Pairing and Pair Breaking of Composite Fermions in the ν=1/2+1/2 Quantum Hall Bilayer

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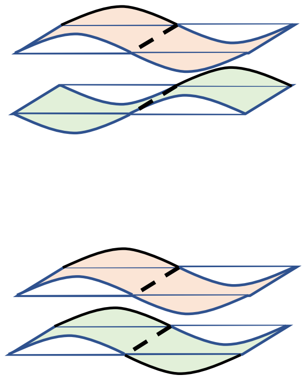
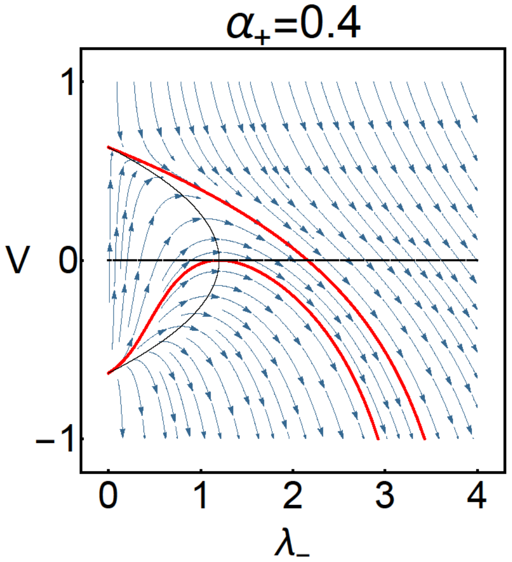
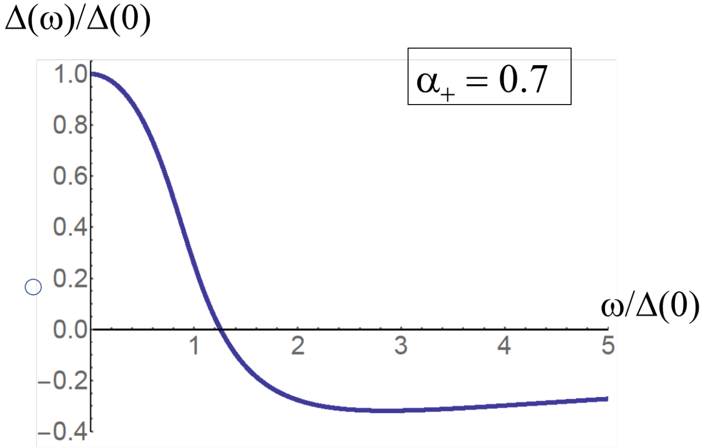
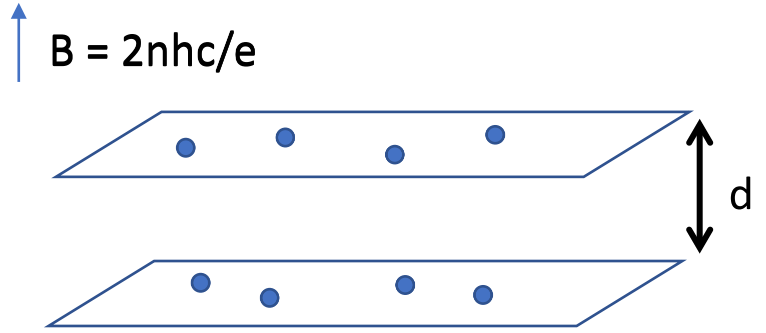


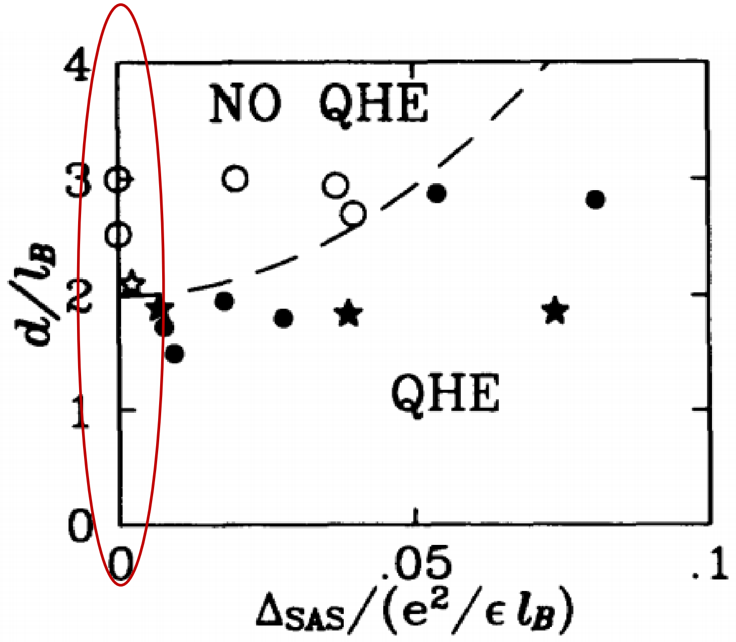




TDLI, 7/20/2021

ν=1/2+1/2 Quantum Hall Bilayer





S.Q. Murphy et al, PRL, 1994

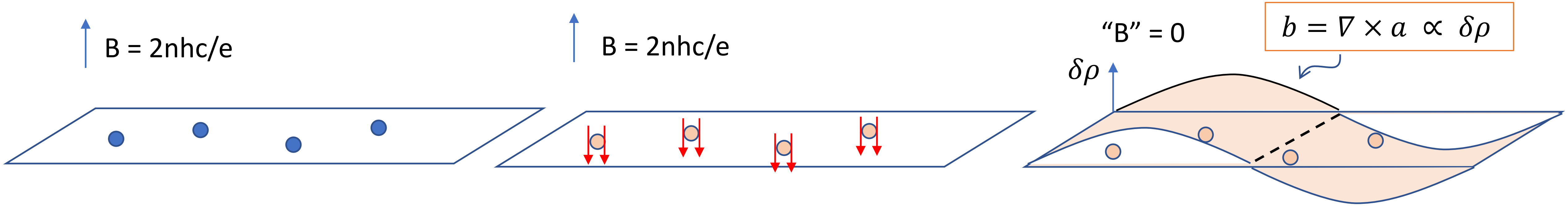
Small d/l_0

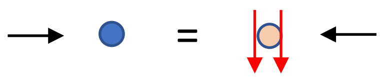
- Halperin 111 state/exciton condensate.
- In absence of tunneling, true long-range order.
- A fascinating state, with decades of ongoing experimental and theoretical work.

Initial theory: K. Moon et al., PRB, 1995

For review, see, J.P. Eisenstein, Annual Review of Condensed Matter Physics, 2014

ν=1/2 Composite Fermion Metal





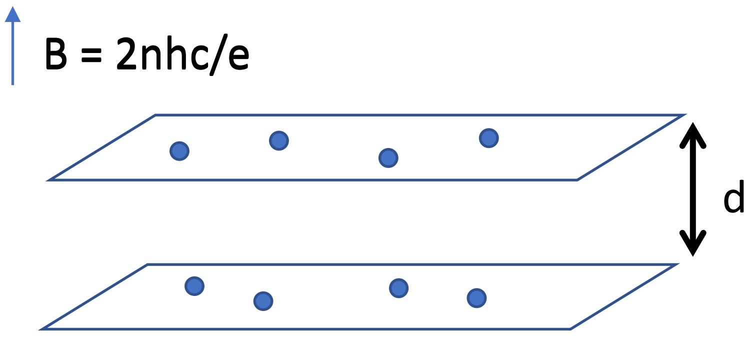
Transverse gauge field *a* tied to particle density

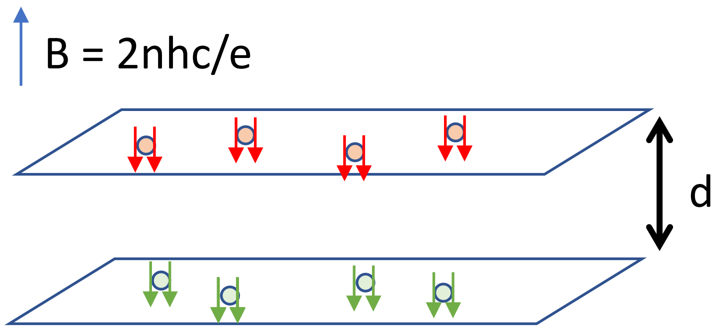
Gauge field propagator: $\langle aa\rangle \sim \dfrac{1}{q + i\omega/q}$ Long-range Coulomb Interaction case

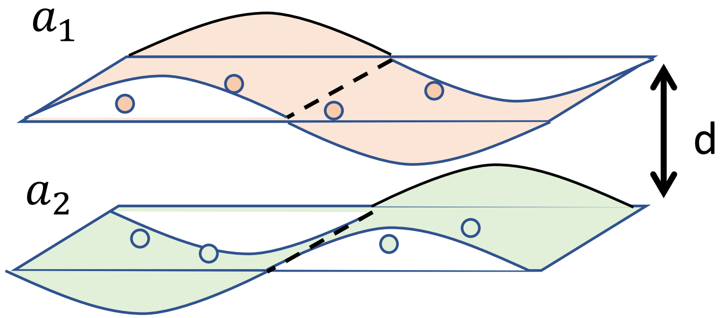
$$\langle aa\rangle \sim \frac{1}{q^2 + i\omega/q}$$ Short-range interaction case

B. Halperin, P. Lee, and N. Read, PRB 1993

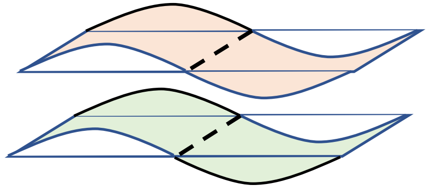
ν=1/2+1/2 Quantum Hall Bilayer







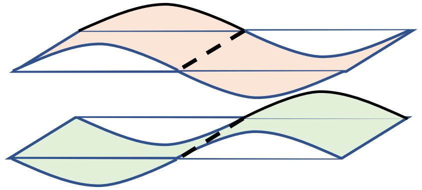
In phase gauge fluctuations



$$a_+ = a_1 + a_2$$

$$\langle a_+ a_+ \rangle \sim \frac{1}{q + i\omega/q}$$

Out of phase gauge fluctuations



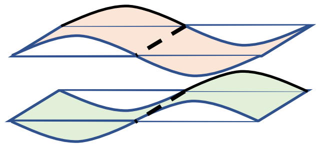
$$a_- = a_1 - a_2$$

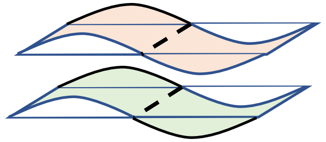
$$\langle a_- a_- \rangle \sim \frac{1}{dq^2 + i\omega/q}$$

NEB, I. McDonald, C. Nayak, PRL, 1996

Interlayer Pairing and Pair Breaking

Fermi surface average of attractive pairing interaction in interlayer Cooper channel:

Out of phase gauge field:  $\lambda_{-}(\omega) = \alpha_{-}\left|\frac{\omega_0}{\omega}\right|^{\gamma}$ More singular, and ***attractive***

In phase gauge field:  $\lambda_{+}(\omega) = \alpha_{+}\ln\left|\frac{\omega_0}{\omega}\right|$ Less singular, and ***repulsive***

$\gamma = 1/3$ $\boxed{\alpha_{-} \sim \left({}^{l_0}/_{d}\right)^{2/3}}$ $\alpha_{+} \sim 1$ $\omega_0 \sim e^2/l_0$

- Possibly instability to interlayer BCS pairing of composite fermions.

NEB, I. McDonald, C. Nayak, PRL, 1996

- Resulting paired quantum Hall state *may* be adiabatically connected to the 111 state.

I. Sodemann, I. Kimchi, C. Wang, and T. Senthil, PRB, 2017

Gap Equation

$$\Delta(\omega) = \frac{1}{2} \int \frac{\Delta(\Omega)}{\sqrt{\Omega^2 + |\Delta(\Omega)|^2}} \lambda(\omega - \Omega)\, \mathrm{d}\Omega$$

$$(T = 0,\ \ \text{imaginary frequency})$$

Gap Equation

$$\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\,\lambda(\omega-\Omega)\,\mathrm{d}\Omega$$

$$\lambda(\omega) = \alpha_- \left|\frac{\omega_0}{\omega}\right|^{\gamma} - \alpha_+ \ln\left|\frac{\omega_0}{\omega}\right| - \mathrm{V}_0$$

Gap Equation

$$\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\lambda(\omega-\Omega)\,d\Omega$$

$$\lambda(\omega) = \underbrace{\alpha_- \left|\frac{\omega_0}{\omega}\right|^\gamma - \alpha_+ \ln\left|\frac{\omega_0}{\omega}\right|}_{\text{Singular pairing and pairbreaking (independent of angular momentum pairing channel)}} - V_0$$

Non singular interaction (dependent on angular momentum pairing channel)

Gap Equation

$$\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\lambda(\omega-\Omega)\,\mathrm{d}\Omega$$

$$\lambda(\omega) = \alpha_{-}\left|\frac{\omega_0}{\omega}\right|^{\gamma}$$

Gap Equation

$$\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2 + |\Delta(\Omega)|^2}} \lambda(\omega - \Omega)\, d\Omega$$

$$\lambda(\omega) = \alpha_- \left|\frac{\omega_0}{\omega}\right|^\gamma$$

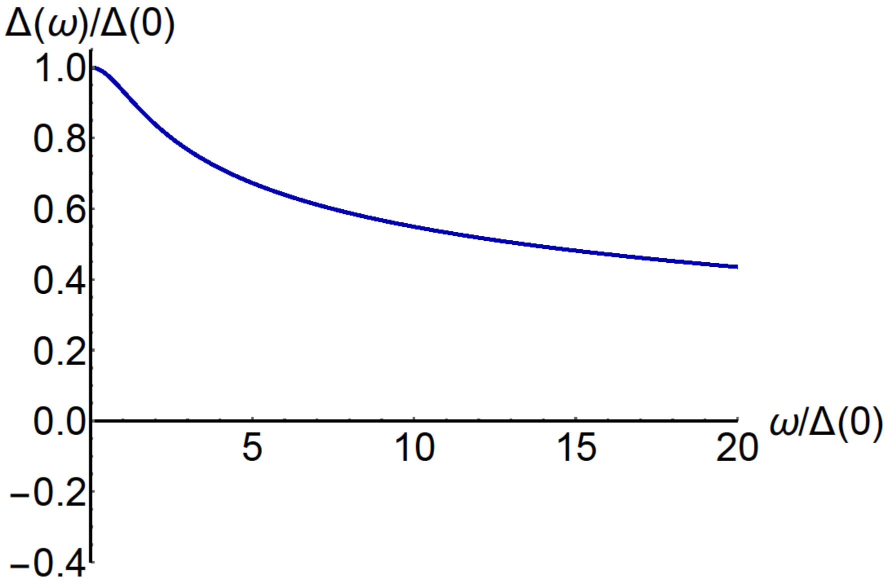
For $\gamma = \frac{1}{3}$: $\Delta(0) = (25.8\cdots)\, \alpha_-{}^3 \omega_0$

$\alpha_- \sim \left({}^{l_0}/_{d}\right)^{2/3} \rightarrow \Delta(0) \sim \omega_0 (l_0/d)^2$

Seems like a big effect!

Solve by simple scaling

NEB, McDonald, Nayak, PRL, 1996



Gap Equation

$$\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\lambda(\omega-\Omega)\,d\Omega$$

$$\lambda(\omega) = \alpha_-\left|\frac{\omega_0}{\omega}\right|^\gamma - \alpha_+\ln\left|\frac{\omega_0}{\omega}\right| - V_0$$

Question: What is the effect of the in-phase pair breaking gauge fluctuations on the energy gap?

Gap Equation

$$\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\lambda(\omega-\Omega)\,\mathrm{d}\Omega$$

$$\lambda(\omega) = \alpha_- \left|\frac{\omega_0}{\omega}\right|^\gamma - \alpha_+ \ln\left|\frac{\omega_0}{\omega}\right| - V_0$$

Previous work.

- Numerical solution:

  Z. Wang, I. Mandal, S.B. Chung, and S. Chakravarty, Annals of Phys., 2014

- Renormalization group approach:

  I. Sodemann, I. Kimchi, C. Wang, and T. Senthil, PRB, 2017

Local Approximation

- Links gap equation and RG approach to pairing
  H. Wang, S. Raghu, & G. Torroba, PRB 2017
- Allows for analytic solution of gap equation

First linearize gap equation: $\Delta(\omega) = \frac{1}{2}\int_{|\Omega|>|\Delta(0)|} \frac{\Delta(\Omega)}{|\Omega|}\lambda(\omega - \Omega)\ d\Omega$

Then apply local approximation:

$$\lambda(\omega - \Omega) = \begin{cases} \underline{\lambda(\omega)} - \lambda'(\omega)\,\Omega + \cdots, & |\Omega| < |\omega| \\ \underline{\lambda(\Omega)} + \lambda'(\Omega)\,\omega + \cdots, & |\Omega| > |\omega| \end{cases}$$

Local Approximation

$$\Delta(\omega) = \int_{|\Omega|>|\omega|} \frac{\Delta(\Omega)}{|\Omega|}\lambda(\Omega)\mathrm{d}\Omega \; + \int_{|\omega|>|\Omega|>|\Delta(0)|}^{\omega_0} \frac{\Delta(\Omega)}{|\Omega|}\lambda(\omega)\mathrm{d}\Omega$$

Local Approximation

$$\Delta(\omega) = \int_{|\Omega|>|\omega|} \frac{\Delta(\Omega)}{|\Omega|} \lambda(\Omega) \mathrm{d}\Omega \ + \int_{|\omega|>|\Omega|>|\Delta(0)|}^{\omega_0} \frac{\Delta(\Omega)}{|\Omega|} \lambda(\omega) \mathrm{d}\Omega$$

Equivalent to a linear second order diff eq.

$$\frac{d}{d\omega}\left(\frac{\Delta'}{\lambda'}\right) - \frac{\Delta}{\omega} = 0$$

Boundary Conditions

$$\left.\frac{d\Delta}{d\omega}\right|_{\omega=\Delta(0)} = 0$$

$$\left.\frac{d}{d\omega}\left(\frac{\Delta}{\lambda}\right)\right|_{\omega=\omega_0} = 0$$

Local Approximation

Linear second-order diff eq.

$$\frac{d}{d\omega}\left(\frac{\Delta'}{\lambda'}\right) - \frac{\Delta}{\omega} = 0$$

Introduce: $V(\omega) = -\lambda'(\omega)\,\frac{\Delta(\omega)}{\Delta'(\omega)}$ → Nonlinear first-order diff eq.

Flow parameter: $l = \ln\frac{\omega_0}{\omega}$

$$\frac{dV}{dl} = -\gamma\lambda_- + \alpha_+ - V^2$$

Simple example of link between the gap equation and the RG approach to singular pairing, e.g., in M. Metlitski, D. Mross, S. Sachdev, T. Senthil, PRB, 2015,

H. Wang, S. Raghu, & G. Torroba, PRB 2017

Flow Equations

$$\boxed{\begin{aligned} \frac{dV}{dl} &= -\gamma\lambda_- + \alpha_+ - V^2 \\ \frac{d\lambda_-}{dl} &= \gamma\lambda_- \end{aligned}}$$

$$V(\omega_0) = \alpha_- + V_0$$

$V \longrightarrow -\infty$ as $\omega \rightarrow \Delta(0)$

Flow parameter: $l = \ln\frac{\omega_0}{\omega}$

Flow Equations

$$\frac{dV}{dl} = -\gamma\lambda_- + \alpha_+ - V^2$$
$$\frac{d\lambda_-}{dl} = \gamma\lambda_-$$

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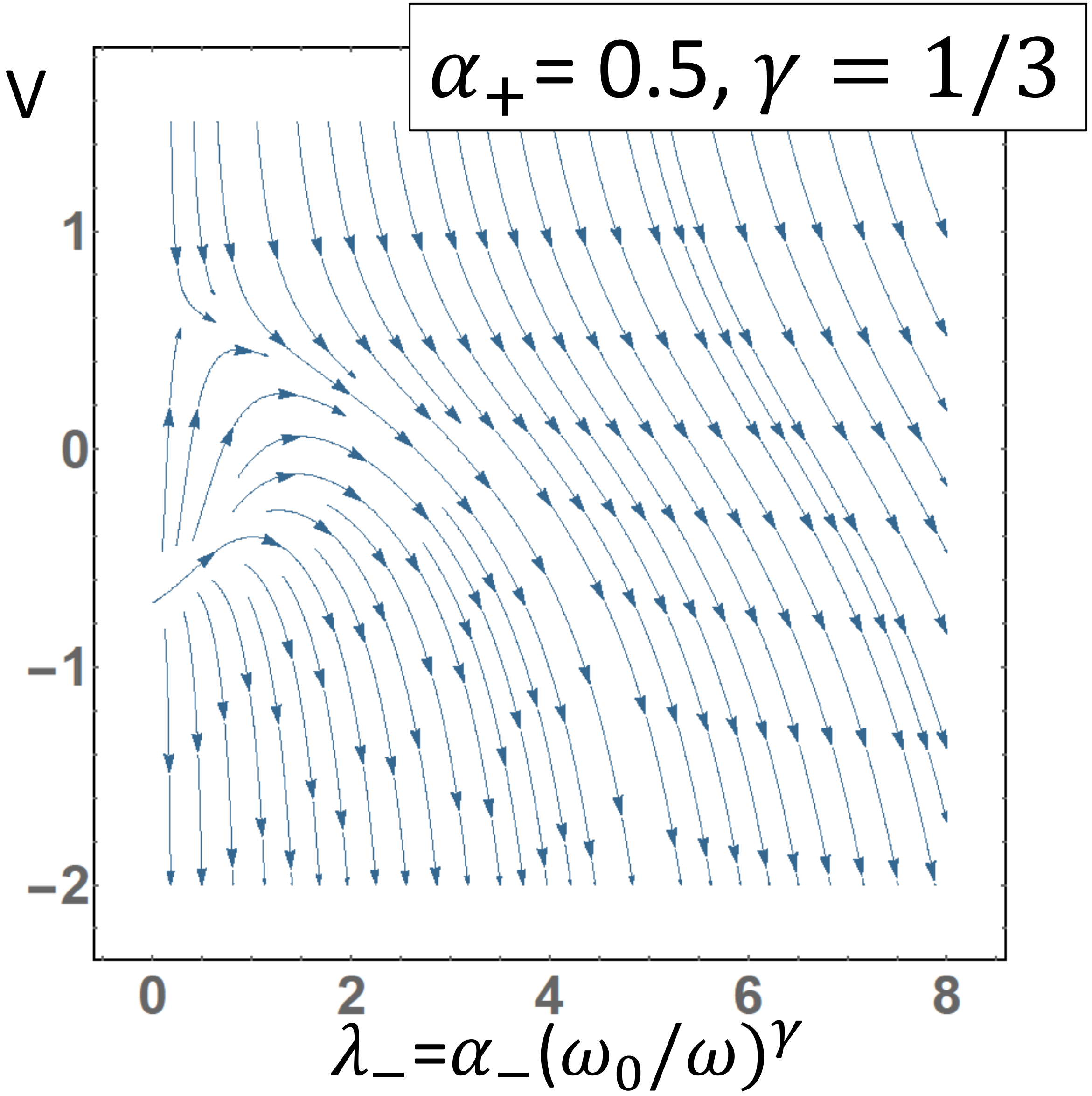

Flow Equations

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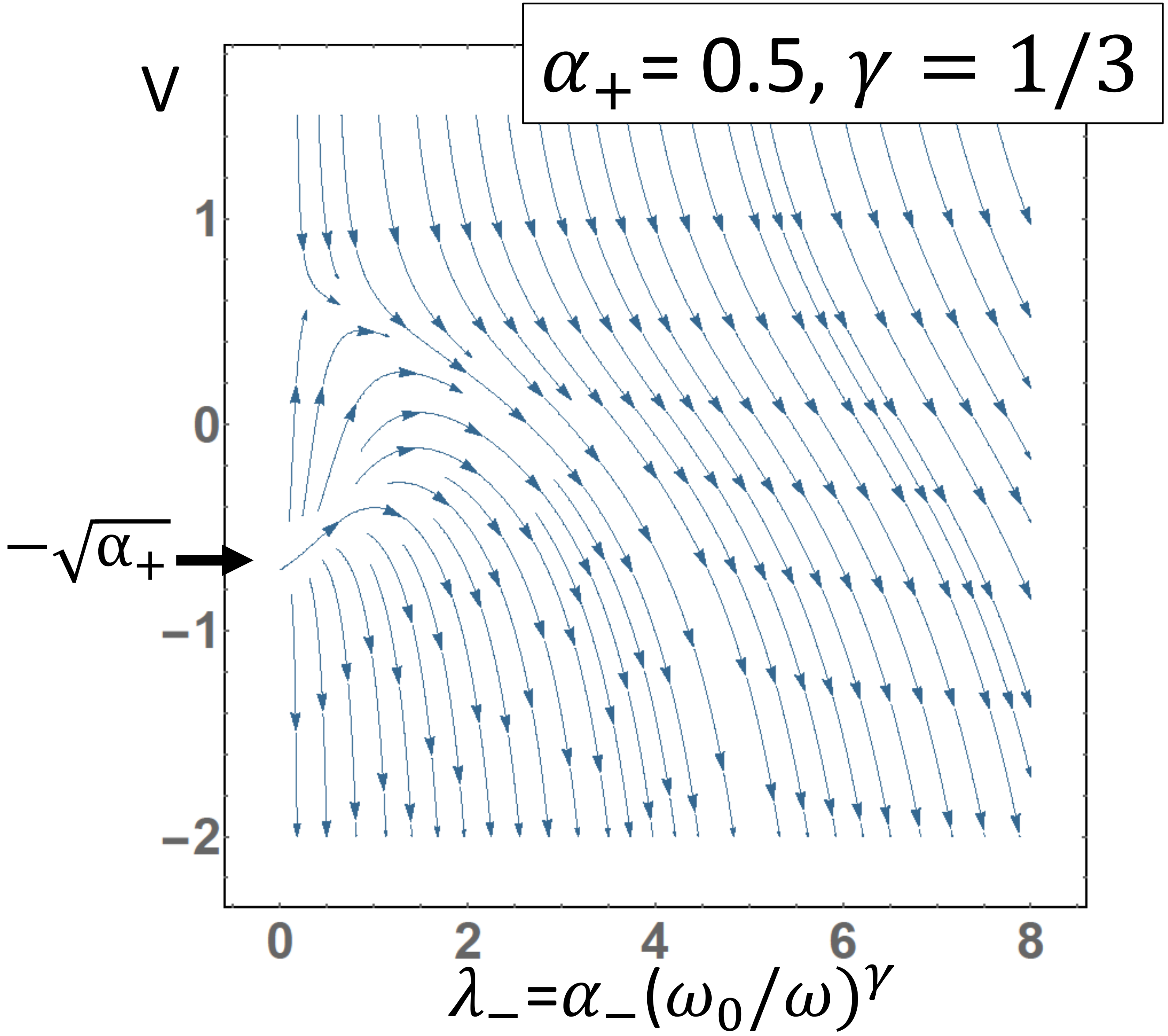

Flow Equations

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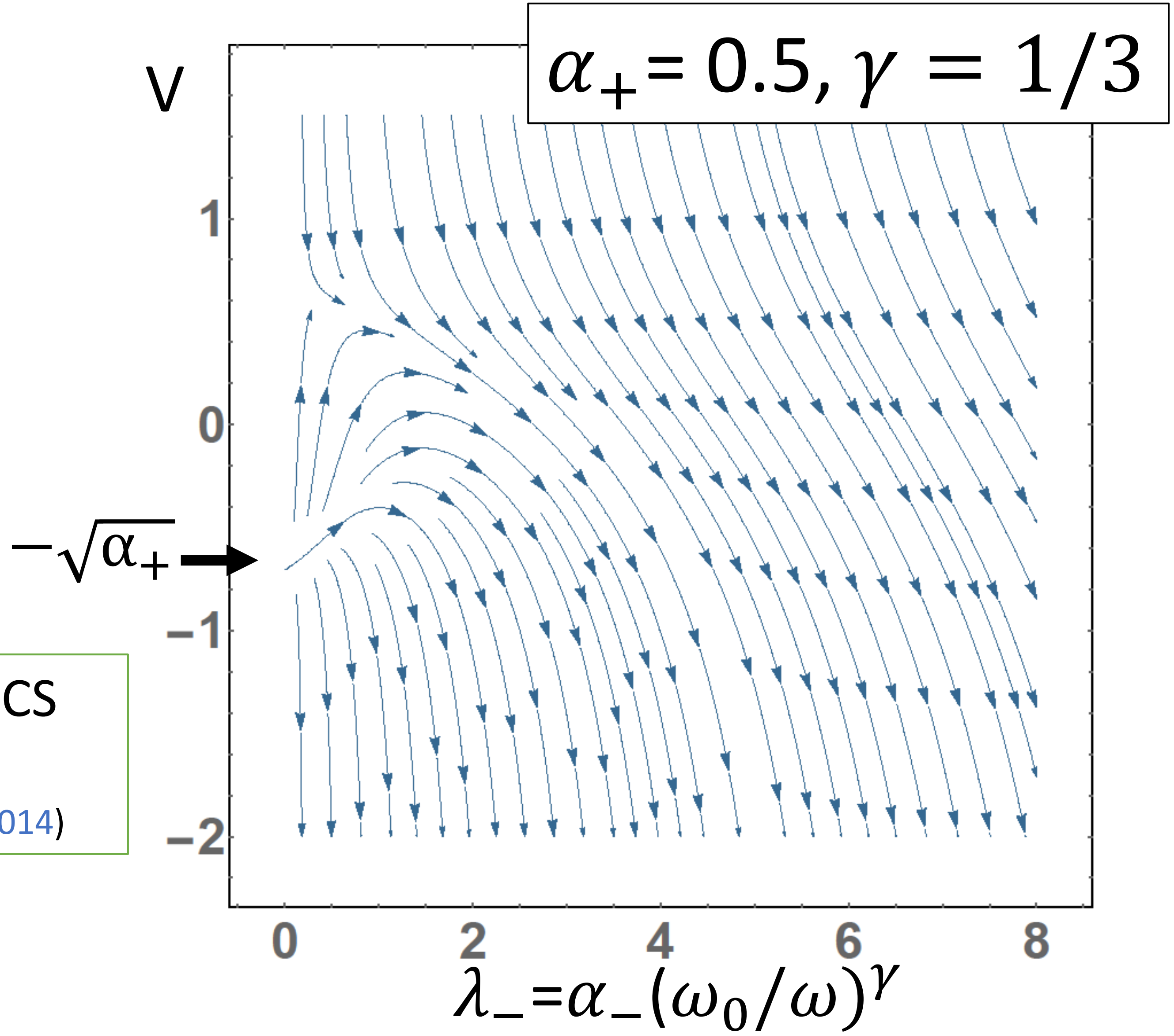


Finite critical value of V_0 needed for BCS pairing instability when $\alpha_- = 0$.
(M. Metlitski, D. Mross, S. Sachdev, and T. Senthil, PRB, 2014)

Flow parameter: $l = \ln\frac{\omega_0}{\omega}$

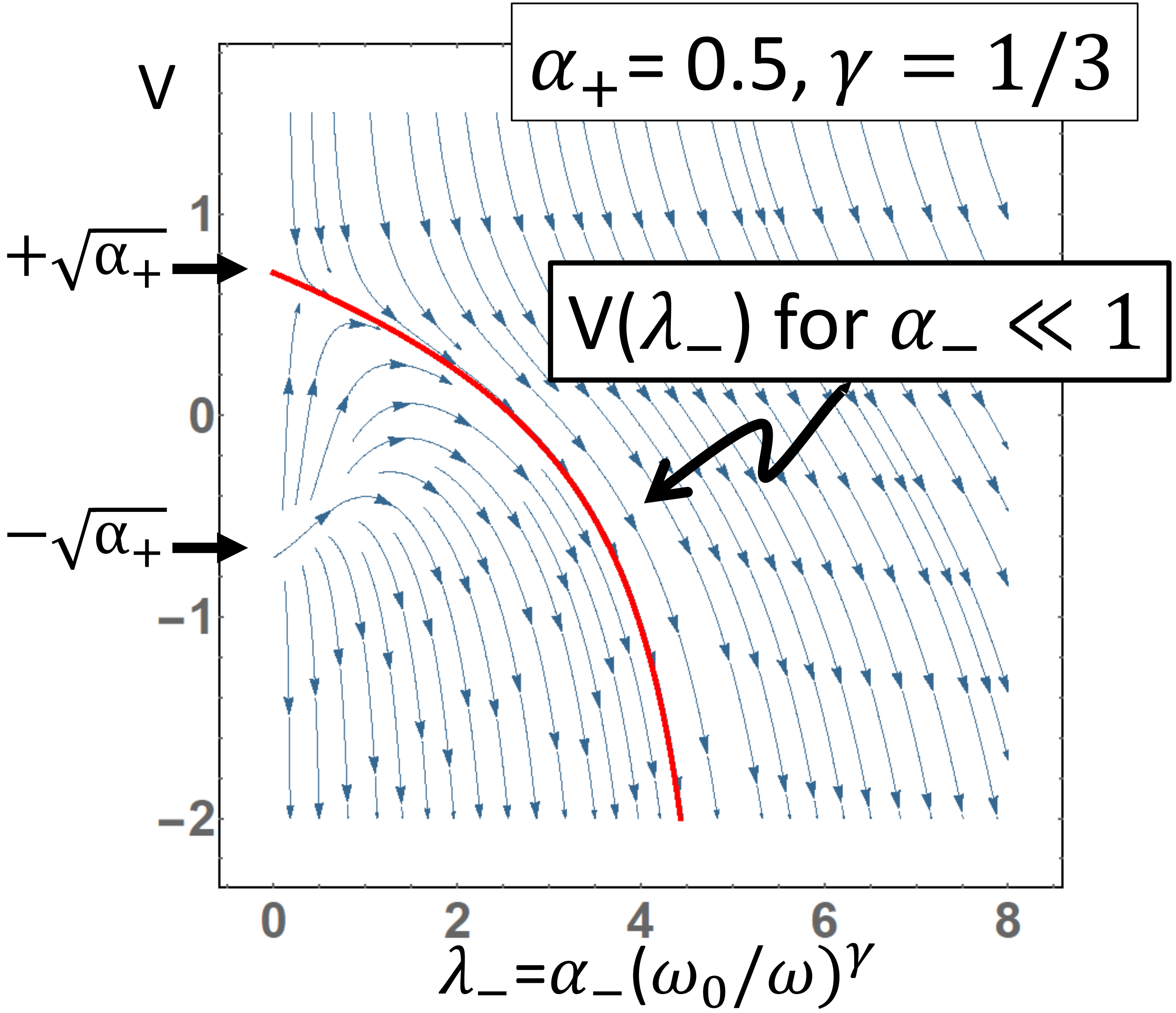
Flow Equations

$$\frac{dV}{dl} = -\gamma\lambda_- + \alpha_+ - V^2$$

$$\frac{d\lambda_-}{dl} = \gamma\lambda_-$$

$$V(\omega_0) = \alpha_- + V_0$$

$V \longrightarrow -\infty$ as $\omega \rightarrow \Delta(0)$



Flow parameter: $l = \ln\frac{\omega_0}{\omega}$

Flow Equations

$$\frac{dV}{dl} = -\gamma\lambda_- + \alpha_+ - V^2$$

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$$V(\omega_0) = \alpha_- + V_0$$

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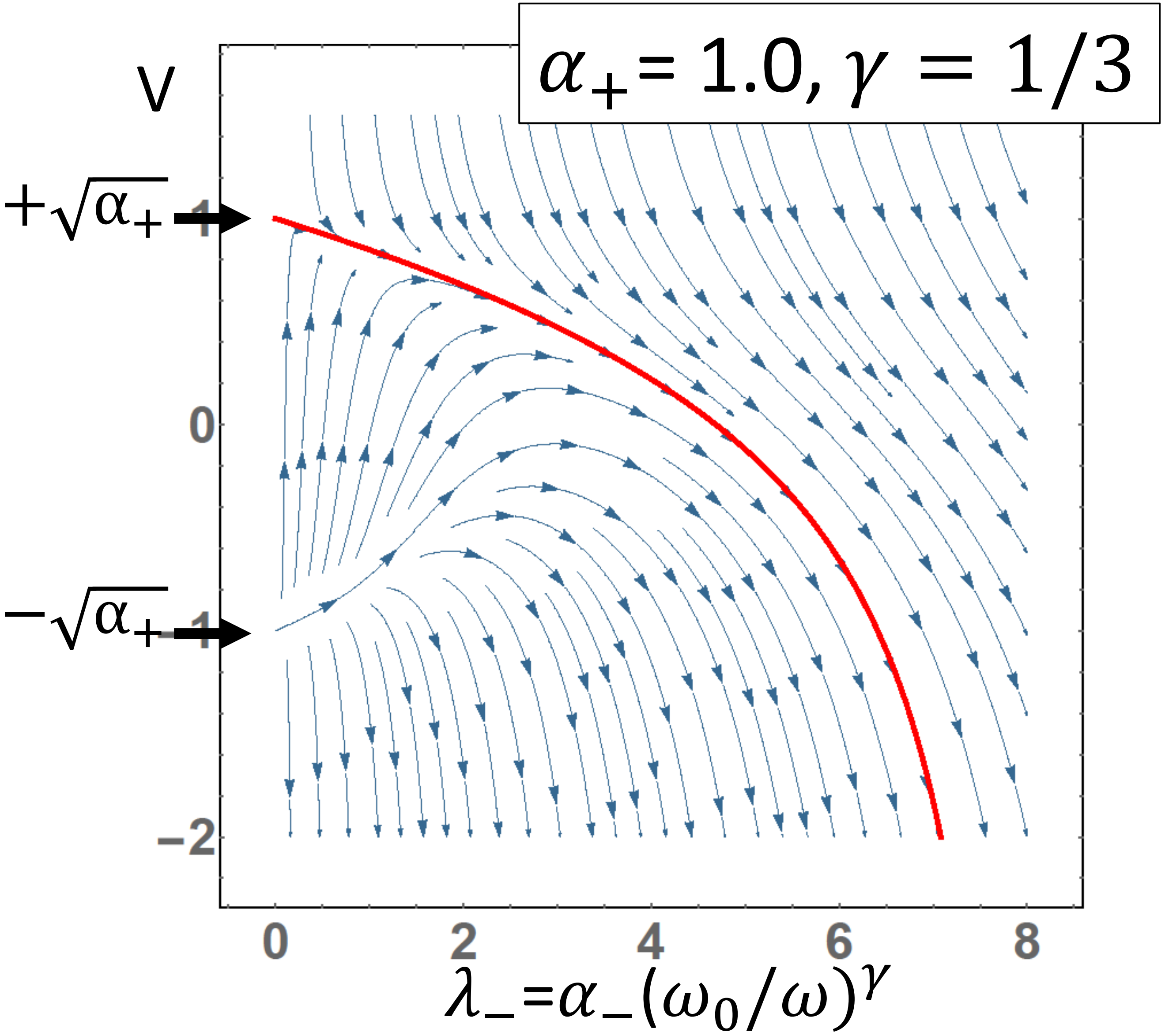
Flow parameter: $l = \ln\frac{\omega_0}{\omega}$



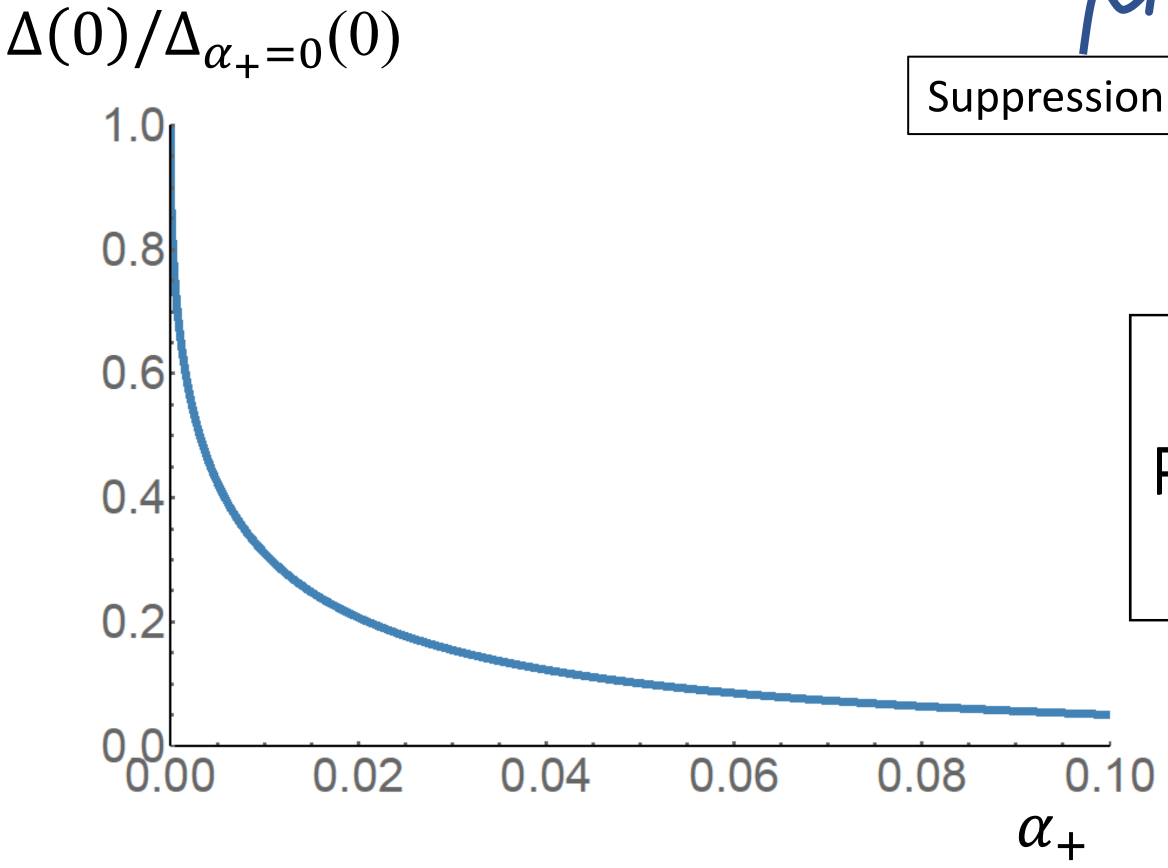
Large Layer Spacing Limit

Result for small α_-:

$$\Delta(0\,) \cong \boxed{e^{-(2.566\cdots)\sqrt{\alpha_+}/\gamma^{3/2}}}\ \boxed{\left(\frac{0.6917\cdots}{\gamma}\right)^{1/\gamma} \alpha_-{}^{1/\gamma}\ \omega_0}$$

Suppression factor due to α_+

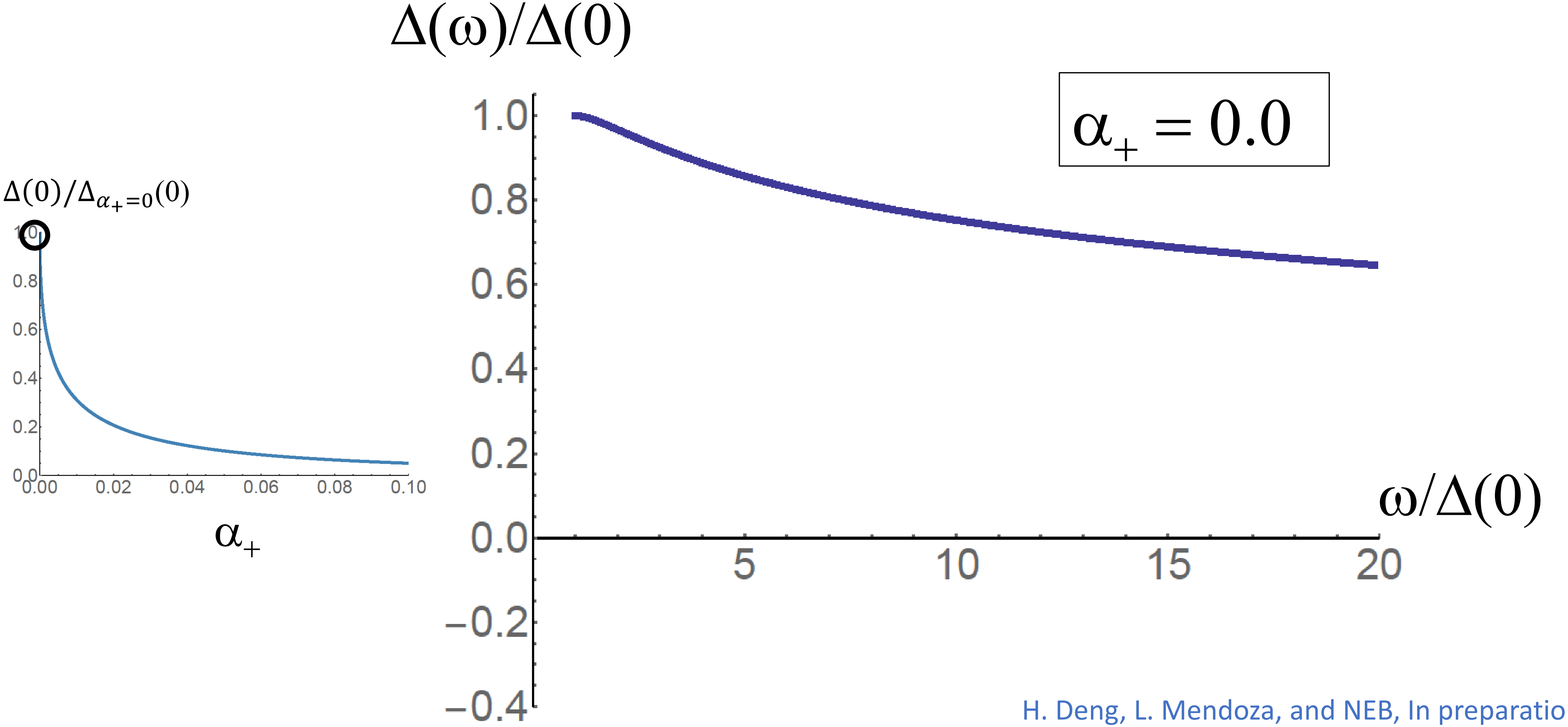
$\alpha_+ = 0$ gap



Pair breaking parameter: $\sqrt{\alpha_+}/\gamma^{3/2}$

H. Deng, L. Mendoza, and NEB, In preparation

Local Approximation $\Delta(\omega)$ for $\alpha\_=.1$



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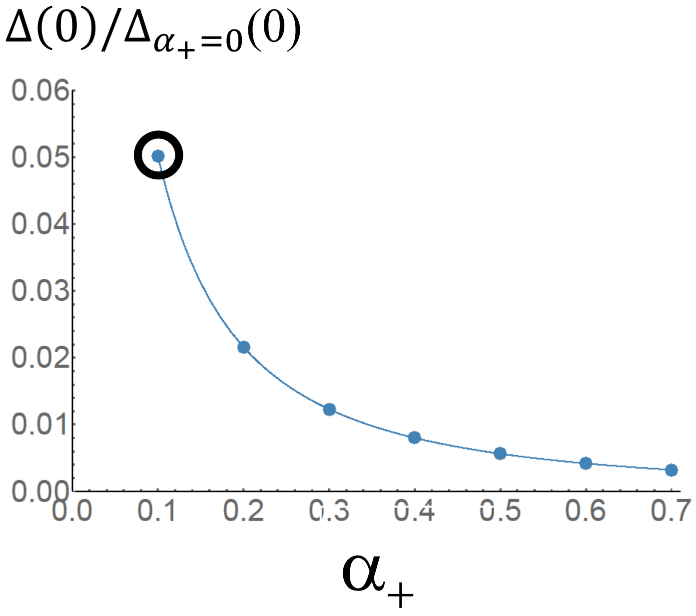

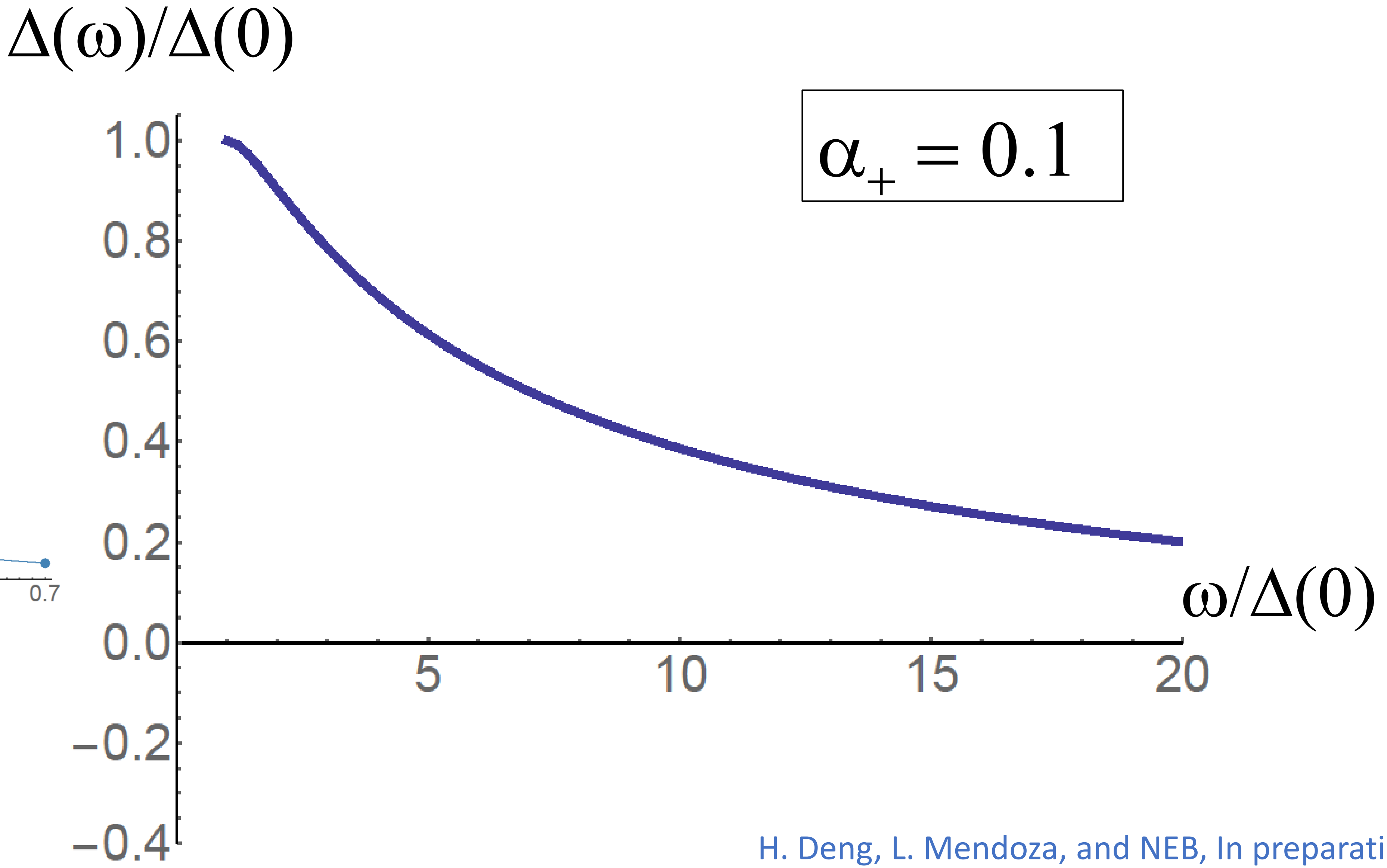

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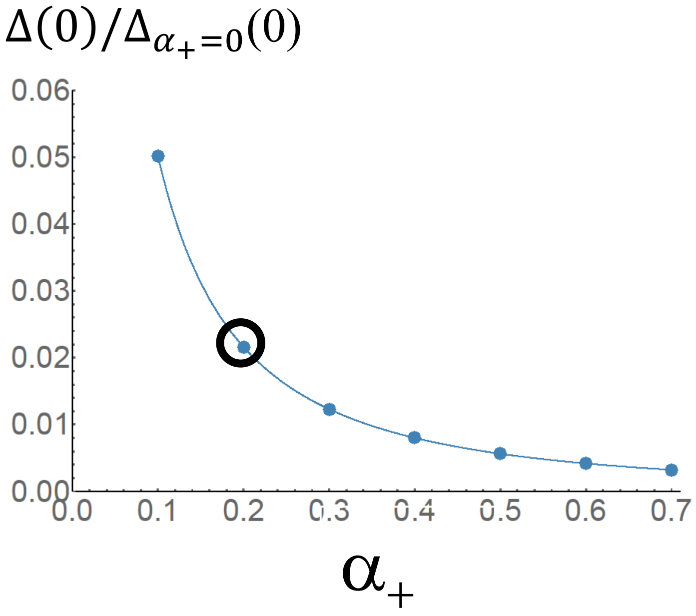
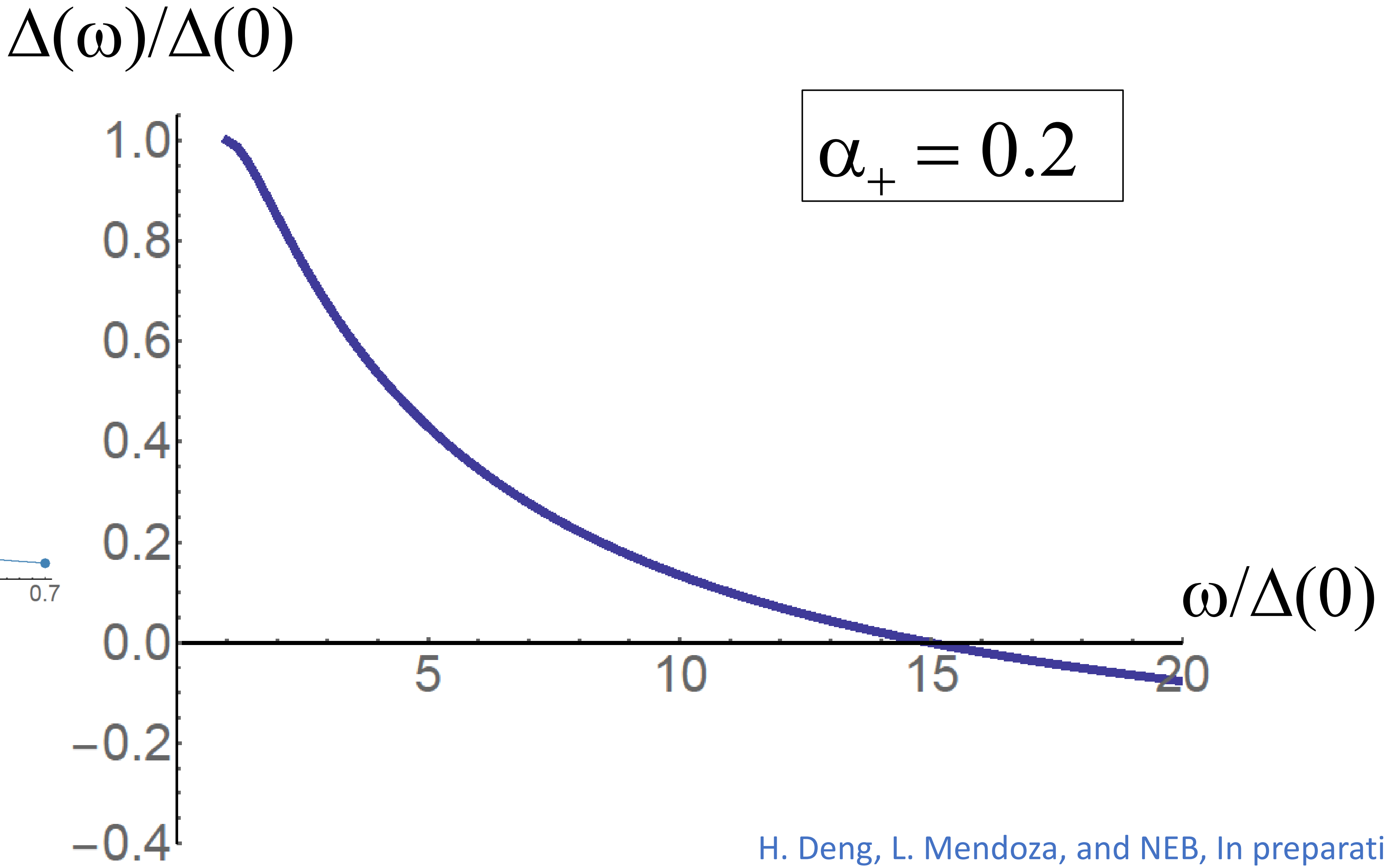
Local Approximation Δ(ω) for $\alpha\_ = .1$





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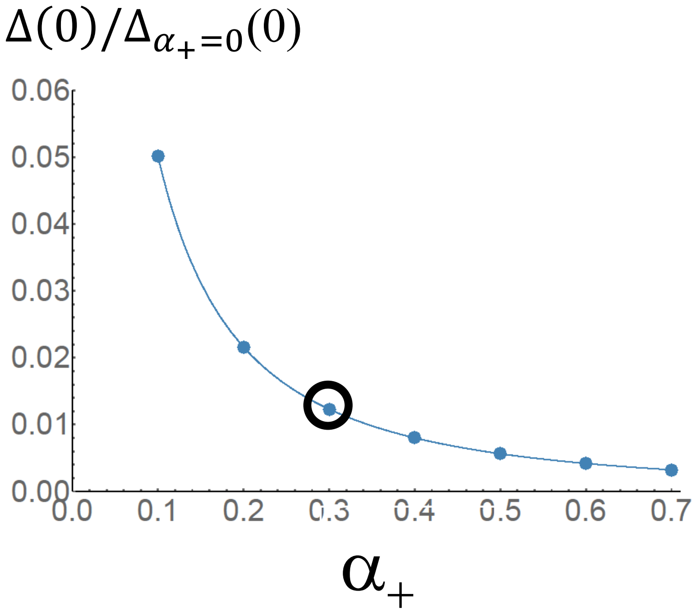

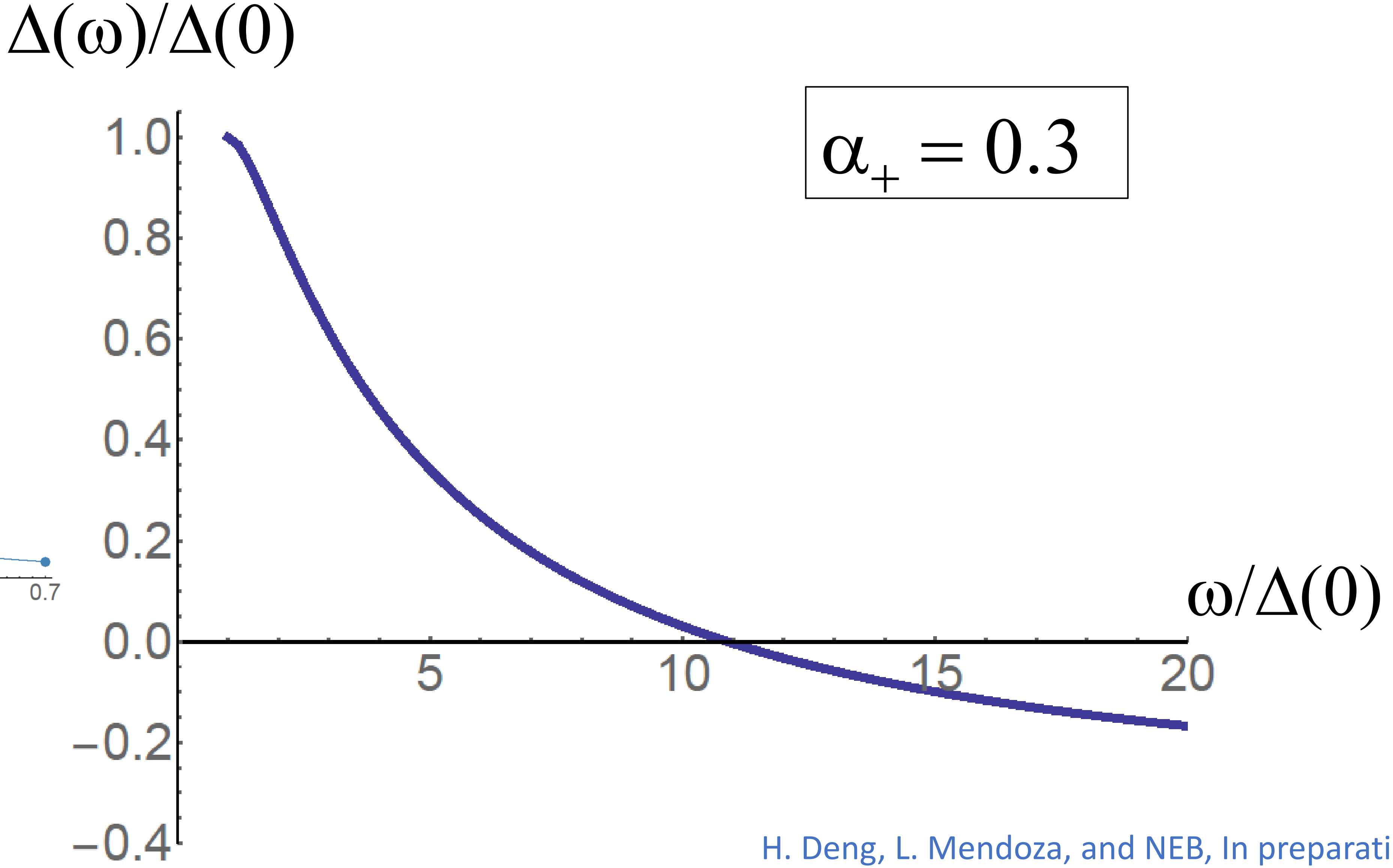

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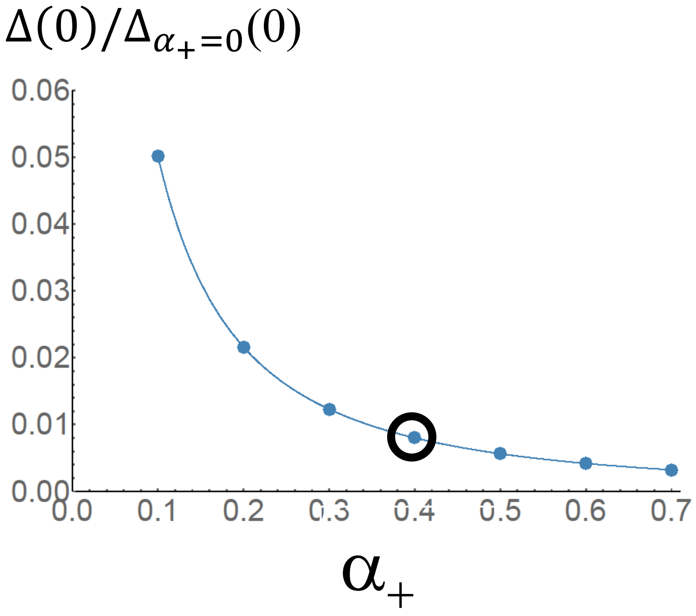
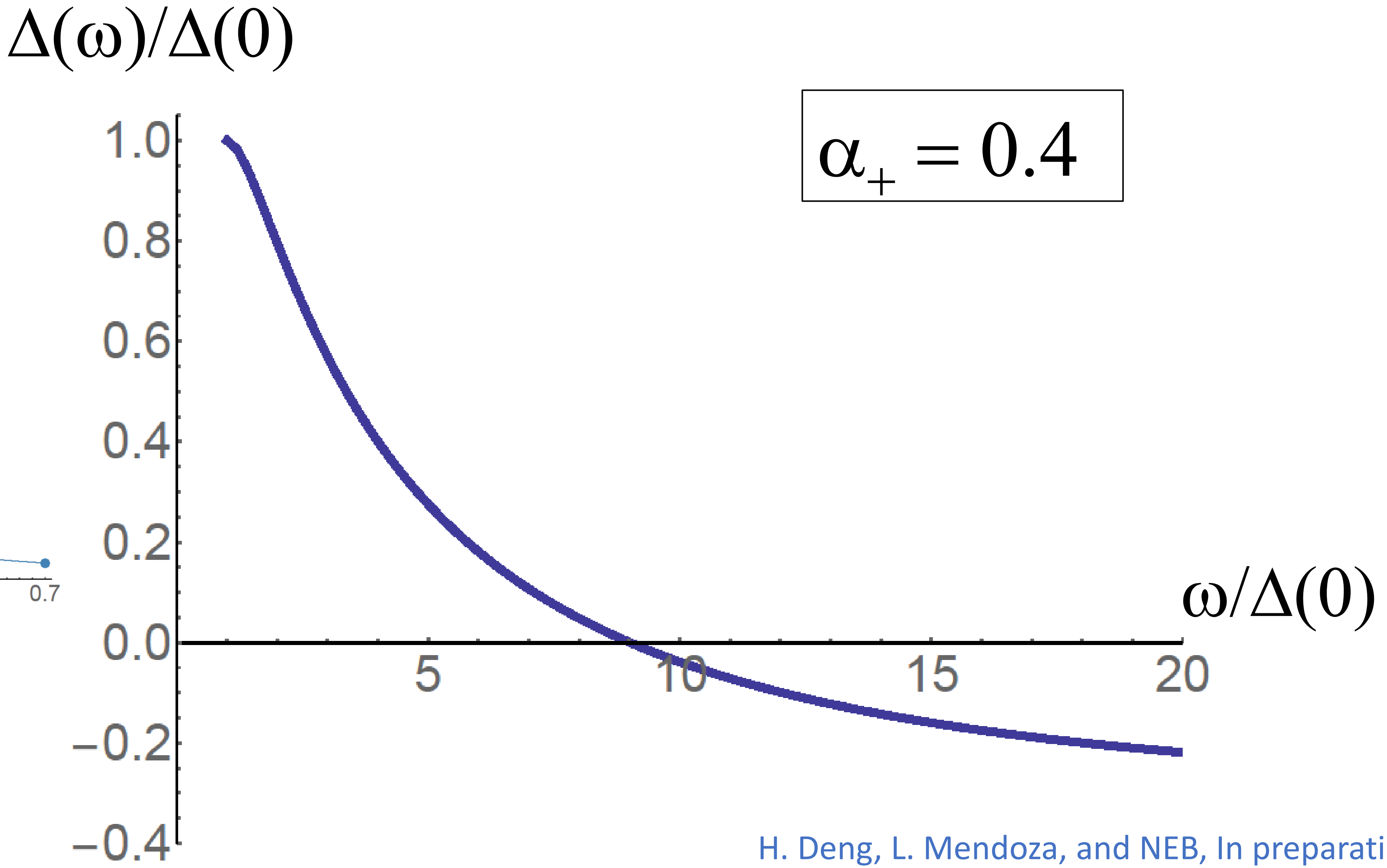
Local Approximation $\Delta(\omega)$ for α_=.1





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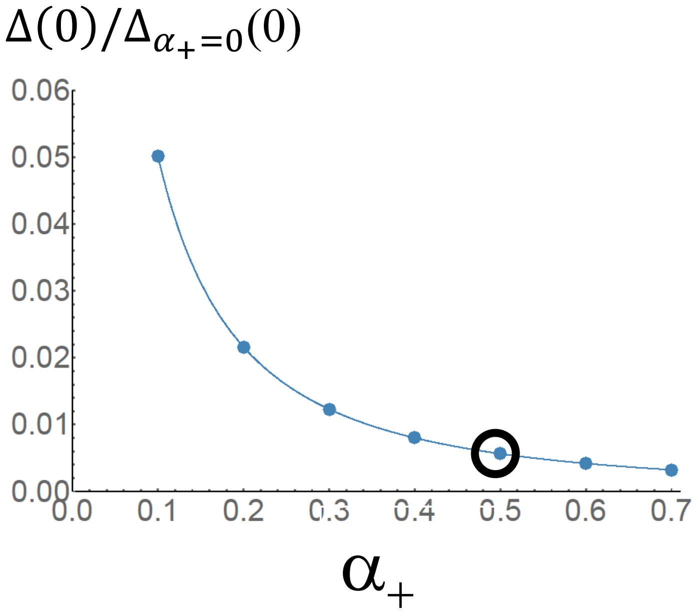
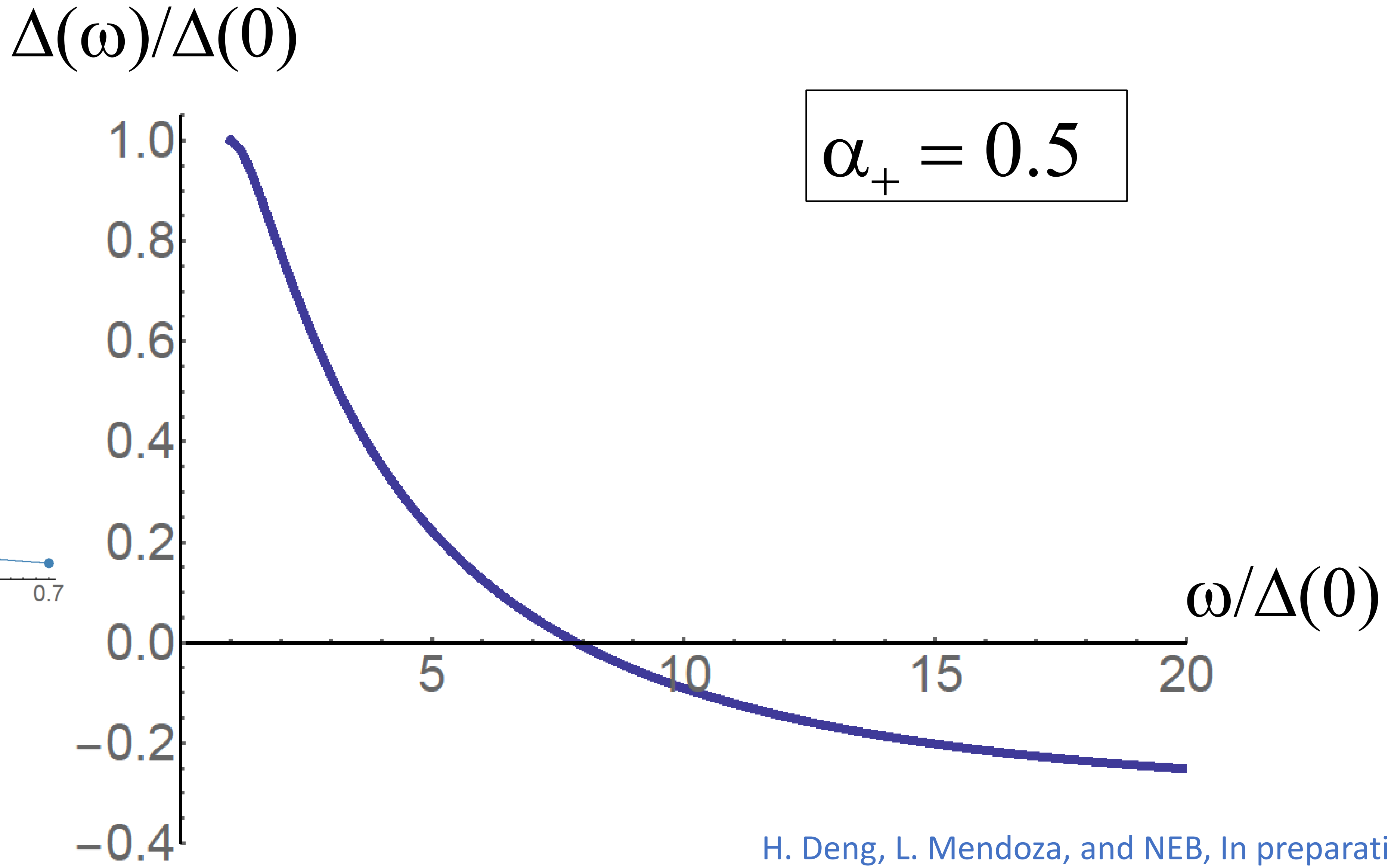
Local Approximation Δ(ω) for $\alpha\_$=.1





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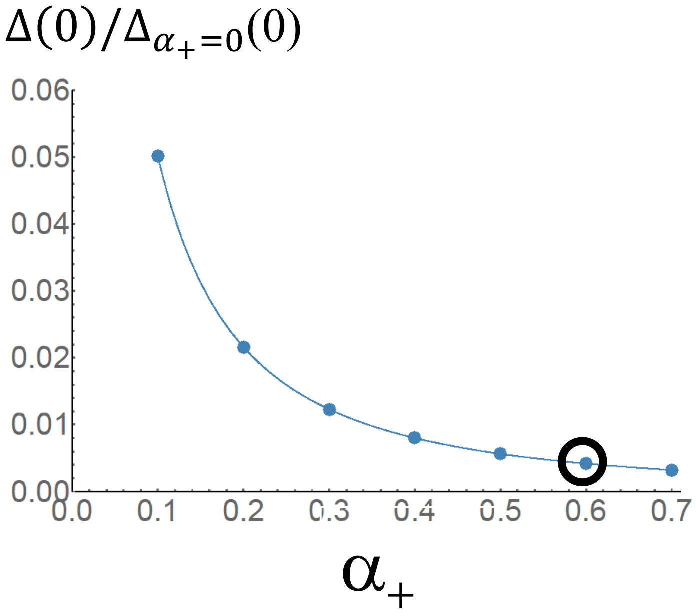
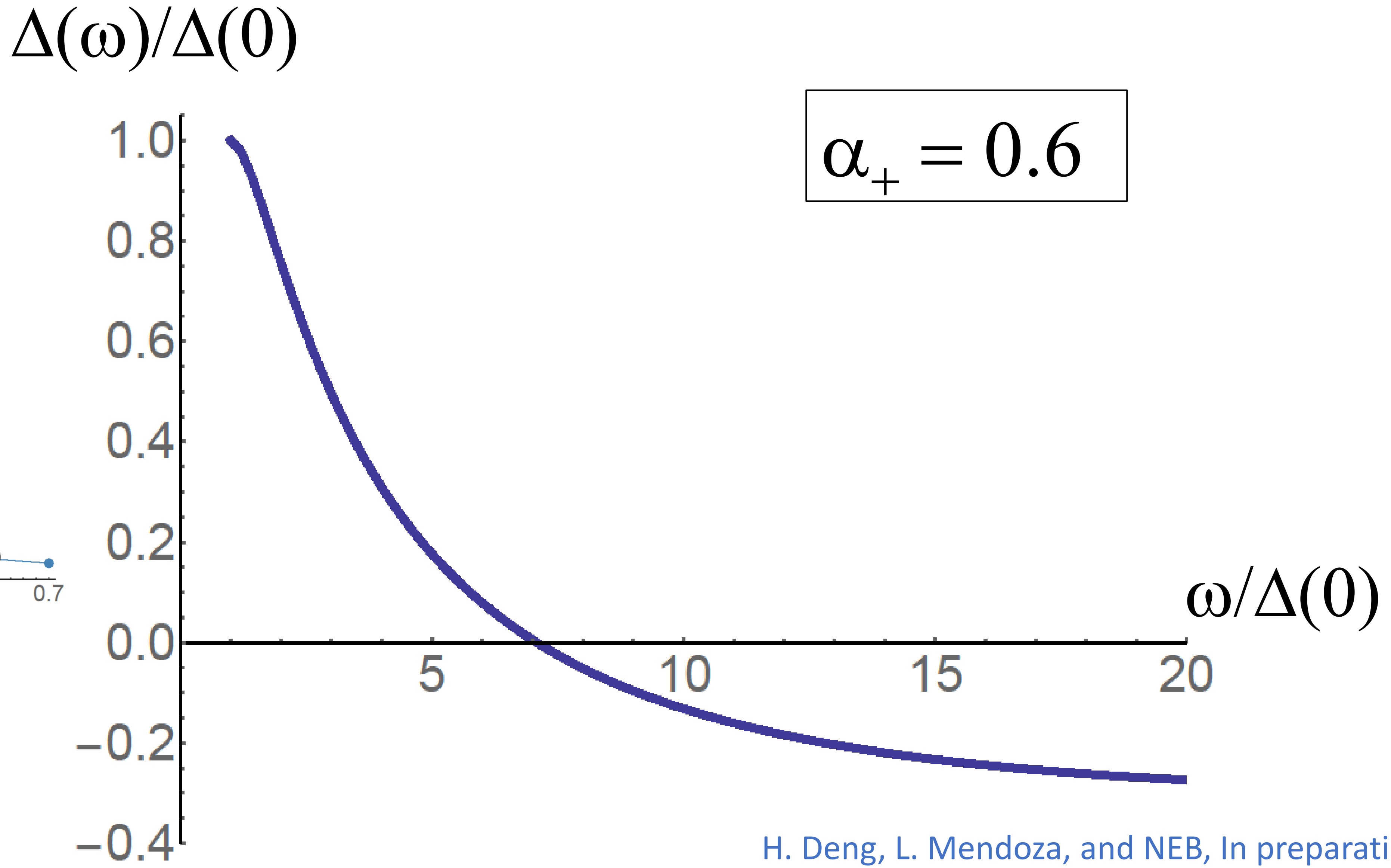
Local Approximation $\Delta(\omega)$ for $\alpha\_=.1$





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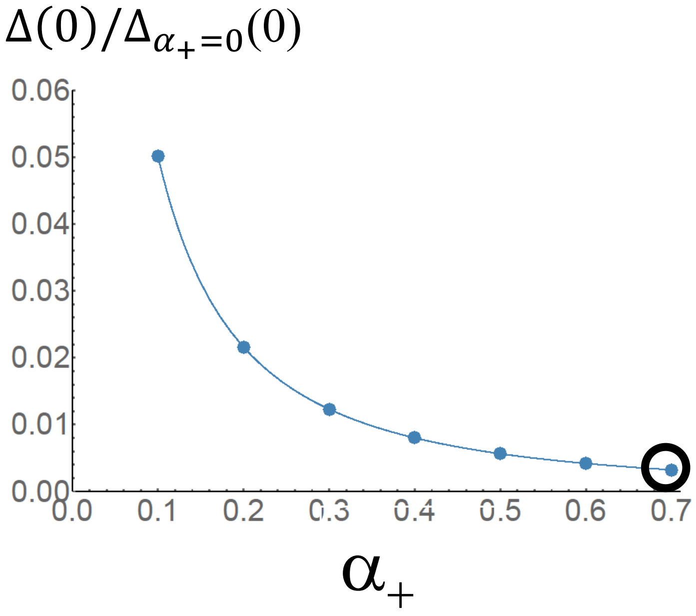

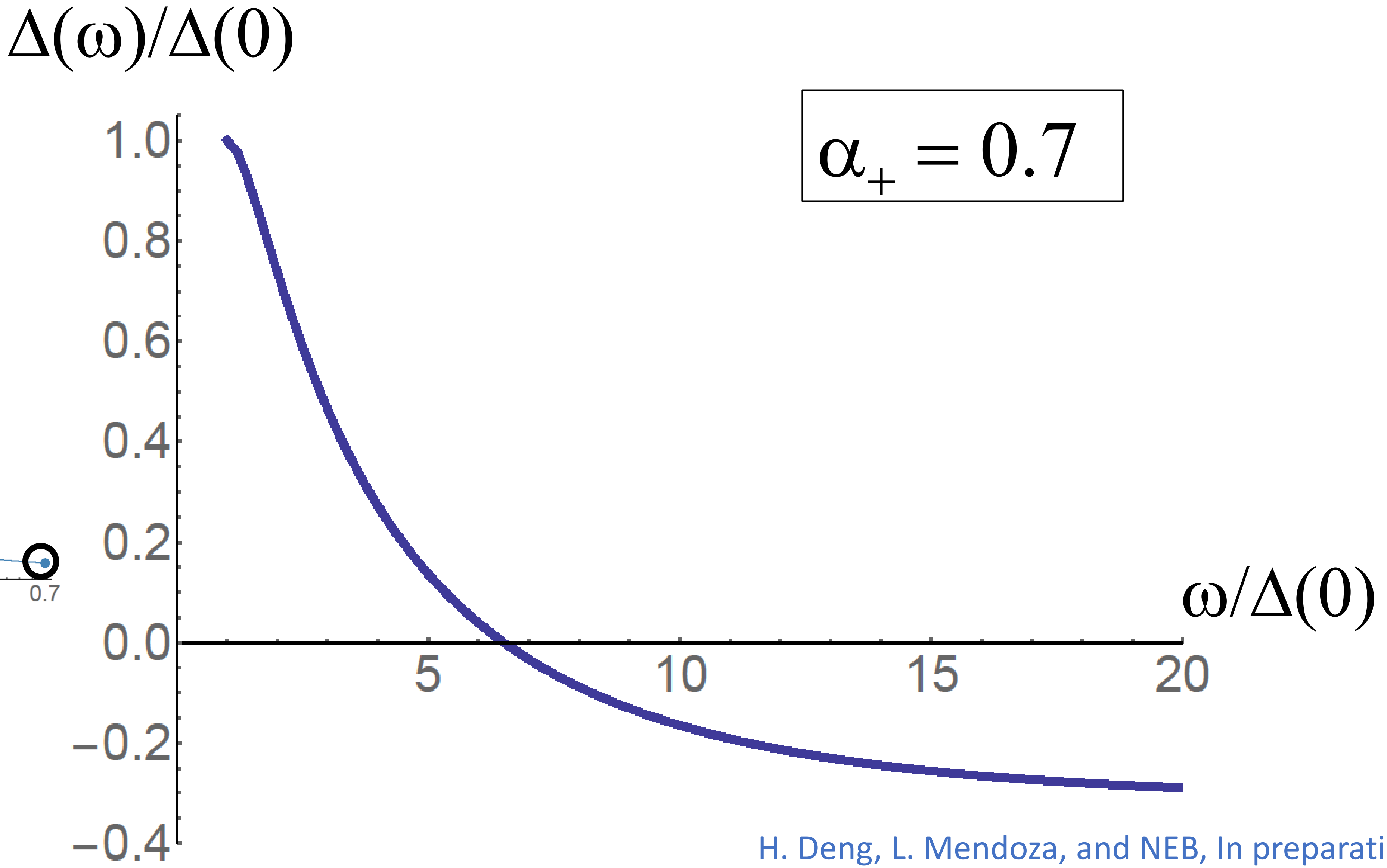

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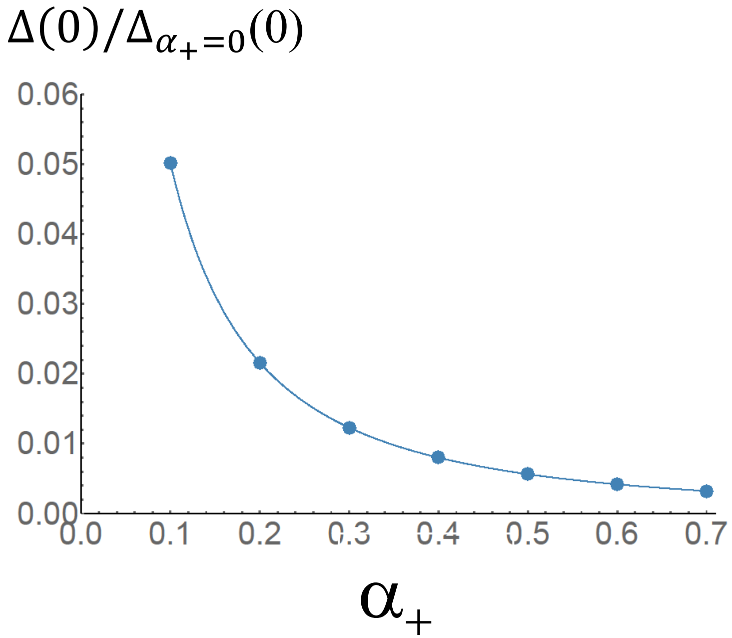
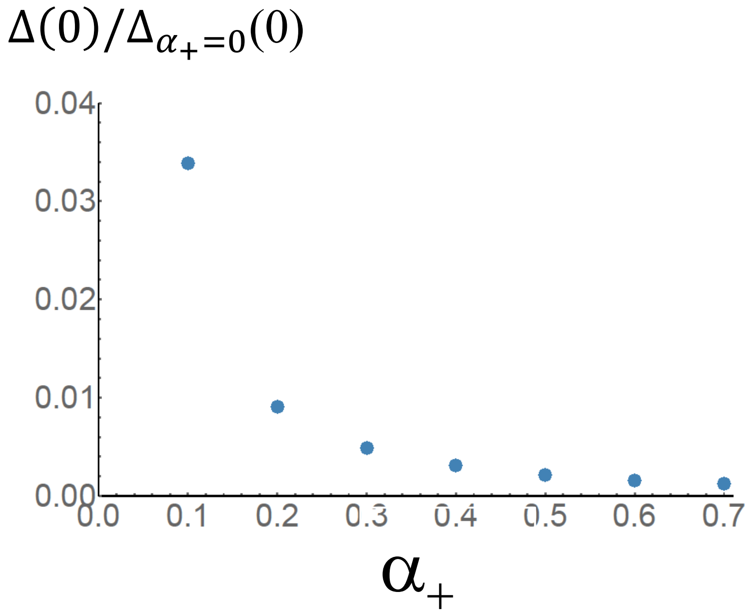
How Good is the Approximation?

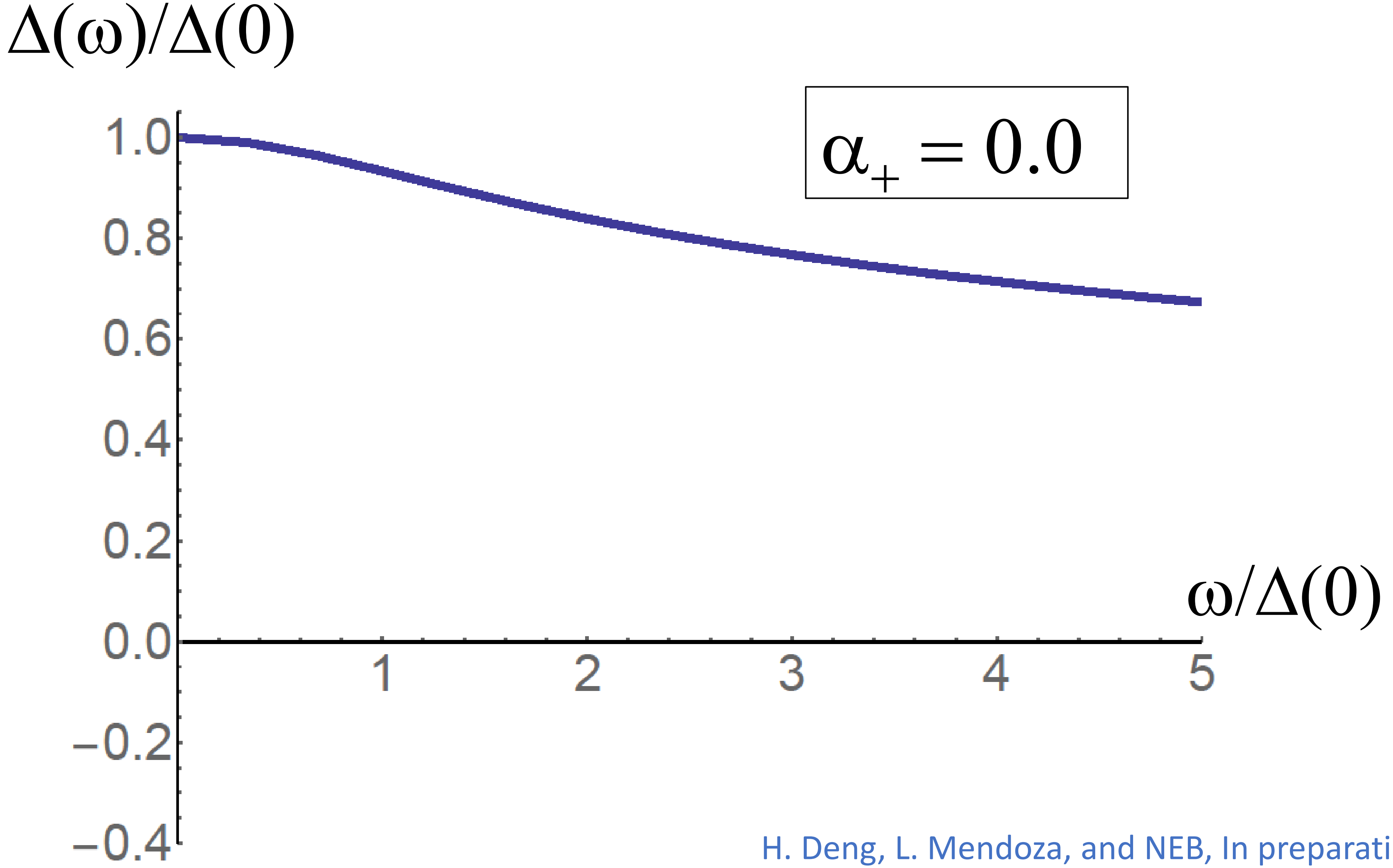
Local approximation provides an analytic solution to the gap equation, and a link to the RG approach. **But how good is it?**

	Local Approximation		Exact Result
For $\alpha_+ = 0$:	$\Delta(0) = 8.93\, \alpha_-{}^3\, \omega_0$	→	$\Delta(0) = 25.8\, \alpha_-{}^3\, \omega_0$
For $\alpha_+ > 0$:		→	





Exact Result for $\Delta(\omega)$ for $\alpha\_=.1$



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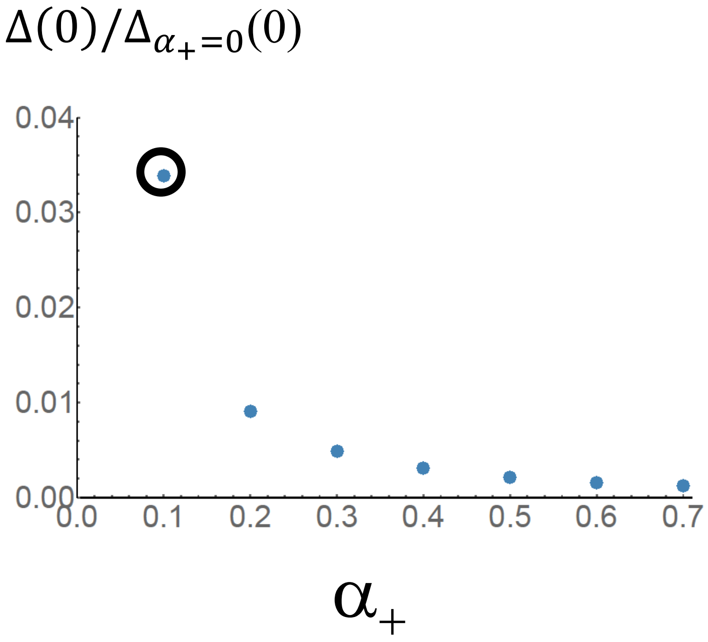
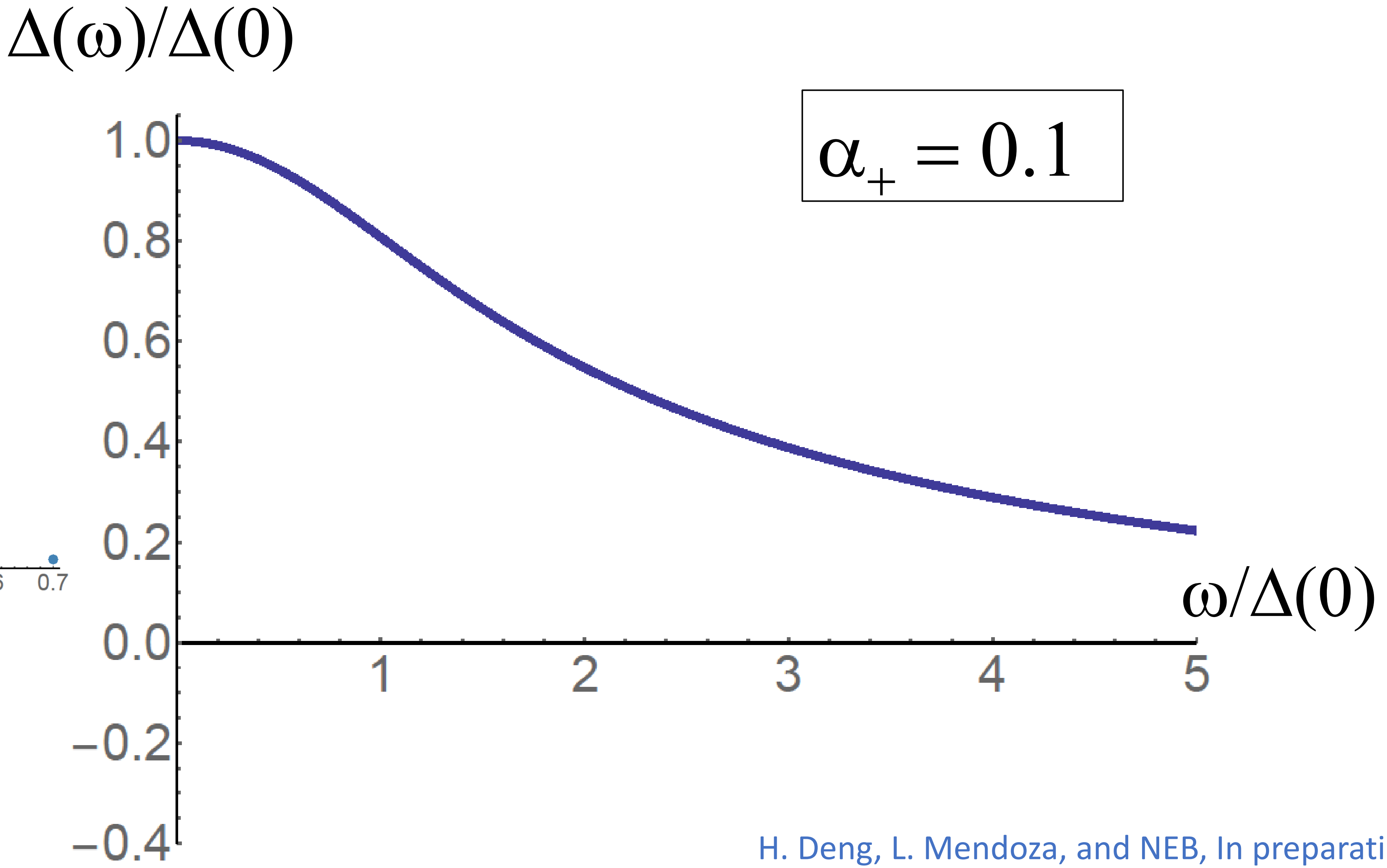
Exact Result for $\Delta(\omega)$ for α_{-}=.1





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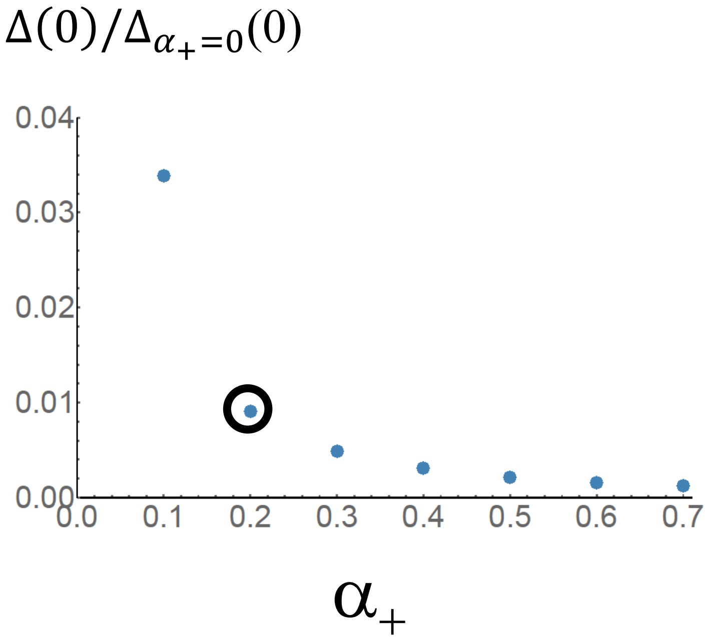
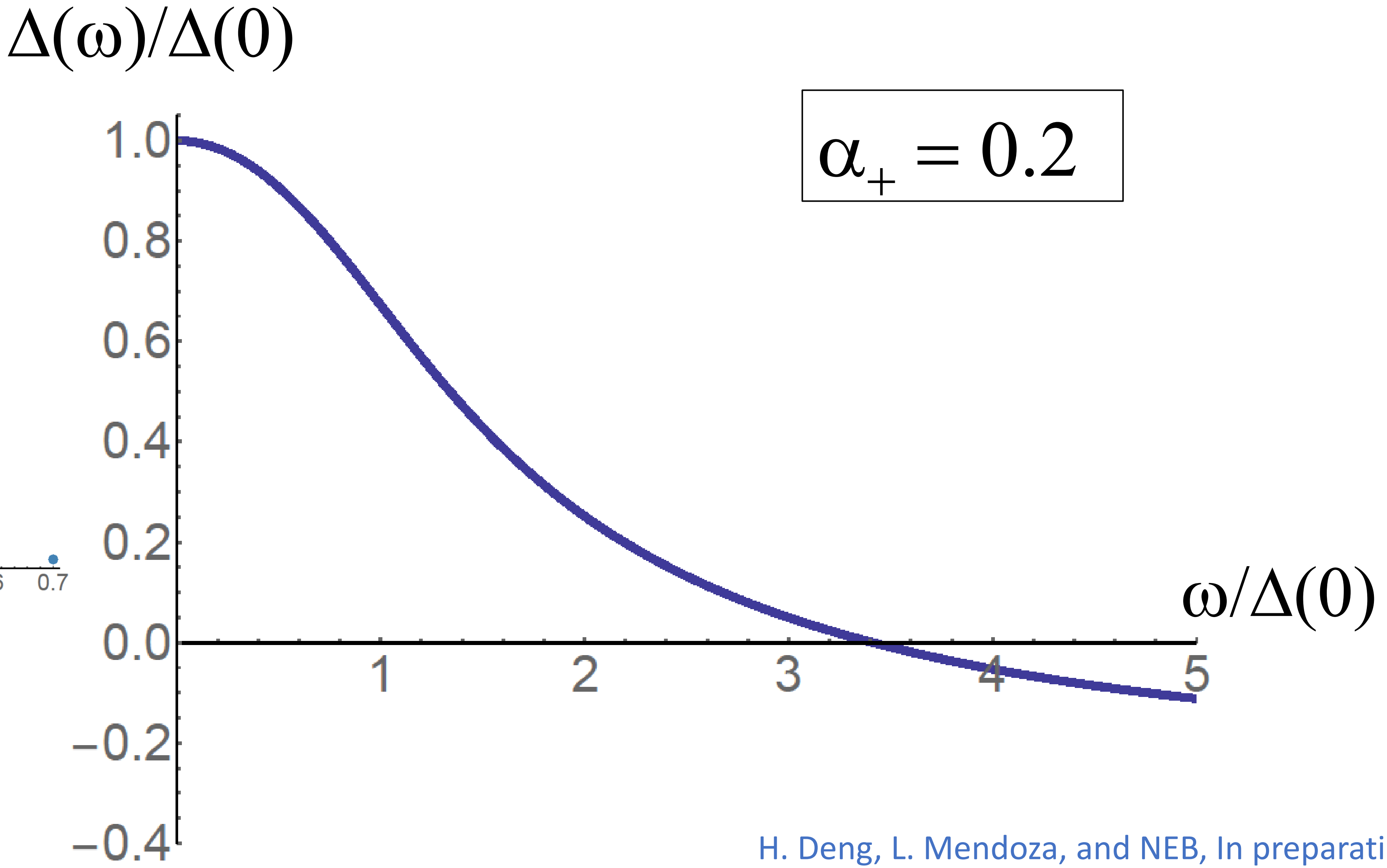
Exact Result for $\Delta(\omega)$ for α_-=.1





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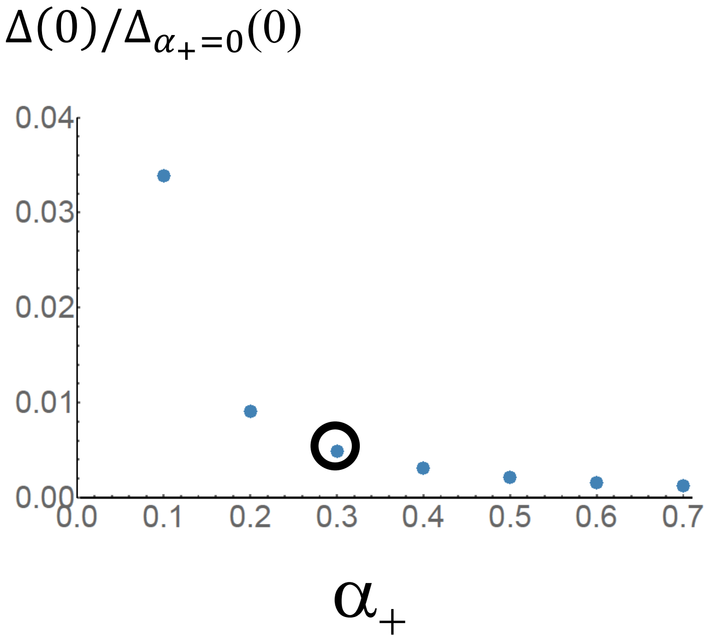
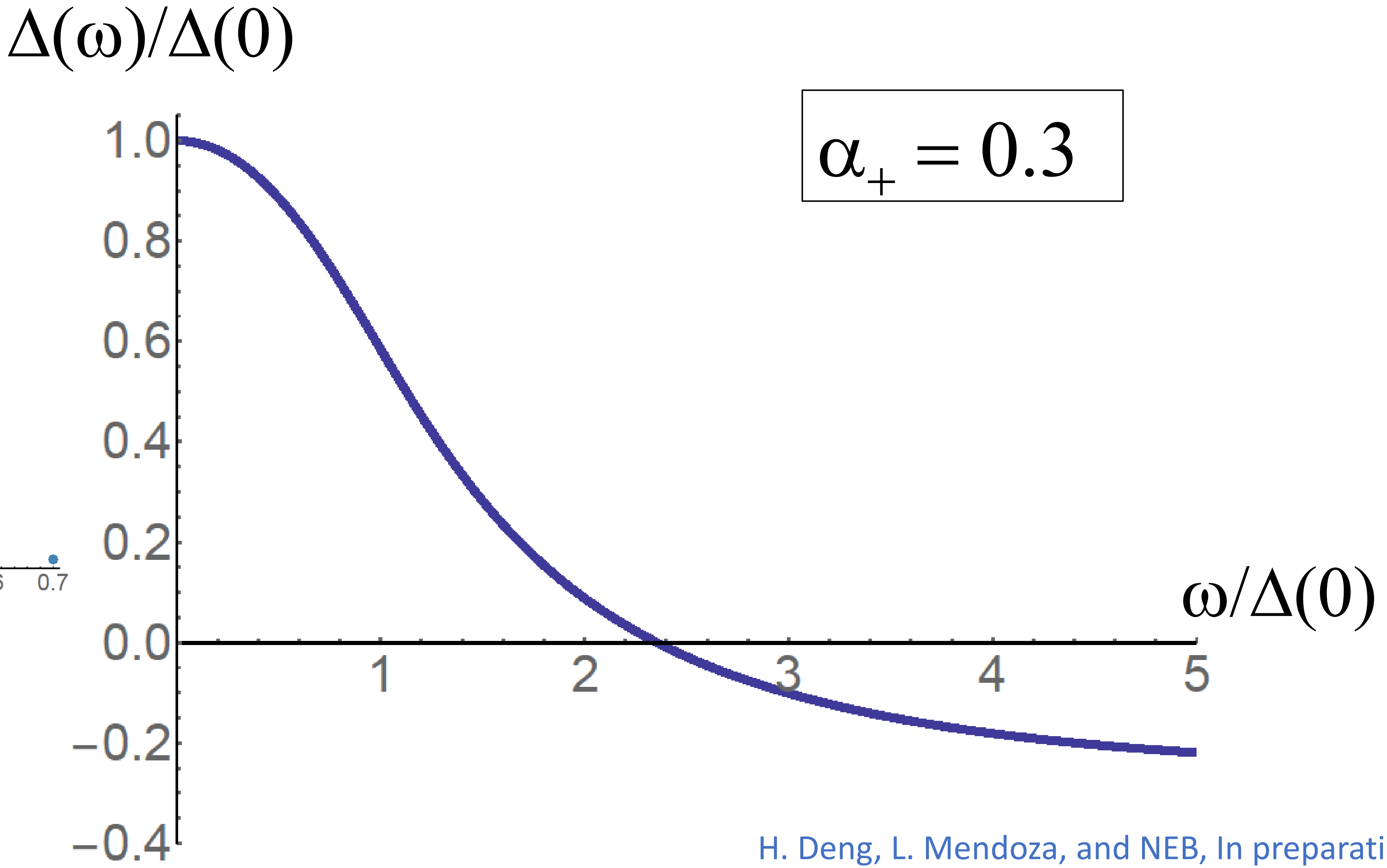
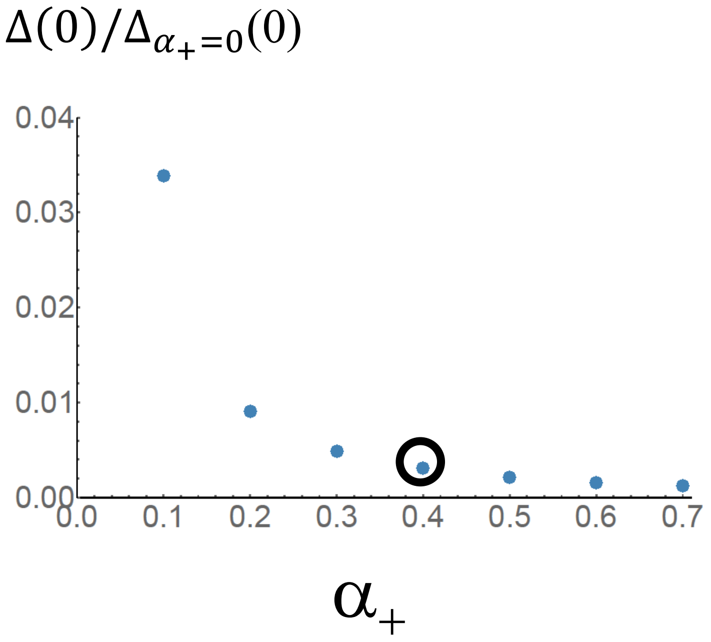
Exact Result for Δ(ω) for $\alpha\_$=.1

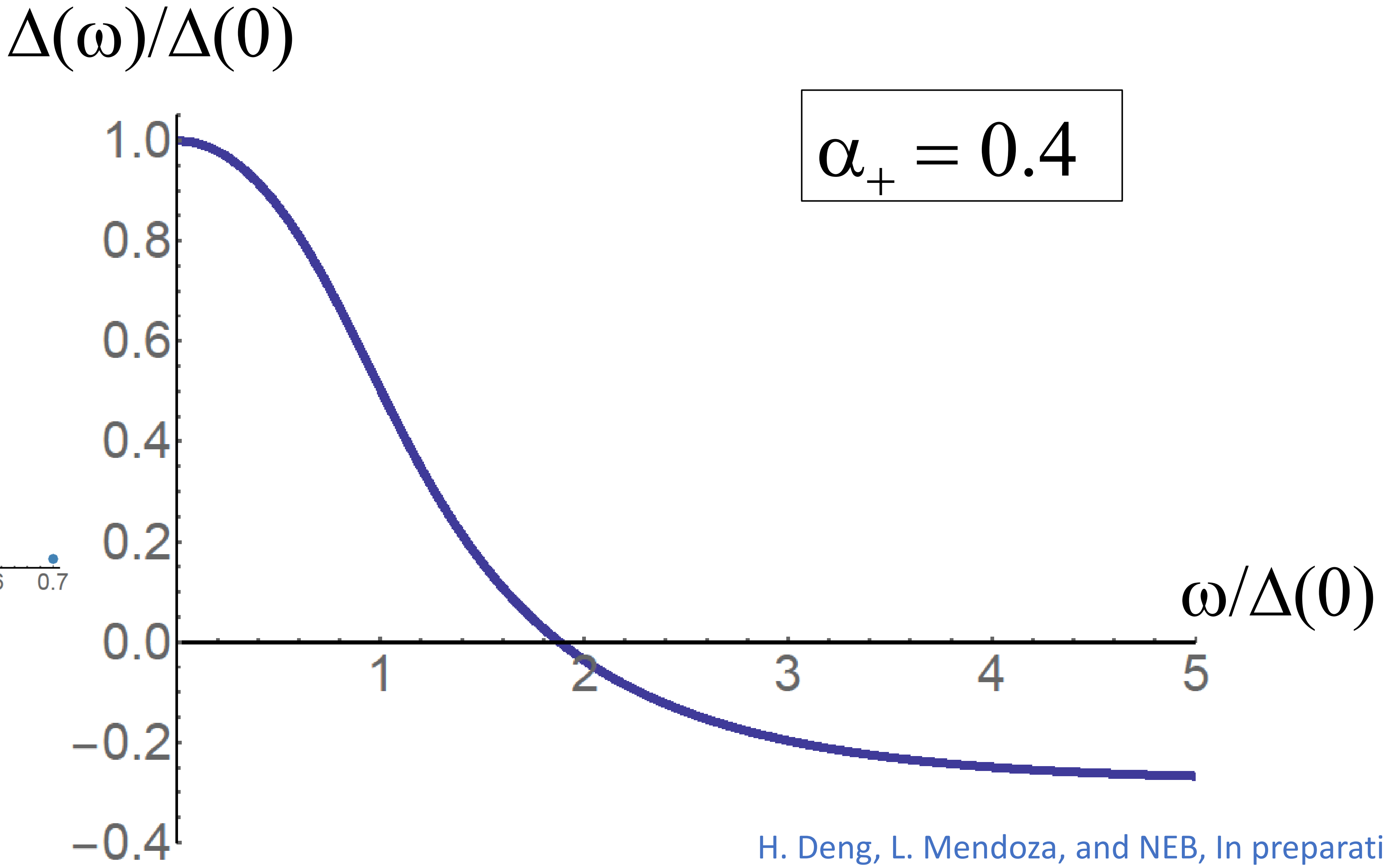




H. Deng, L. Mendoza, and NEB, In preparation

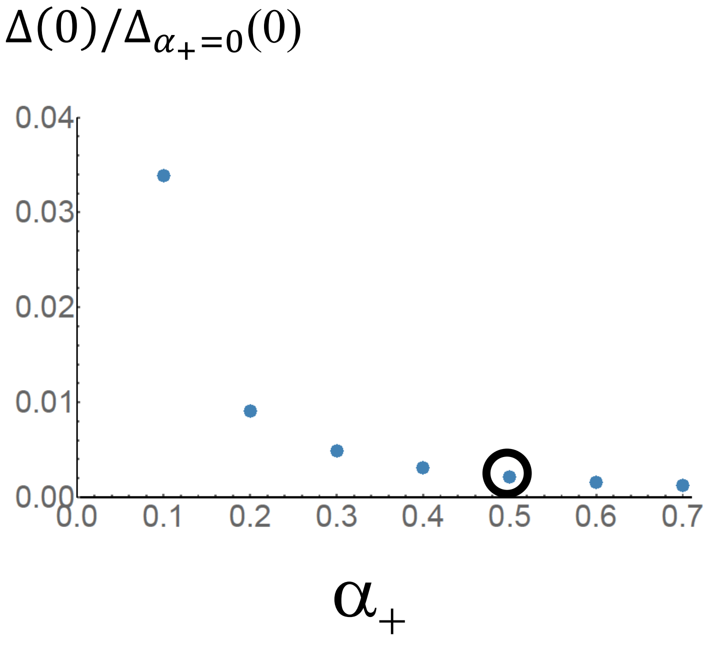
Exact Result for $\Delta(\omega)$ for α_{-}=.1

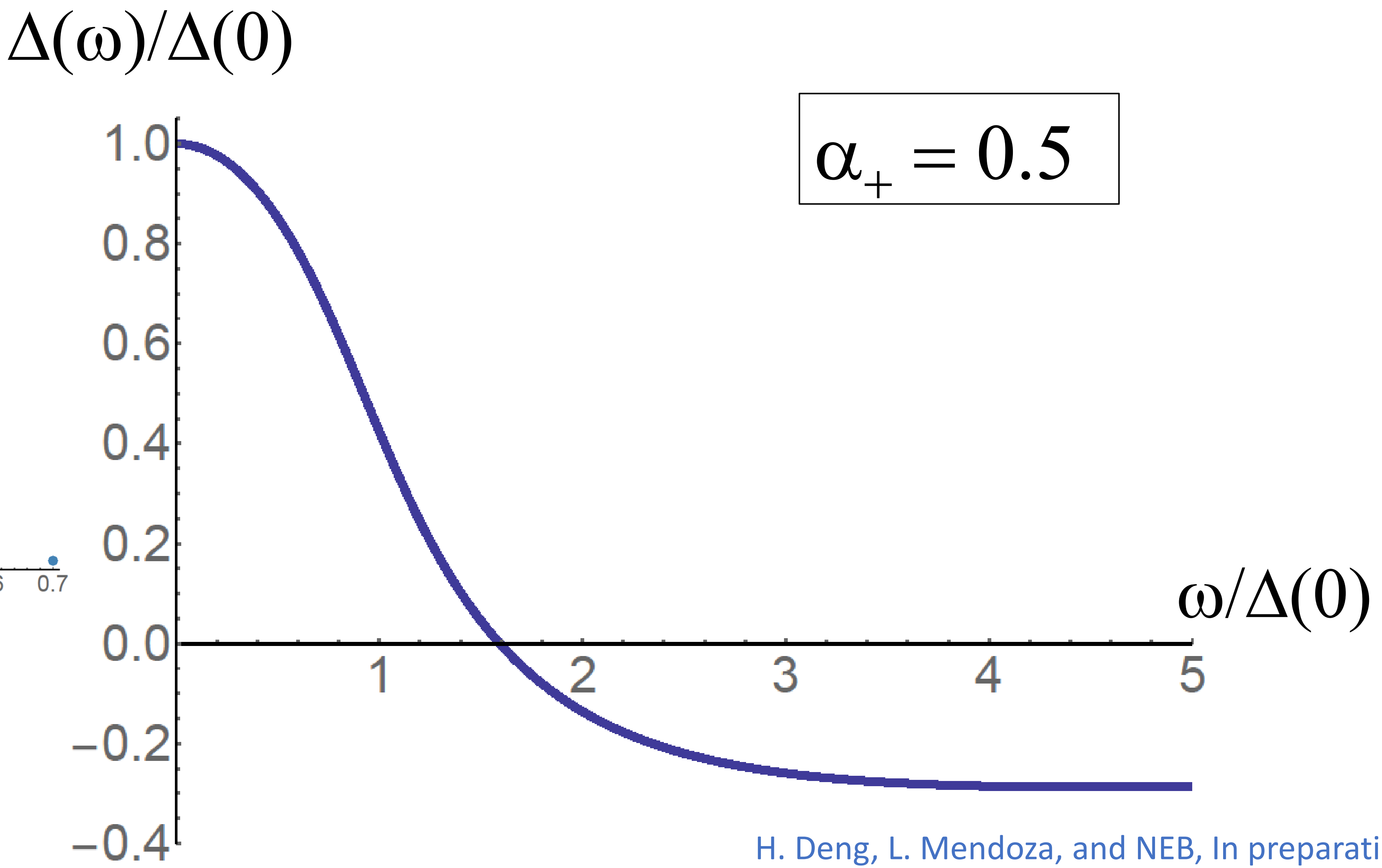




H. Deng, L. Mendoza, and NEB, In preparation

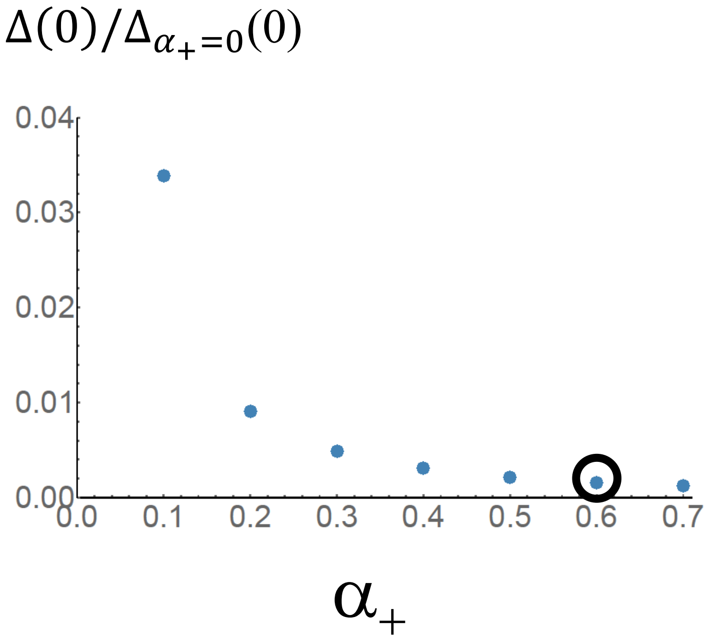
Exact Result for $\Delta(\omega)$ for α_{-}=.1

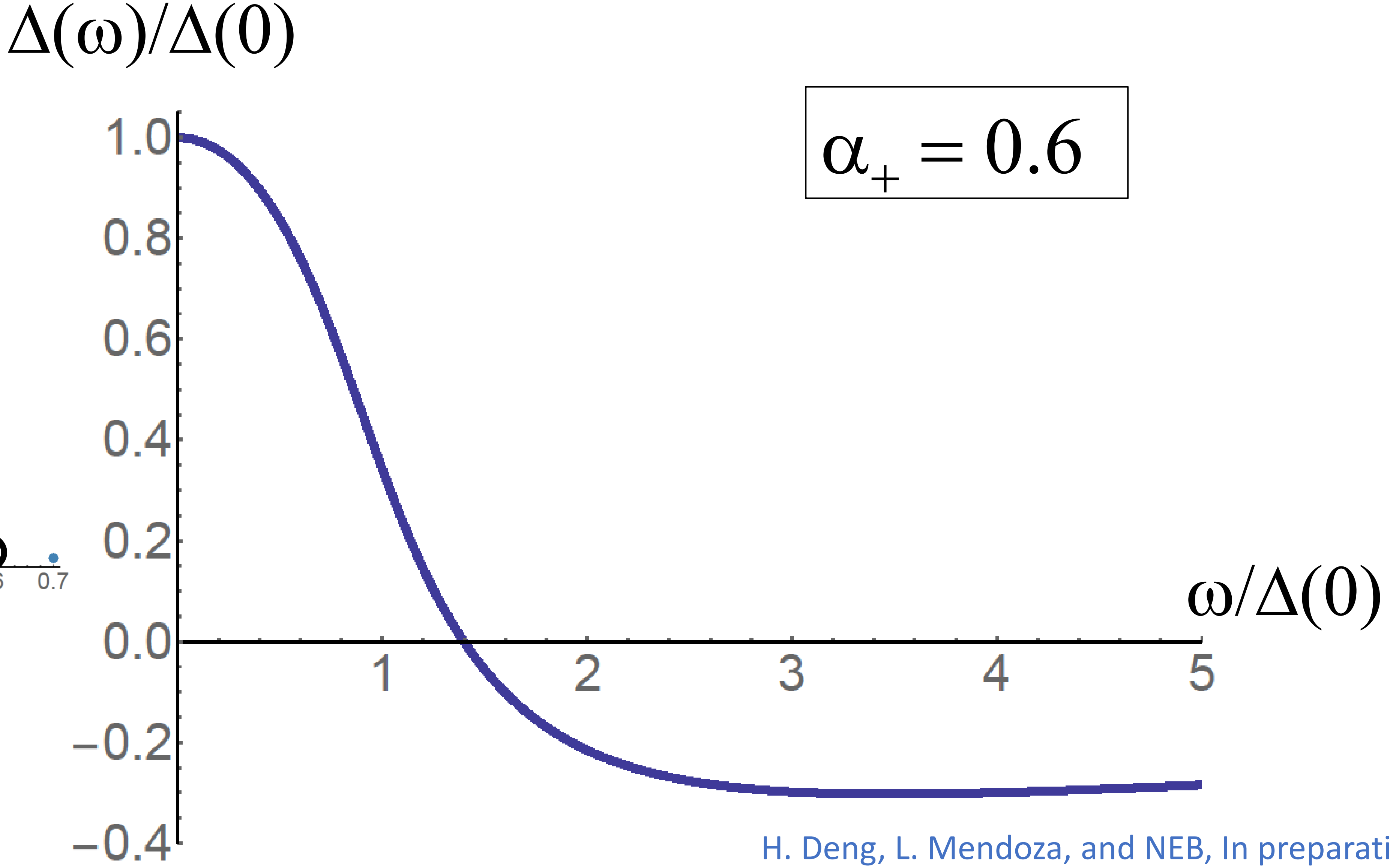




H. Deng, L. Mendoza, and NEB, In preparation

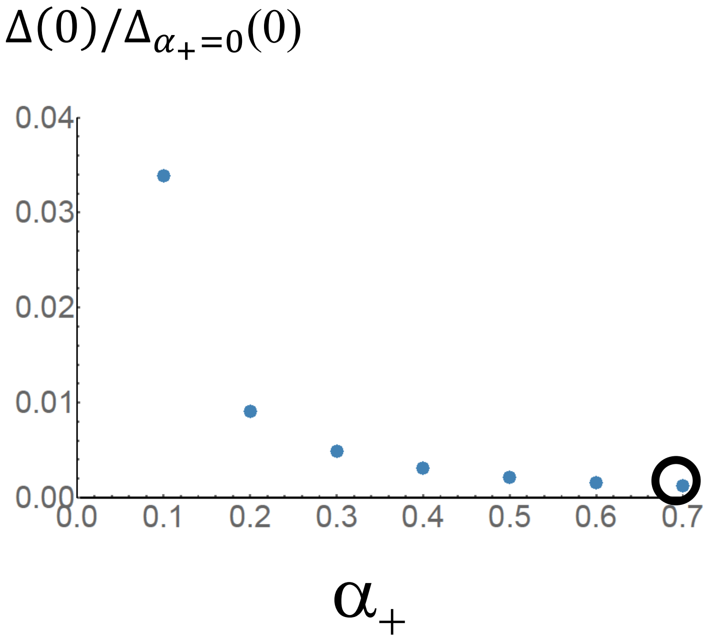
Exact Result for $\Delta(\omega)$ for $\alpha\_=.1$

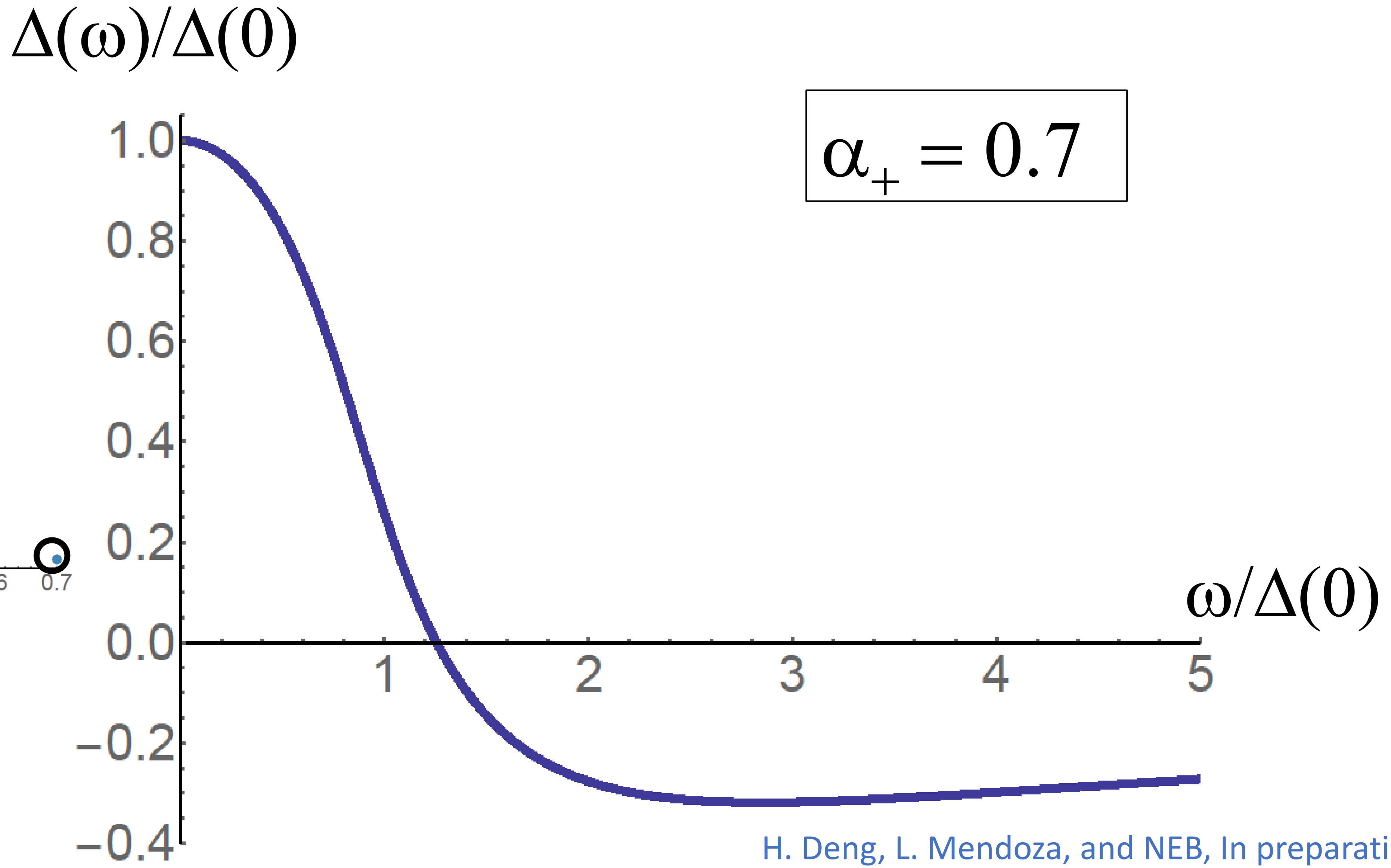




H. Deng, L. Mendoza, and NEB, In preparation

Exact Result for $\Delta(\omega)$ for $\alpha\_=.1$





H. Deng, L. Mendoza, and NEB, In preparation

Crossover from BCS to Gauge Pairing

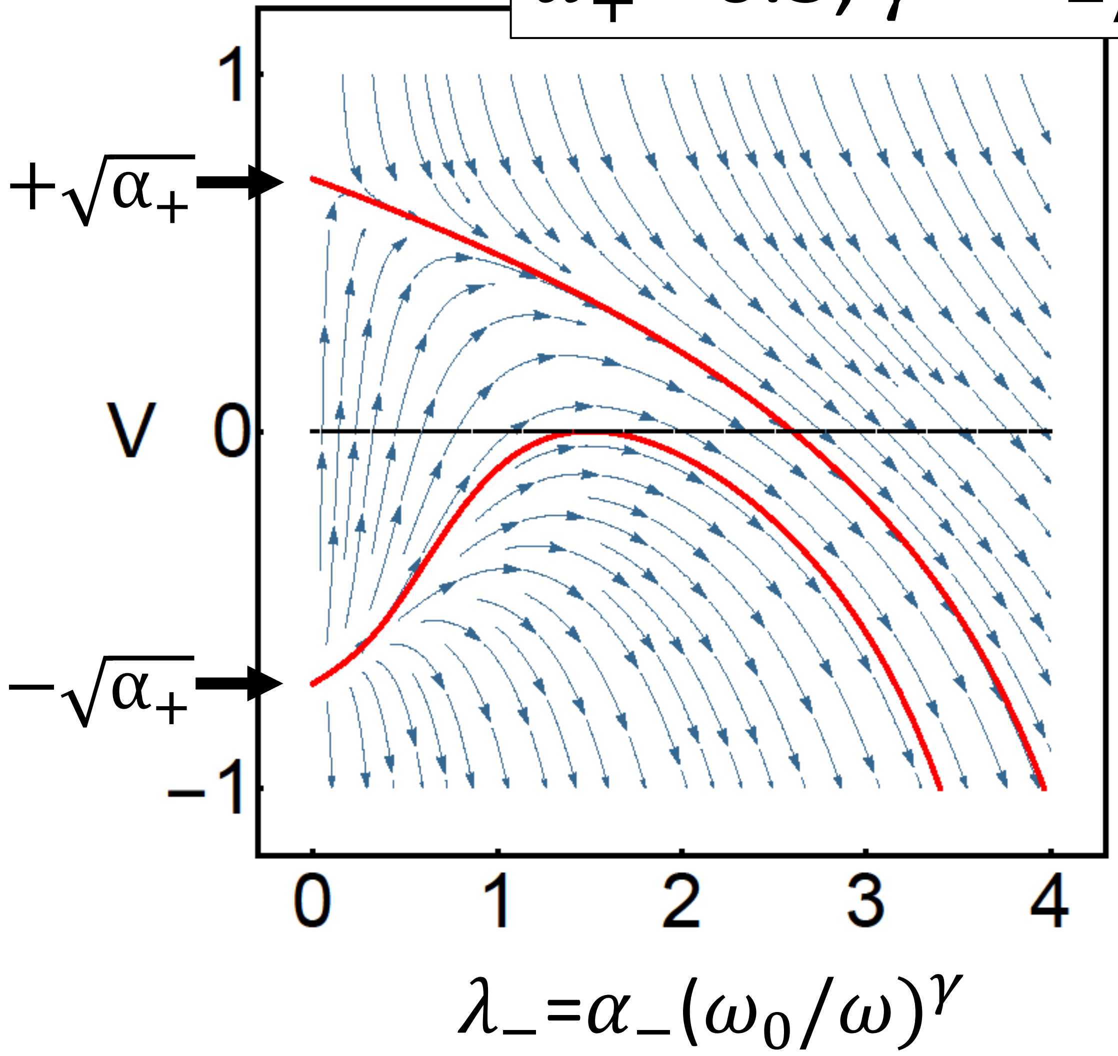
$$\frac{dV}{dl} = -\gamma\lambda_- + \alpha_+ - V^2$$

$$\frac{d\lambda_-}{dl} = \gamma\lambda_-$$

$$V(\omega_0) = \alpha_- + V_0$$

$V \longrightarrow -\infty$ as $\omega \to \Delta(0)$

$\alpha_+ = 0.5, \gamma = 1/3$



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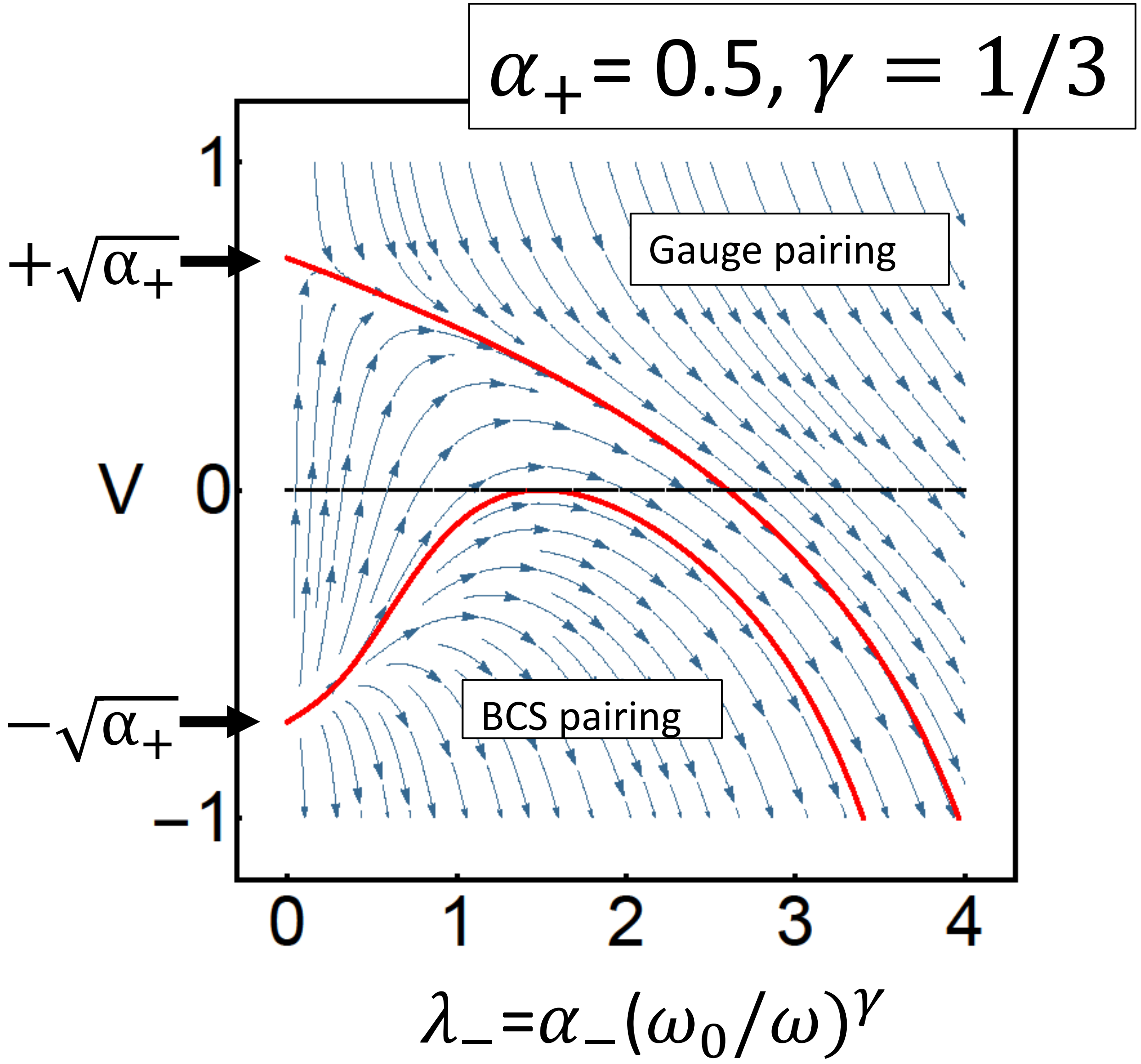

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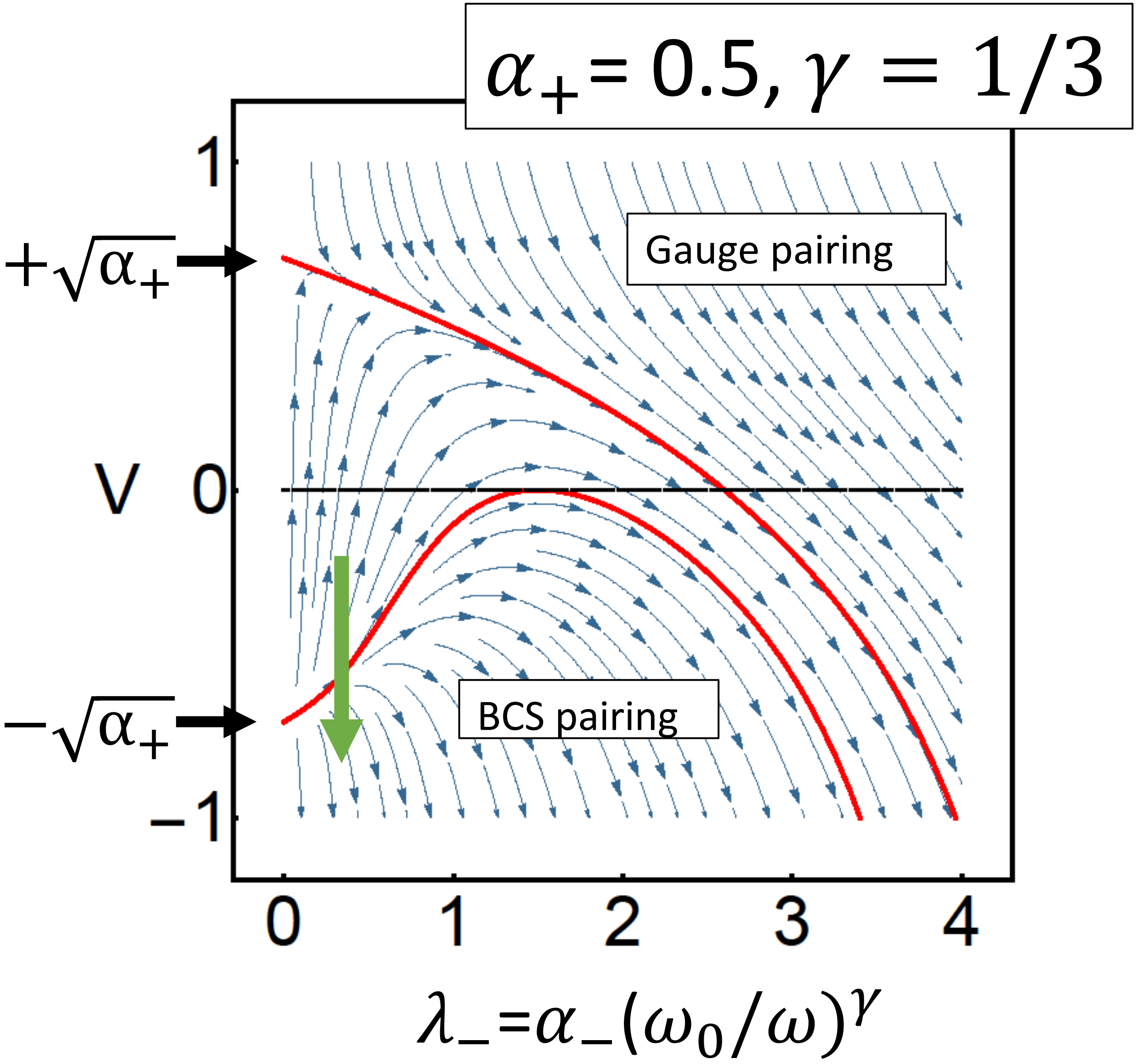
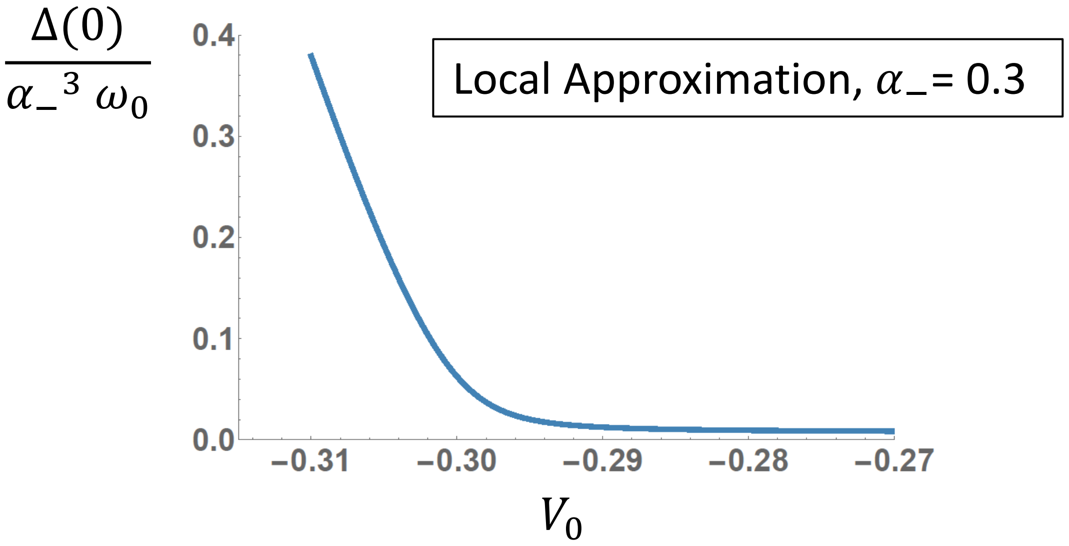
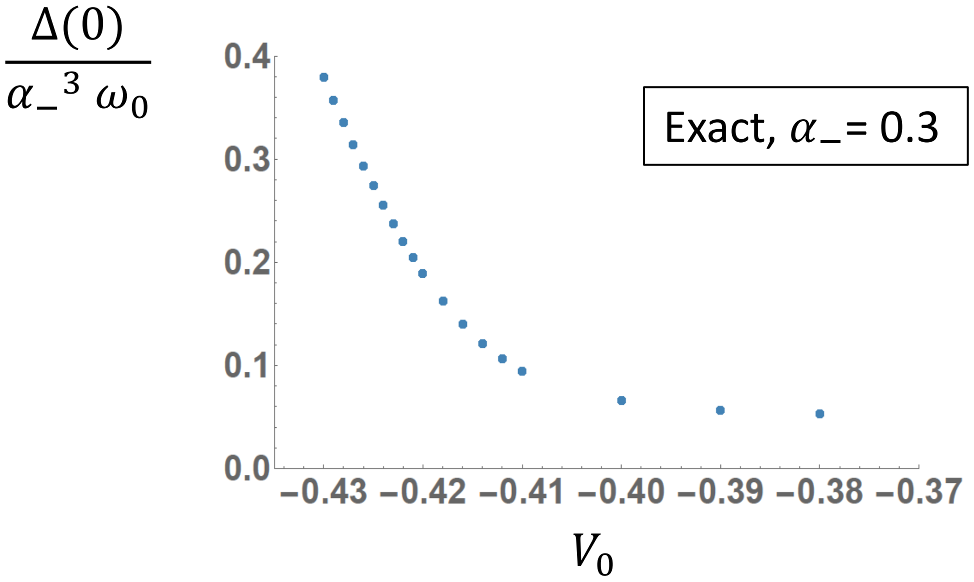
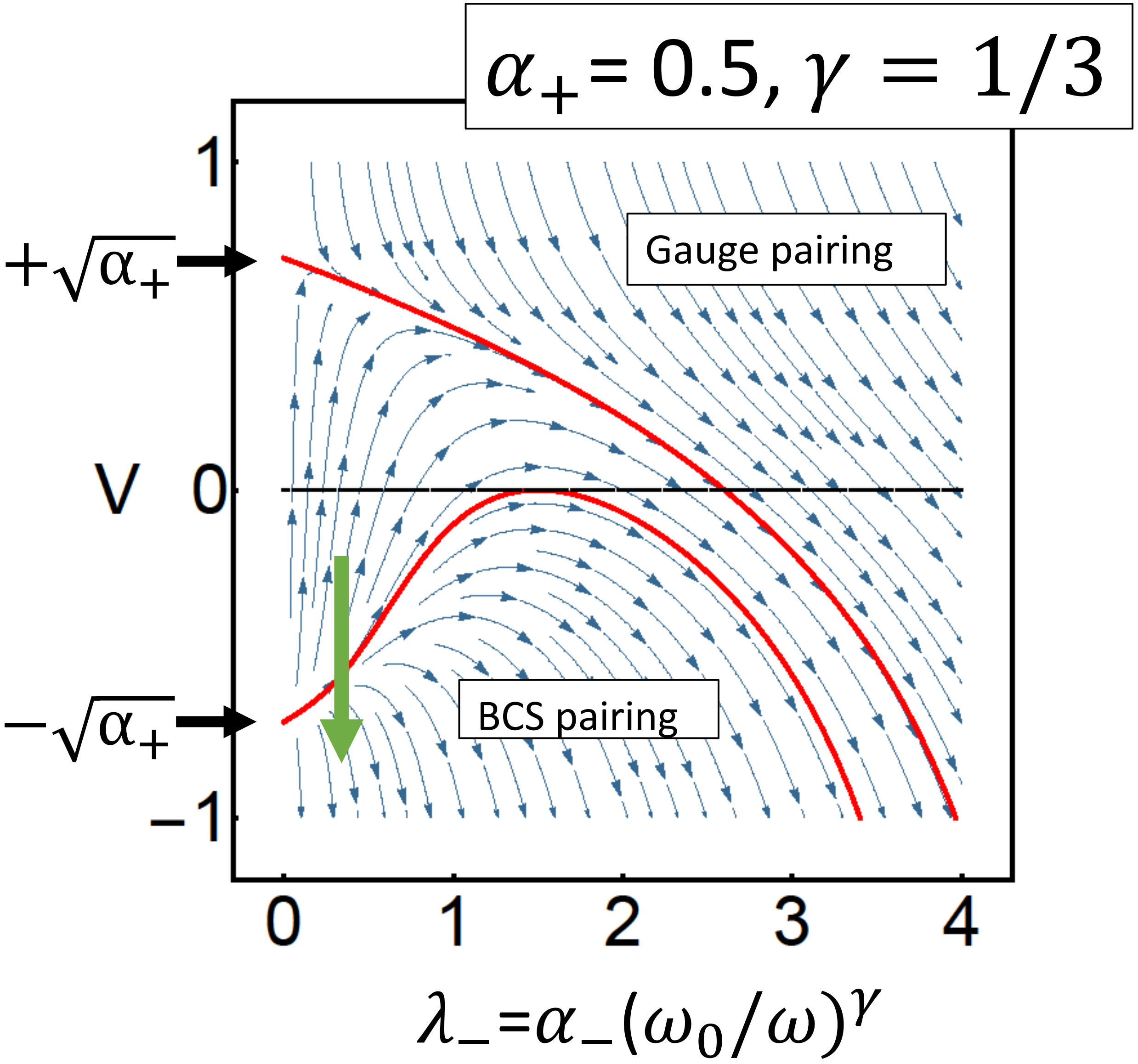


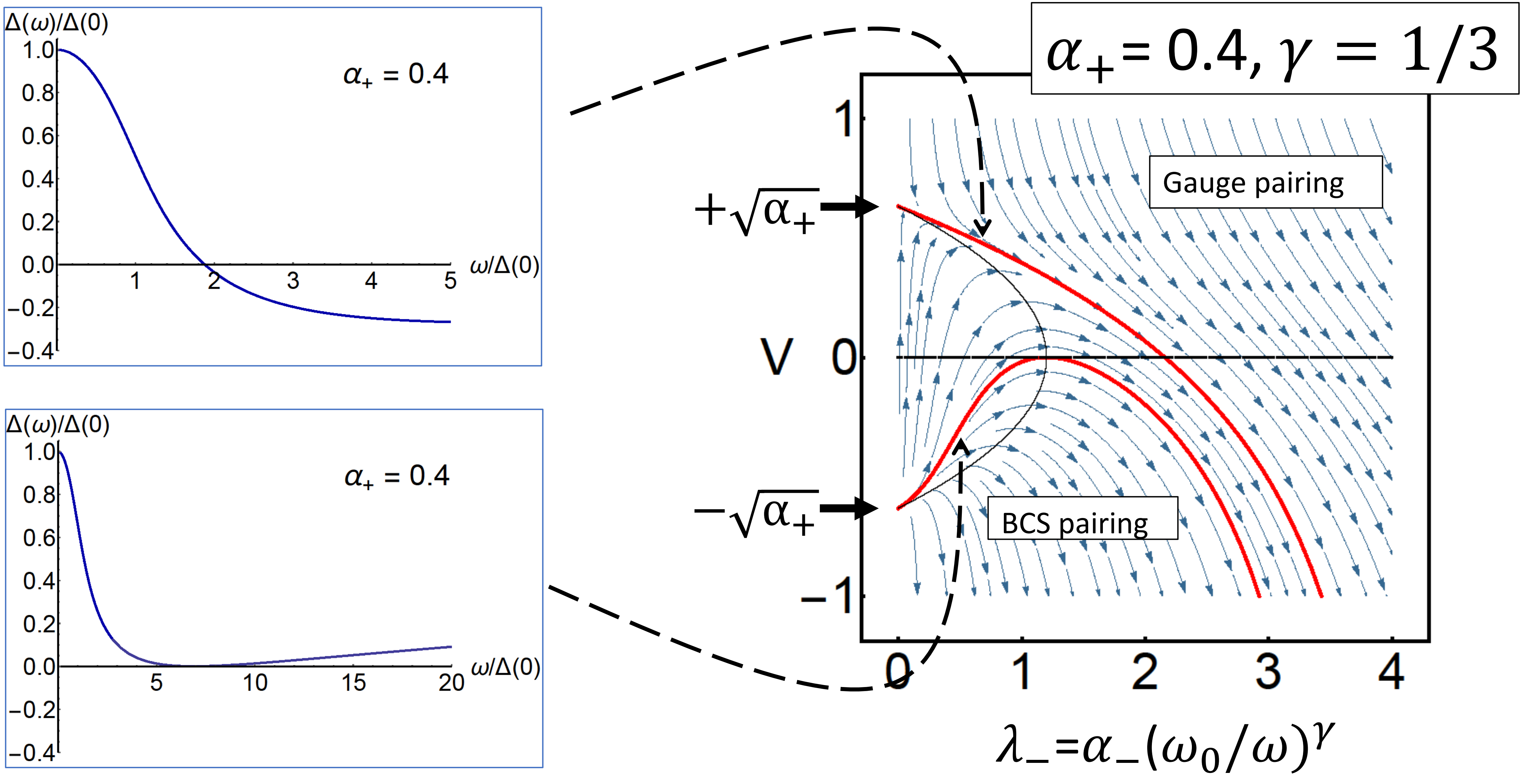
Crossover from Gauge Pairing to BCS







Crossover from Gauge Pairing to BCS



Self-Energy Effects

Eliashberg Equations:

$$Z(\omega)\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\left(\lambda_-(\omega-\Omega) - \lambda_+(\omega-\Omega)\right)\mathrm{d}\Omega$$

$$(Z(\omega)-1)\omega = \frac{1}{2}\int \frac{\Omega}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\left(\lambda_-(\omega-\Omega) + \lambda_+(\omega-\Omega)\right)\mathrm{d}\Omega$$

$$\boxed{\lambda_-(\omega) = \alpha_-\left|\frac{\omega_0}{\omega}\right|^{\gamma}}$$

$$\boxed{\lambda_+(\omega) = \alpha_+\ln\left|\frac{\omega_0}{\omega}\right|}$$

Self-Energy Effects

For $\alpha_+ = 0$, again solve by scaling

Local Approximation

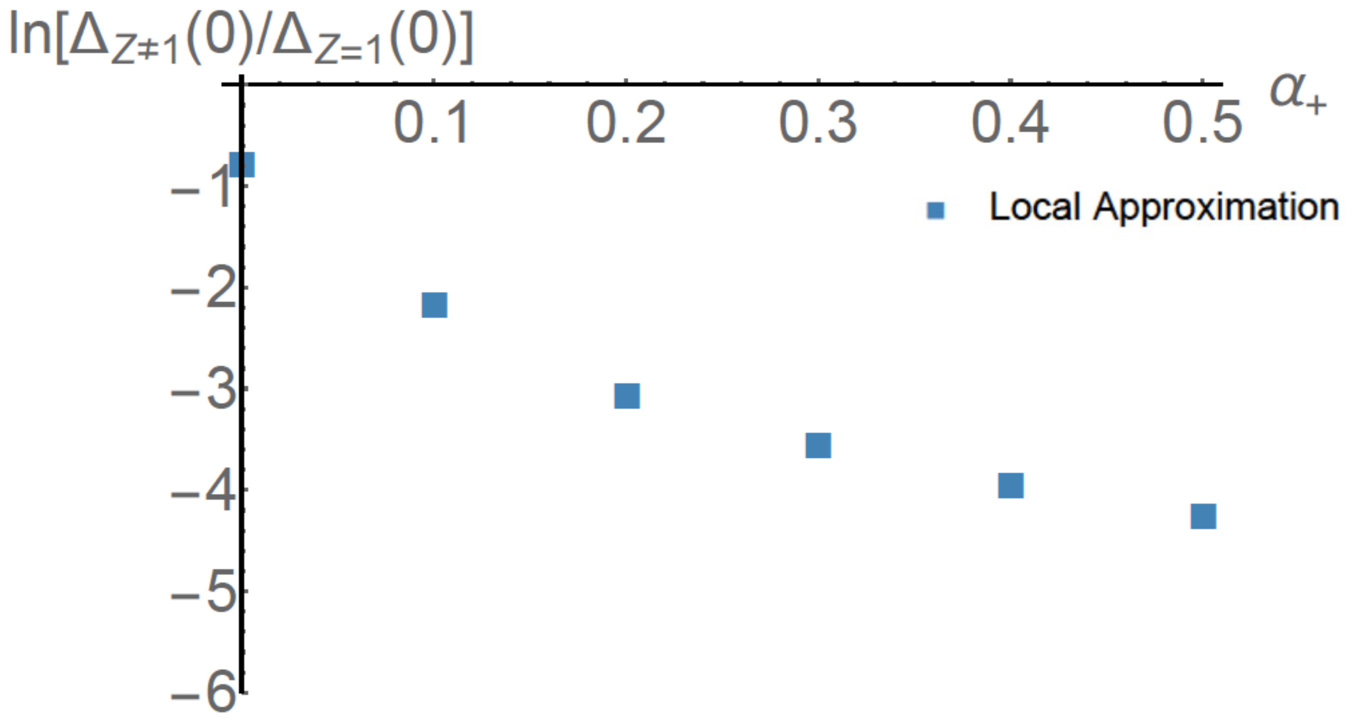
$Z = 1$		$Z \neq 1$
$\Delta(0) = 8.93\, \alpha_-^{\,3}\, \omega_0$	$\longrightarrow$	$\Delta(0) = 4.0\, \alpha_-^{\,3}\, \omega_0$

Exact

$Z = 1$		$Z \neq 1$
$\Delta(0) = 25.8\, \alpha_-^{\,3}\, \omega_0$	$\longrightarrow$	$\Delta(0) = 8.1\, \alpha_-^{\,3}\, \omega_0$

Self-Energy Effects

Additional suppression of energy gap due to self-energy effects is ***enhanced*** with increasing α_+



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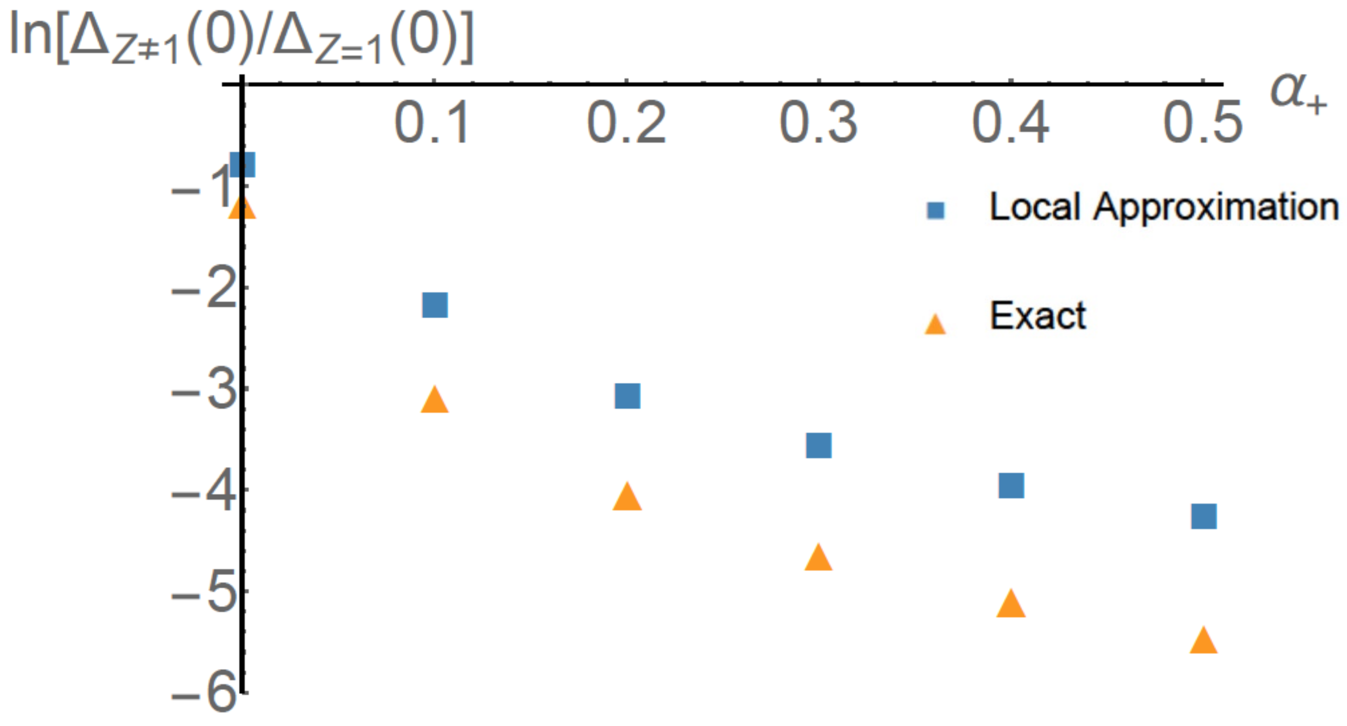
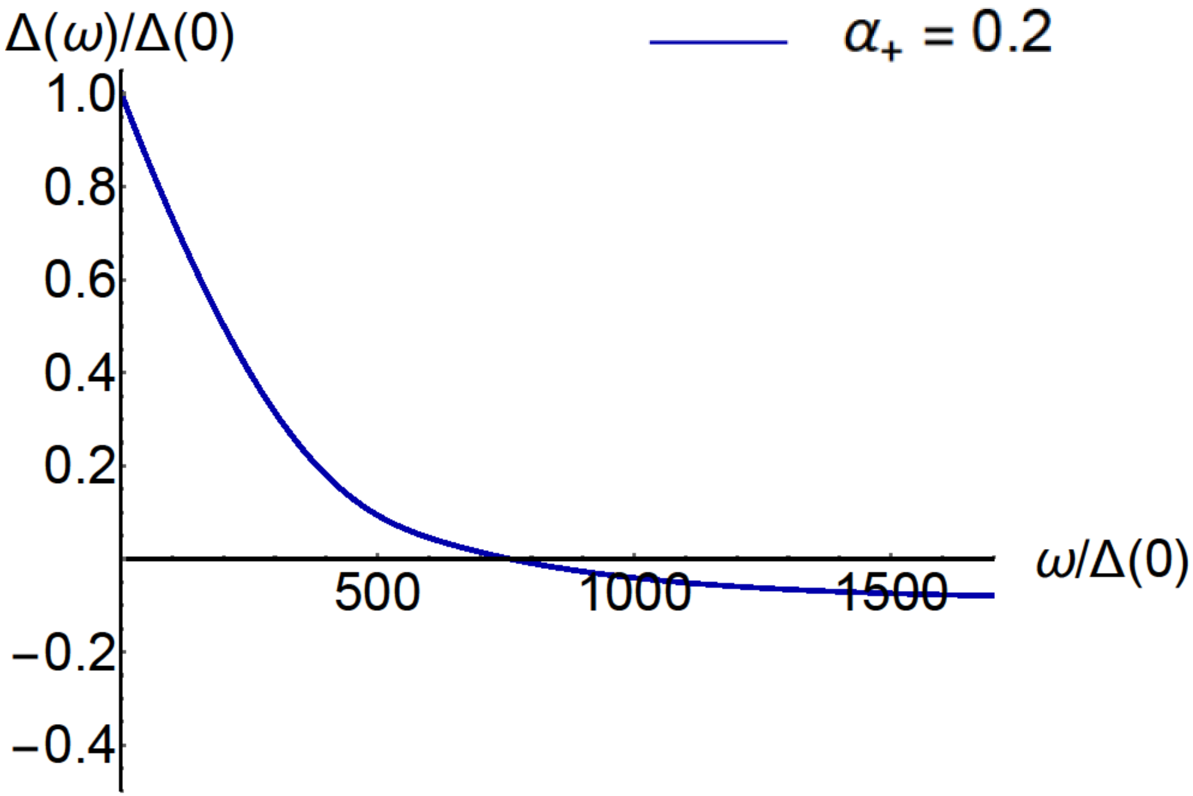
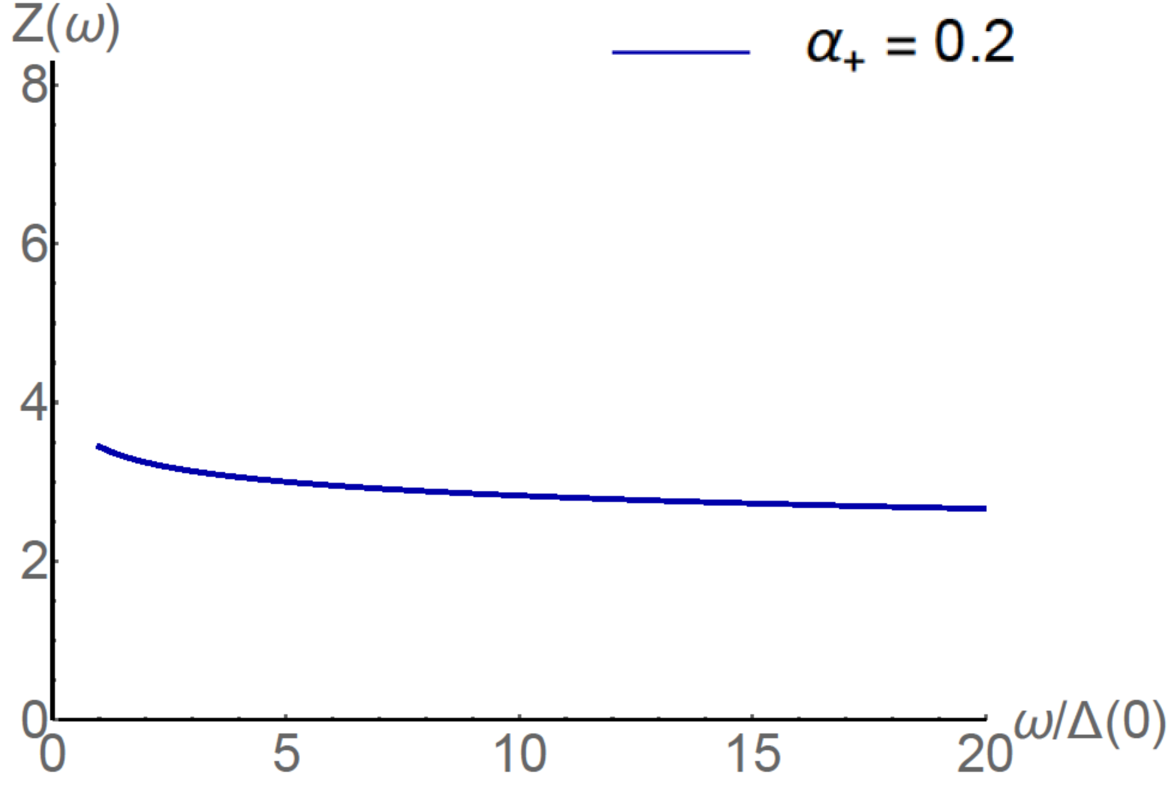


Self-Energy Effects

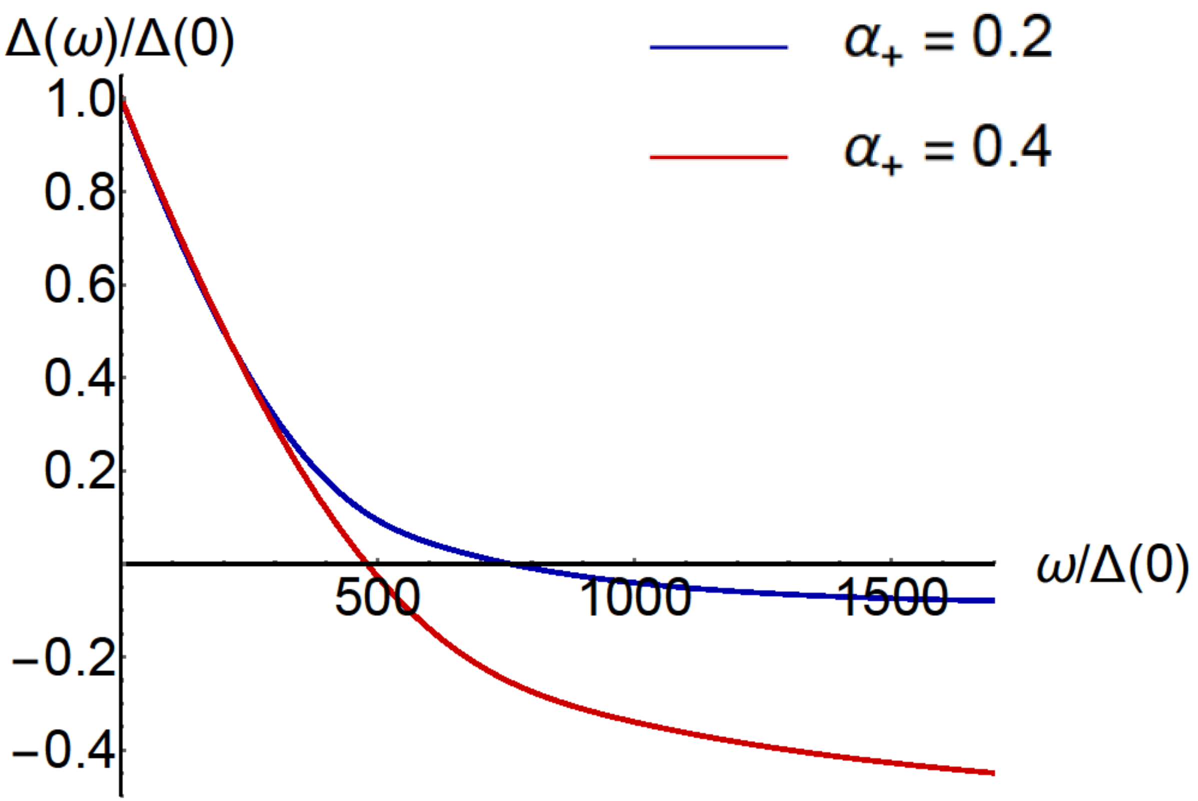
Local Approximation

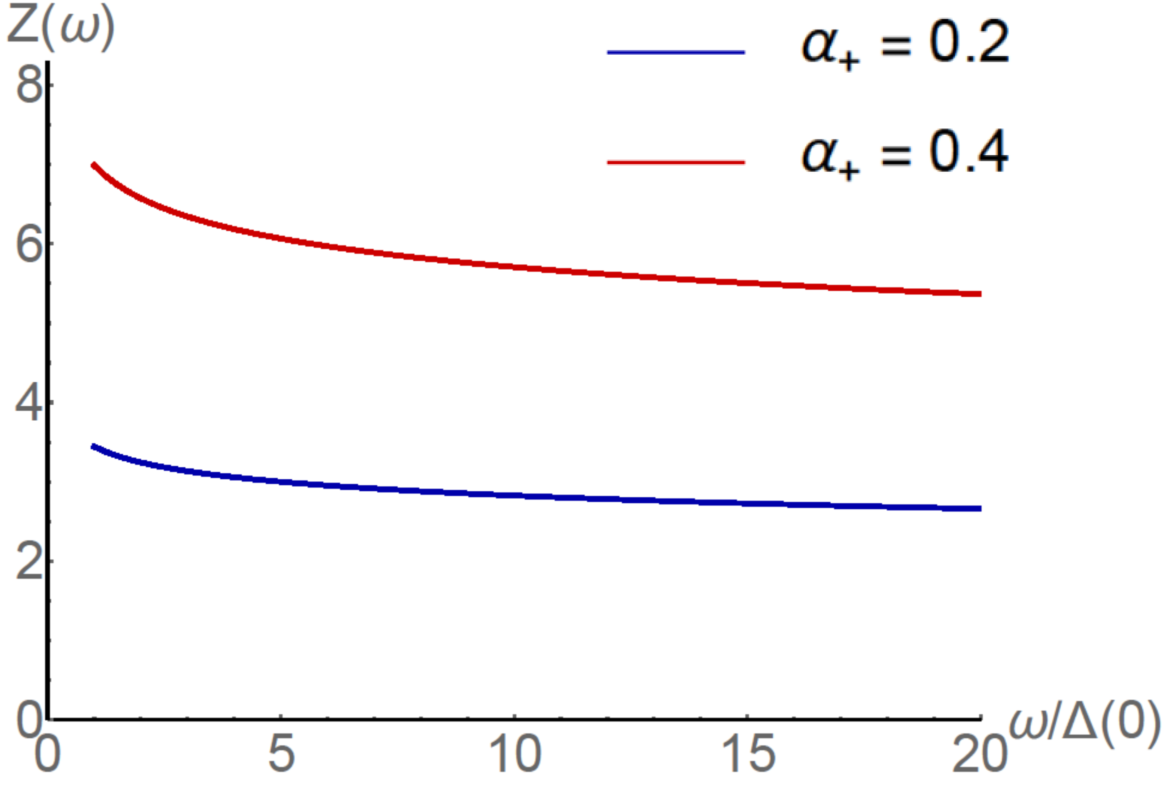




Self-Energy Effects

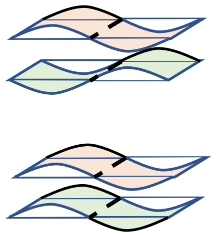
Local Approximation





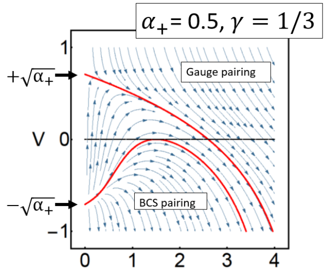
Conclusions

- We have revisited the idea that pairing due to gauge fields in a bilayer composite fermion metal could be a route to the total ν =1 bilayer quantum Hall effect.



$$\boxed{\Delta(\omega) = \frac{1}{2}\int \frac{\Delta(\Omega)}{\sqrt{\Omega^2+|\Delta(\Omega)|^2}}\,\lambda(\omega-\Omega)\, d\Omega}$$

$$\boxed{\lambda(\omega) = \alpha_- \left|\frac{\omega_0}{\omega}\right|^{\gamma} - \alpha_+ \ln\left|\frac{\omega_0}{\omega}\right| + V_0}$$



- Old result: Singular **out-of-phase** gauge fluctuations lead to a pairing instability with $\Delta(0)\sim\frac{1}{d^2}$
- New result: In-phase fluctuations, while less singular, are strongly pair breaking and ***very*** effective at suppressing the gap.

- Any experimentally observed transition to a paired quantum Hall state is likely better thought of in terms of a crossover from gauge pairing to BCS pairing driven by short-range interactions.