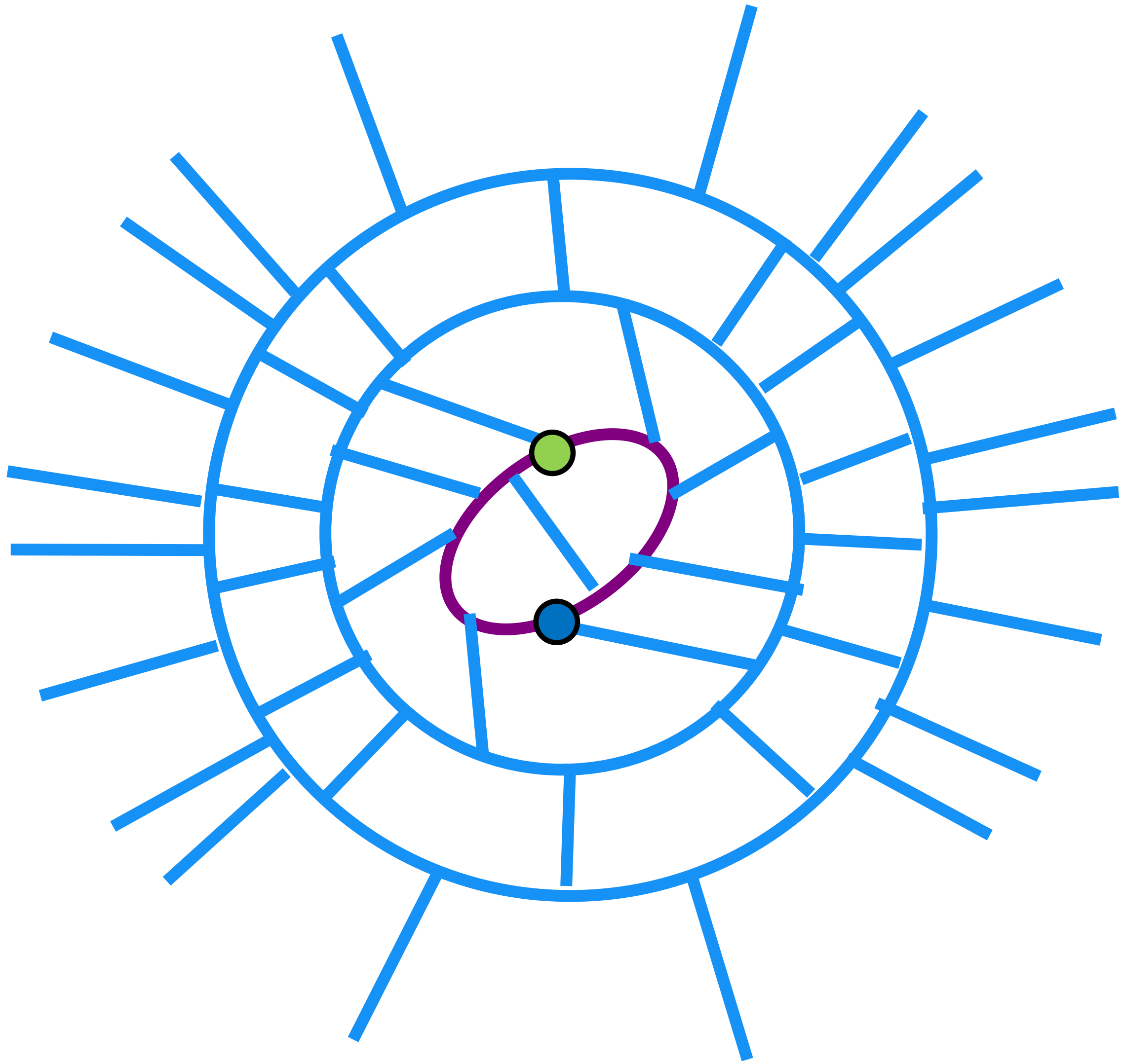
Konig, Kuperberg, Reichardt, Ann. Phys. 2010

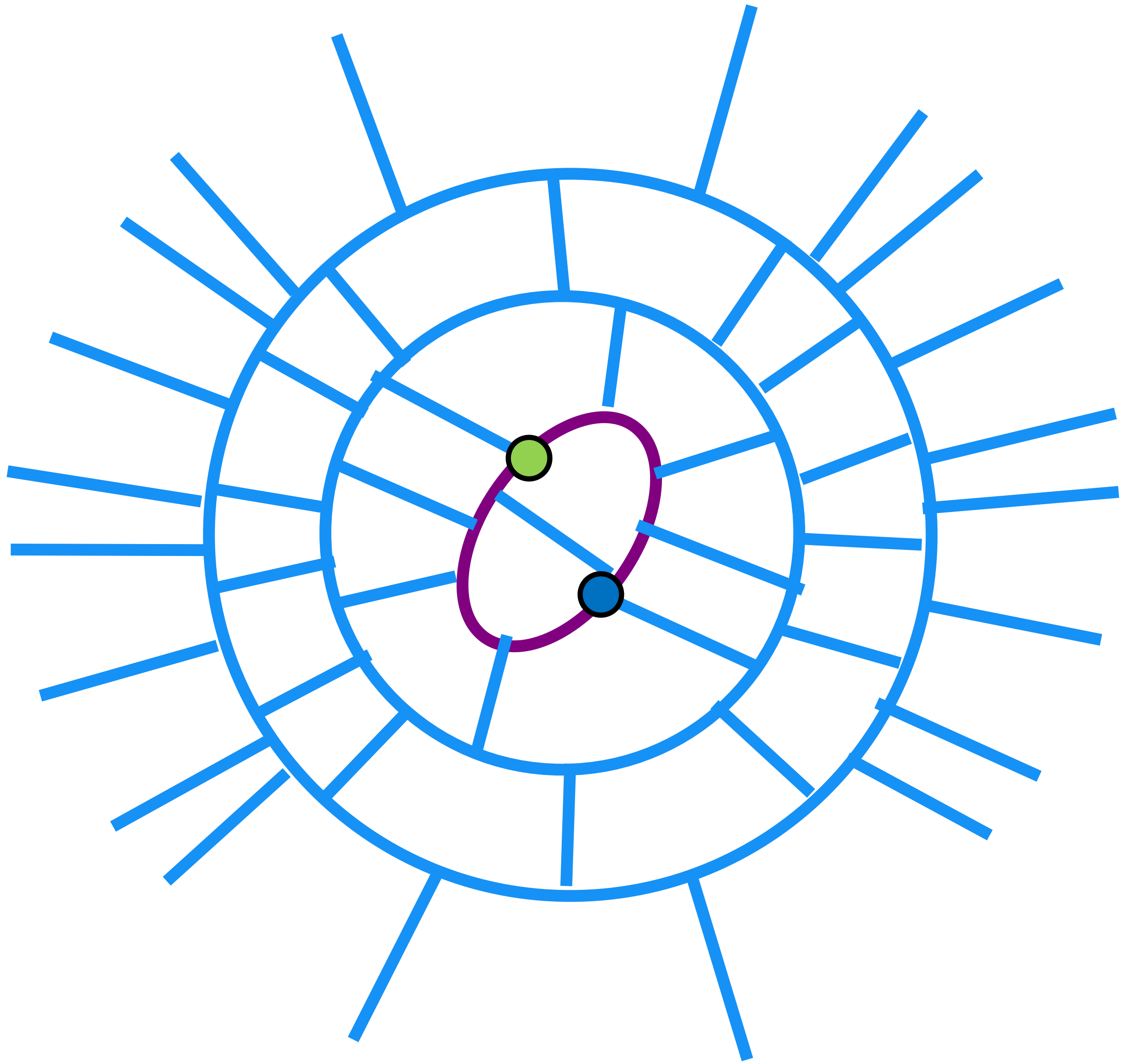
Braid

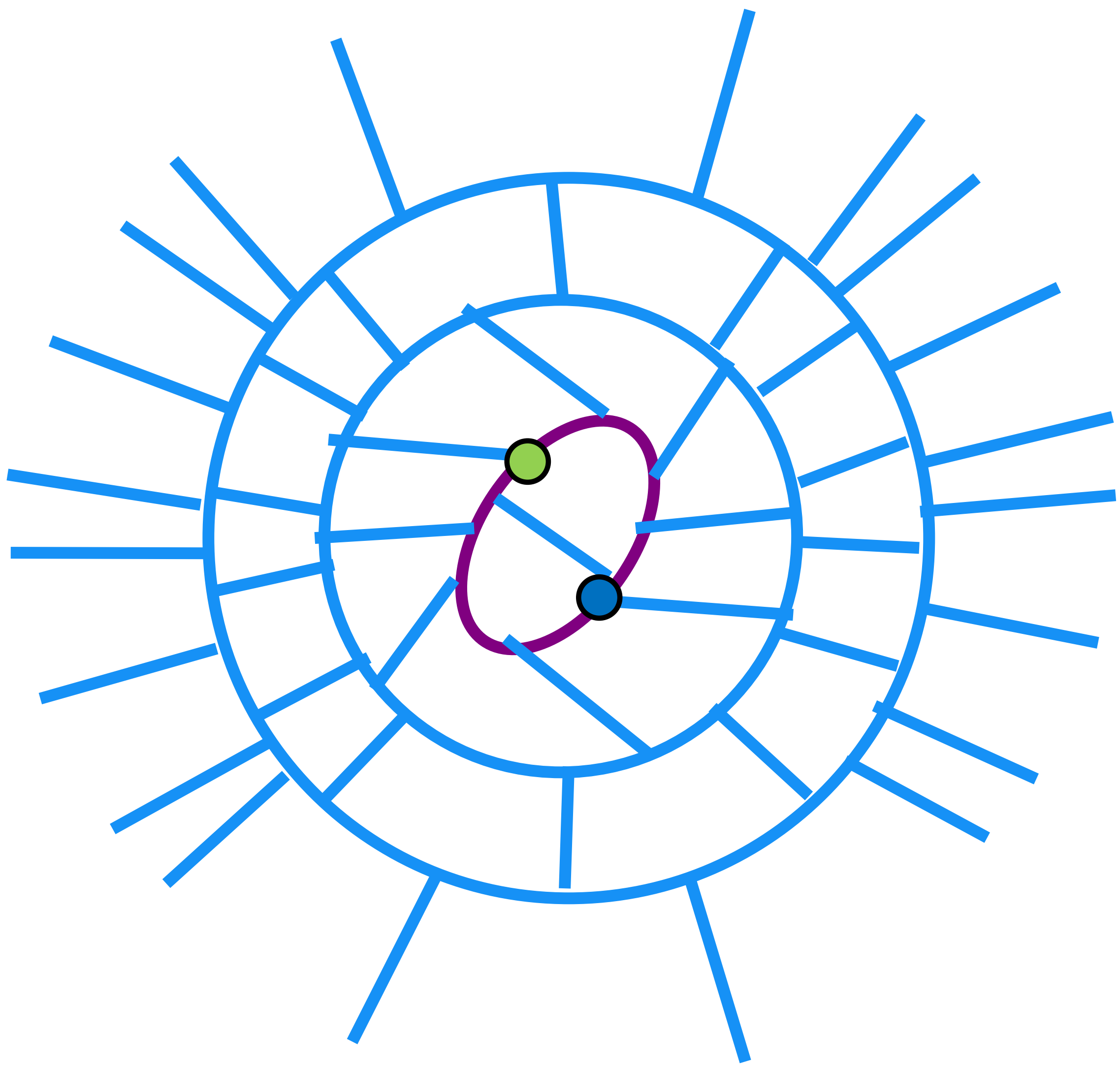


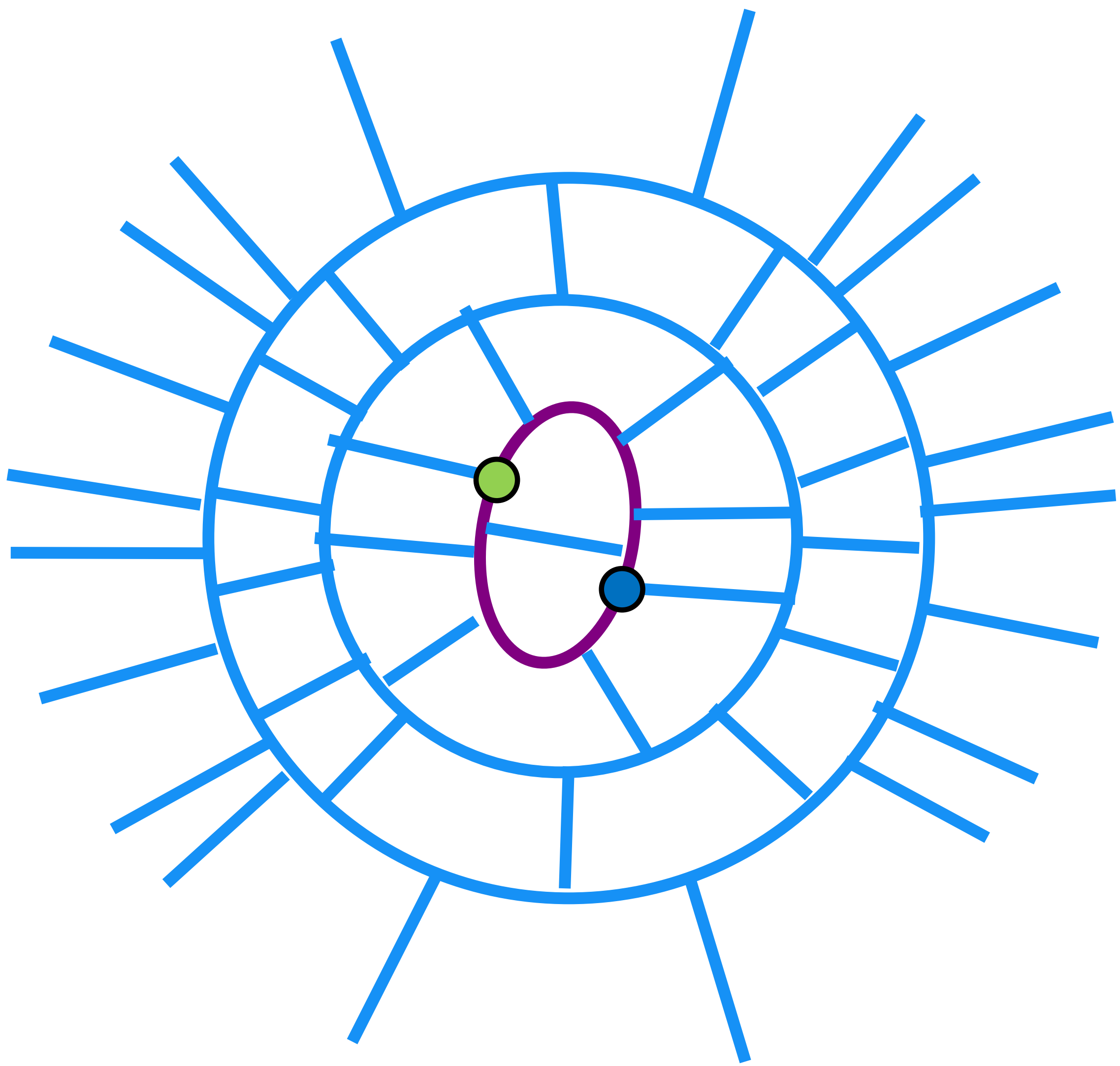


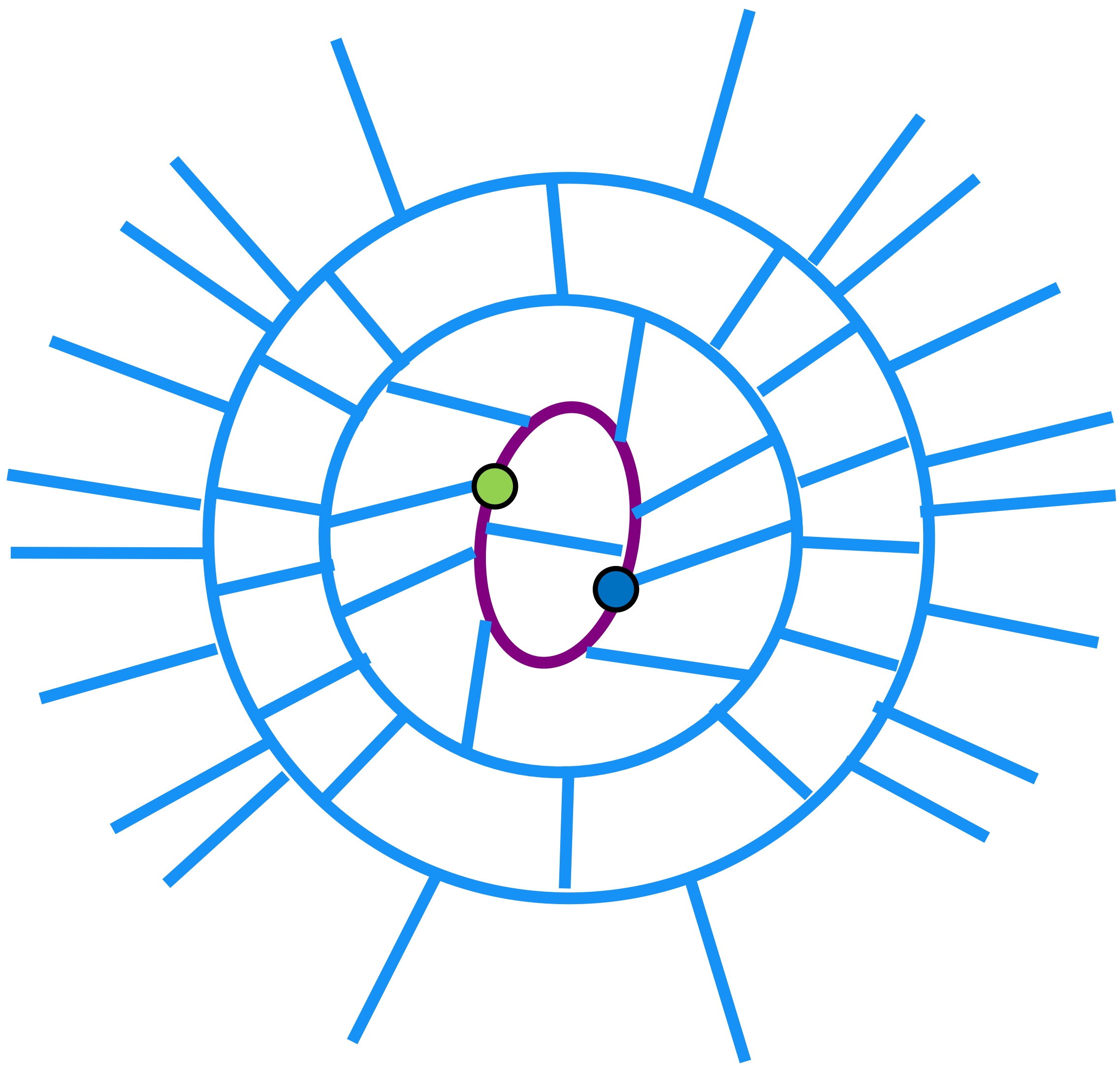


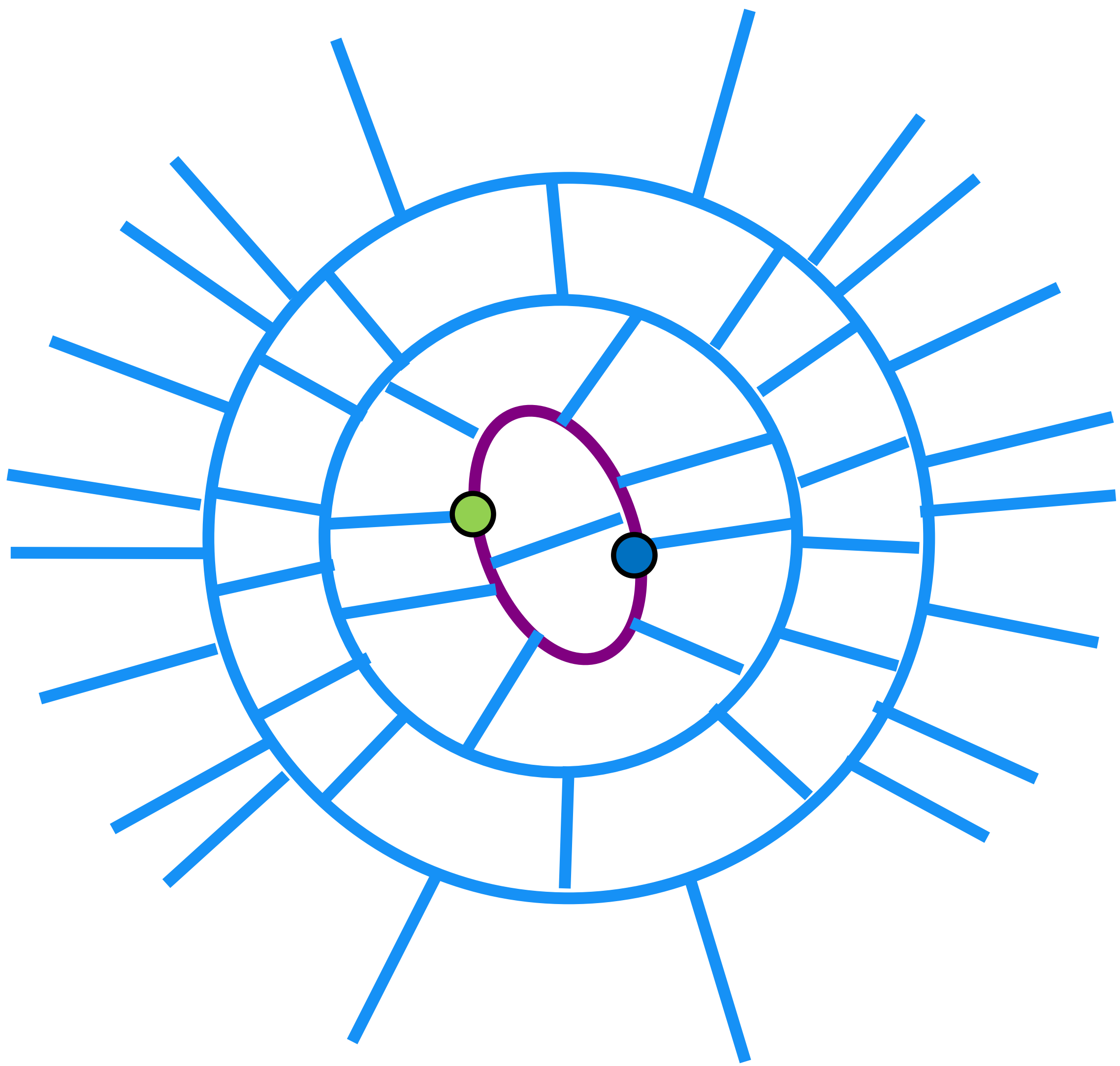


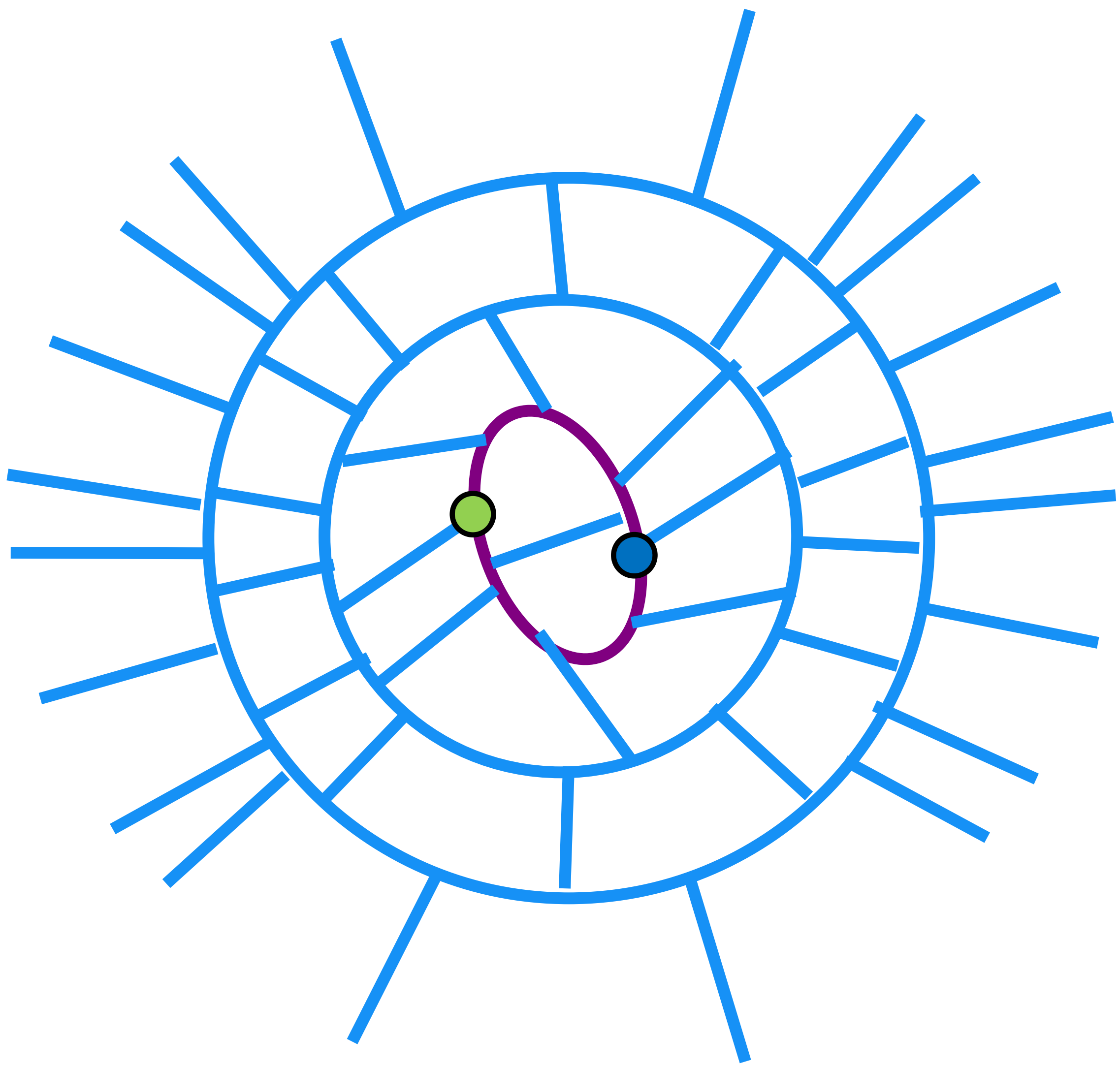


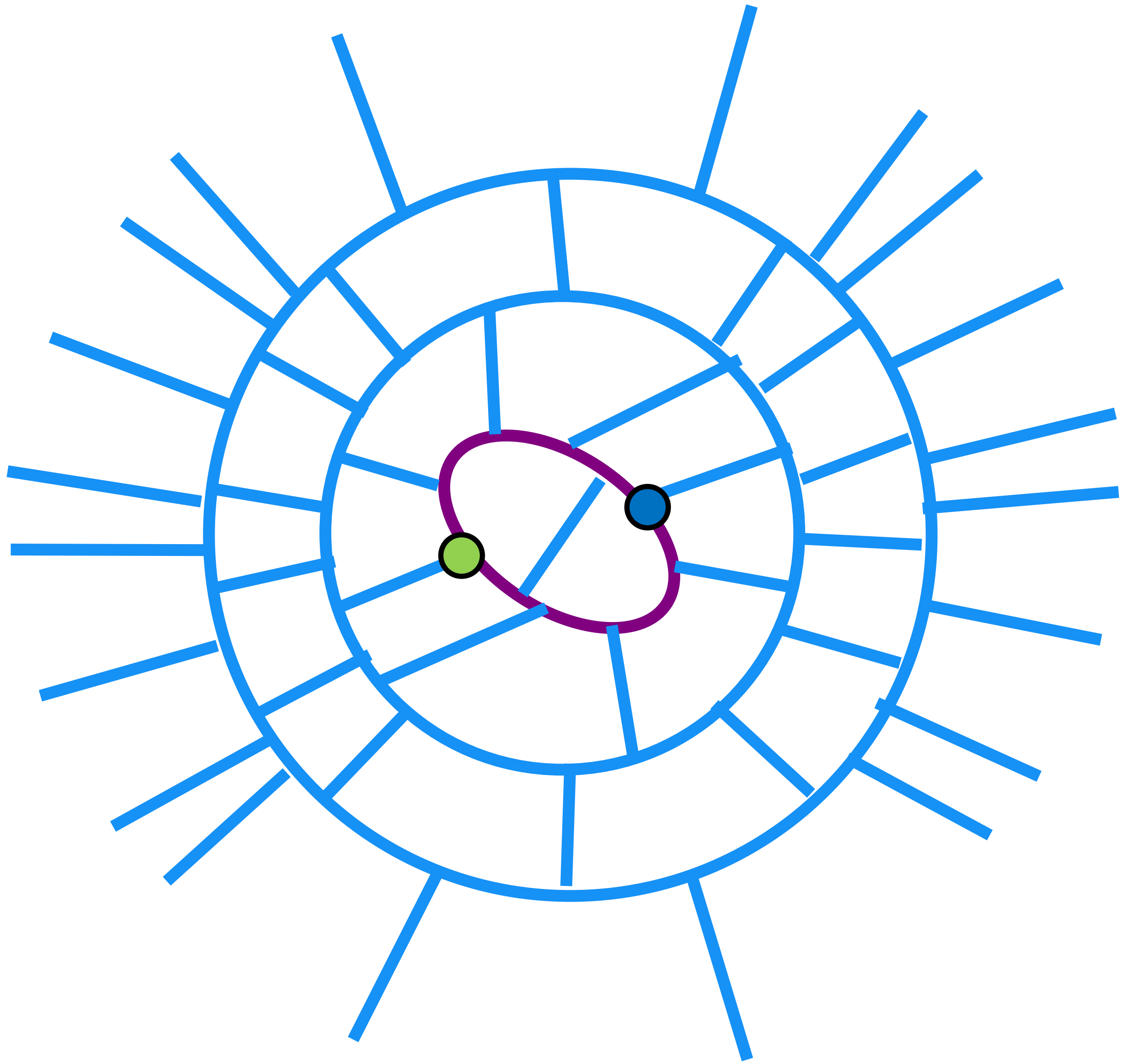


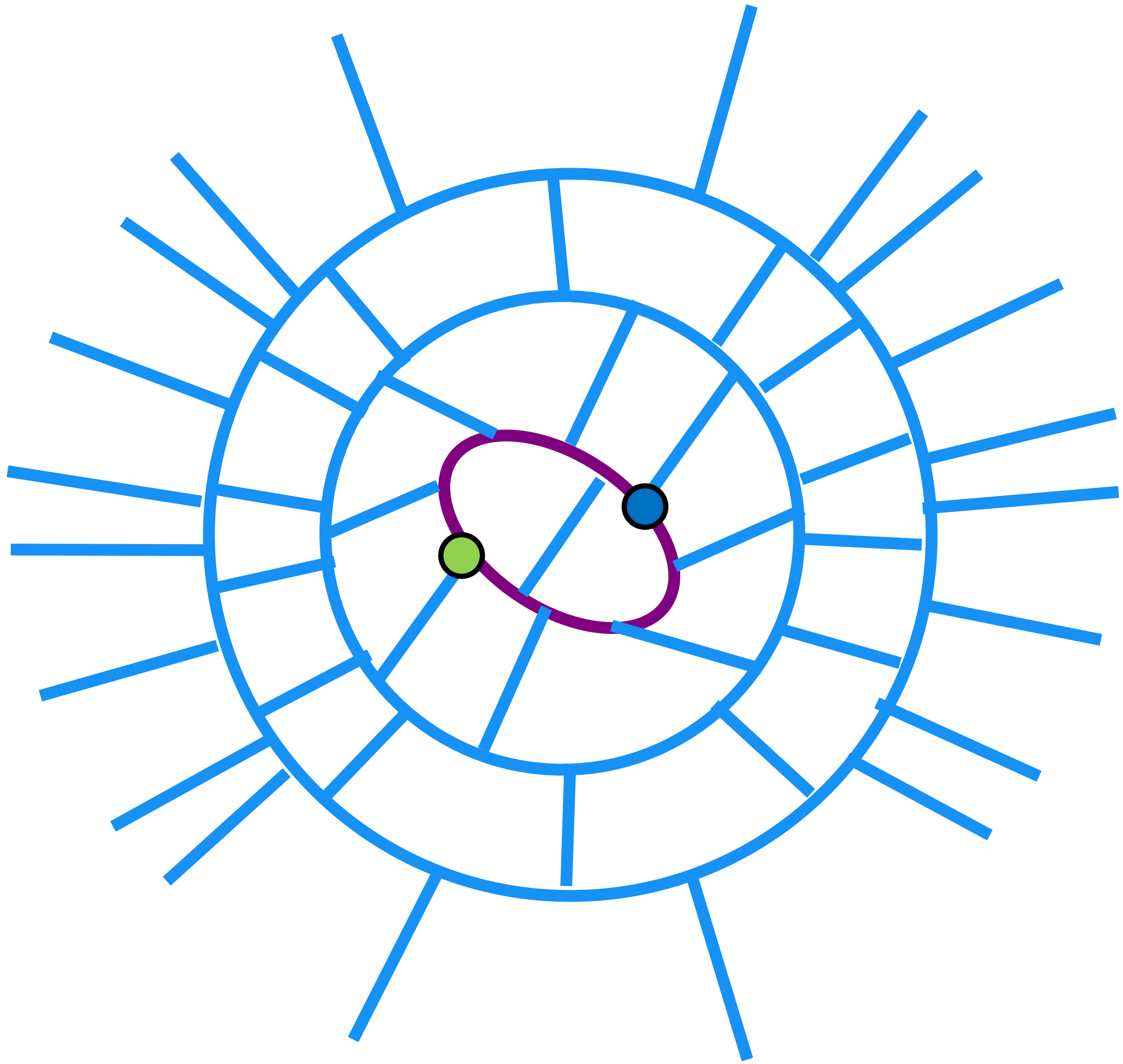


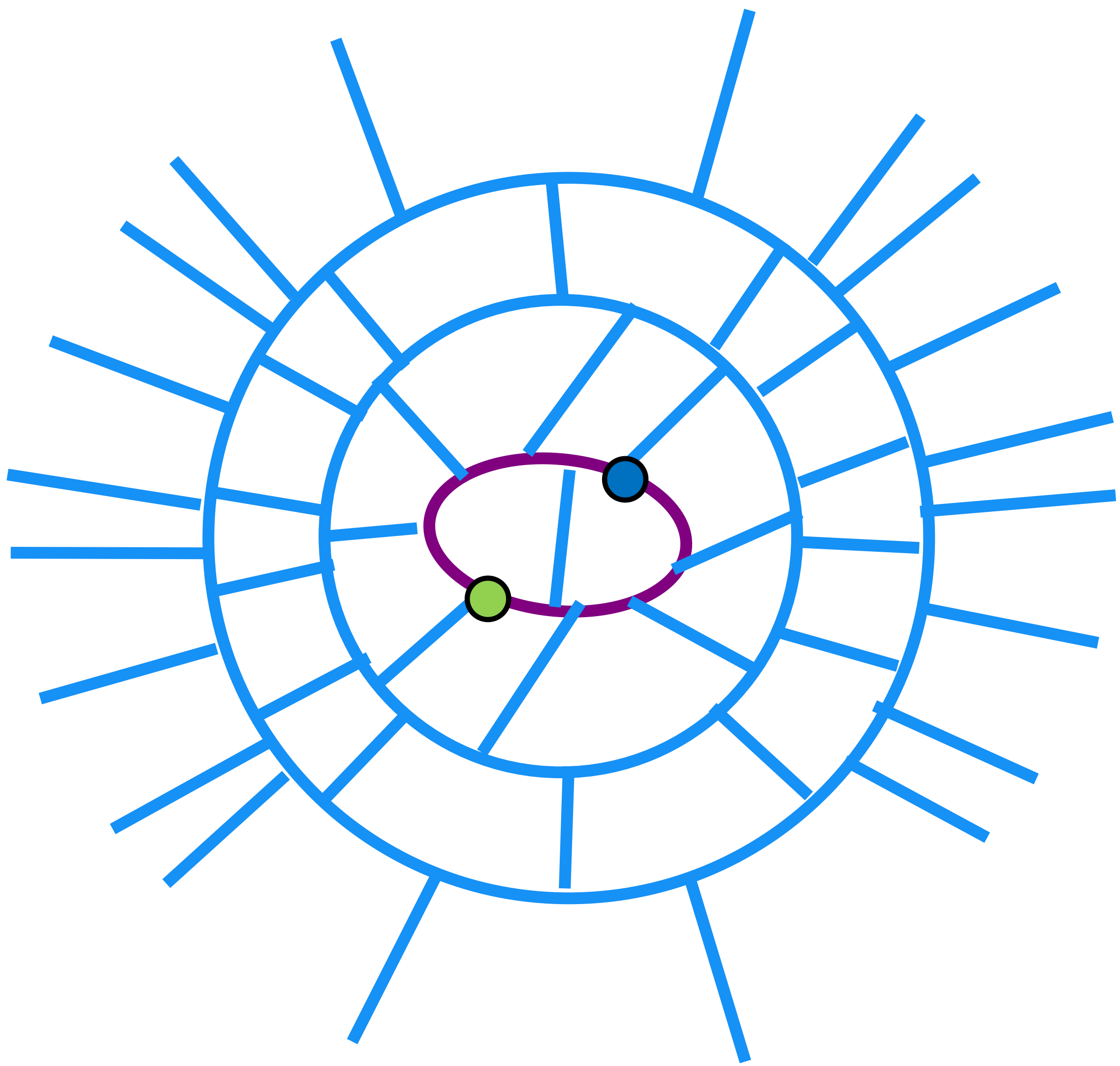


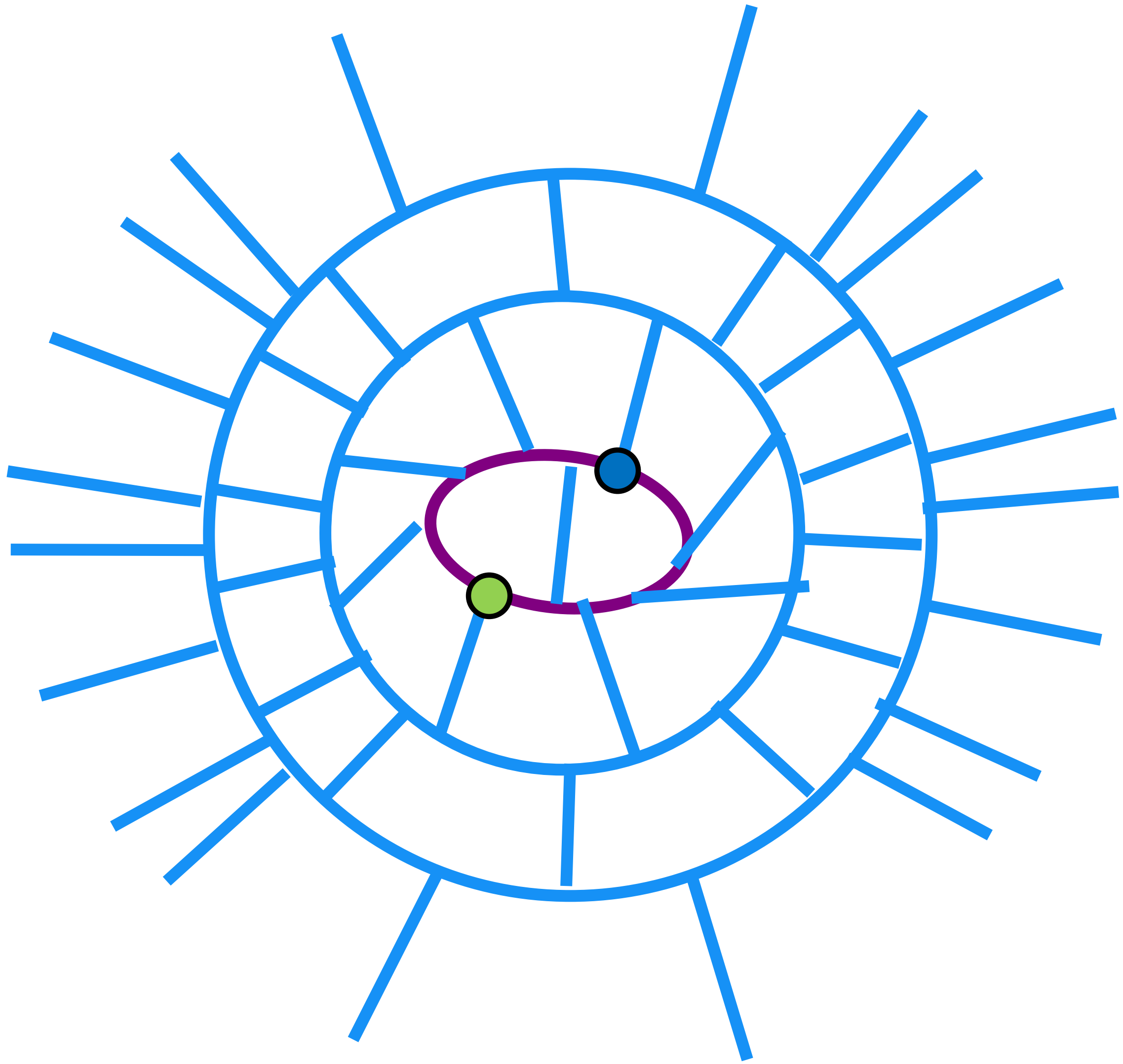


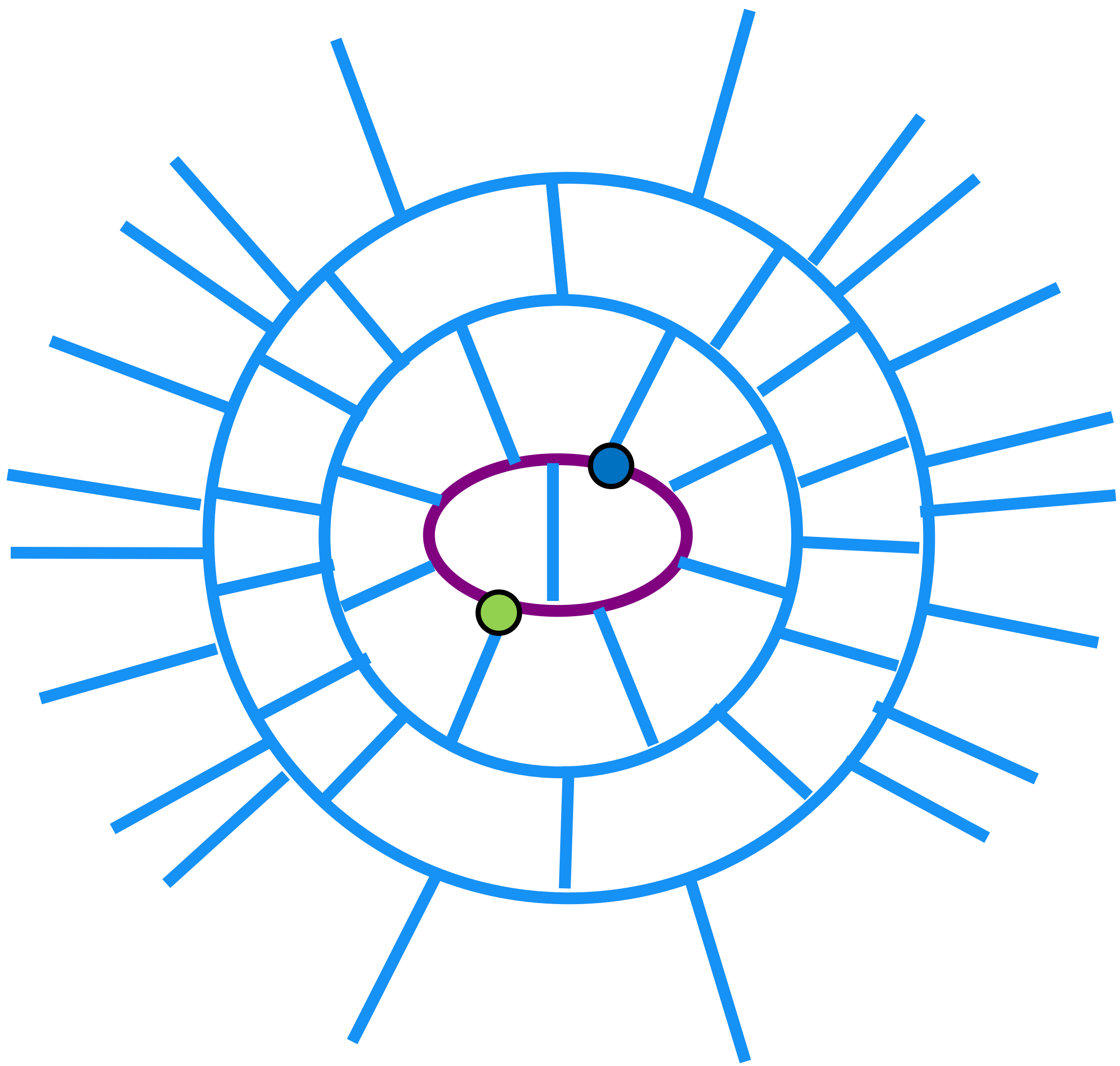






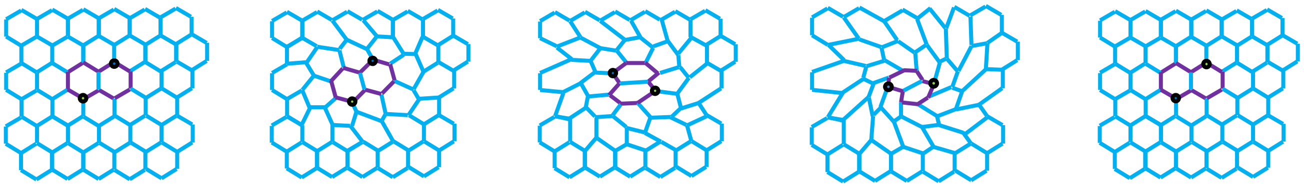






Conclusions

A “conventional” quantum computer can be used to “simulate” a topological quantum computer based on braiding anyons.



The ability to redraw the abstract lattice of the anyon theory using the **F move** is a fundamental resource for this kind of quantum computation.

Thanks to:

	Collaborators		Former Students	
	David DiVincenzo	Juelich/Aachen	Layla Hormozi	BNL
	Steve Simon	Oxford	Georgios Zikos	ASML

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