NAME:

ELECTRICITY AND MAGNETISM I

First Midterm Exam (Fall 2016)

Show all your work to receive full credit

 # 1 : (10 points)

A thick spherical shell carries charge density

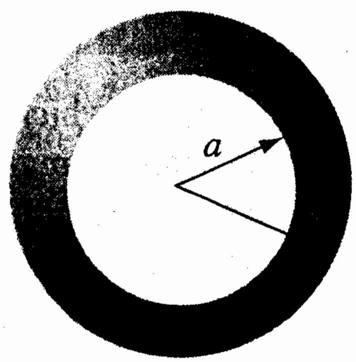
$$\rho = \frac{k}{r^2} \qquad (a \leq r \leq b)$$

(see figure below).

(a) Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, and (iii) $r > b$. (4 pts.)

(b) Plot $|\vec{E}|$ as a function of r, for the case $b = 2a$. (3 pts.)

(c) Find the potential at the center, using infinity as your reference point. (3 pts.)



 # 2 : (10 points)

Consider two concentric spherical shells of radii a and b. Suppose the inner one carries a charge q, and the outer one a charge $-q$ (both of them uniformly distributed over the surface).

(a) Obtain the electric field for (i) $r < a$, (ii) $a < r < b$, and (iii) $r > b$. (4 pts.)

(b) Calculate the energy of this configuration by using $W = (\epsilon_0/2) \int E^2 d\tau$. (2 pts.)

(c) Obtain the potential for the three regions, using infinity as your reference point. (2 pts.)

(d) The configuration can be considered a capacitor, find the capacitance. (2 pts.)

Solution of problem #1

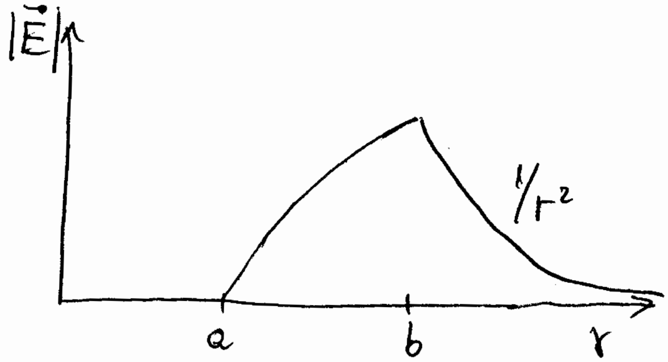
(a) $r<a$ $\quad Q_{enc}=0$, $\quad \boxed{\vec{E}=0}$

$a<r<b$ $\quad Q_{enc}=\int \rho\, d\tau = 4\pi\int_a^r dr'\, r'^2 \frac{k}{r'^2} = 4\pi k(r-a)$,

$$4\pi r^2 E = \frac{4\pi}{\varepsilon_0} k(r-a) \qquad \boxed{\vec{E}=\frac{k}{\varepsilon_0}\frac{r-a}{r^2}\,\hat{r}}$$

$r>b$ $\quad Q_{enc}=4\pi k(b-a) \qquad \boxed{\vec{E}=\frac{k}{\varepsilon_0}\frac{b-a}{r^2}\,\hat{r}}$

(b)

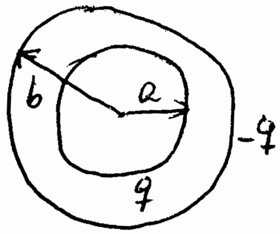


(c) $$V(0) = -\int_\infty^0 d\vec{l}\cdot\vec{E} = -\int_\infty^b dr\,\frac{k}{\varepsilon_0}\frac{b-a}{r^2} - \int_b^a dr\,\frac{k}{\varepsilon_0}\frac{r-a}{r^2} =$$

$$= -\frac{k}{\varepsilon_0}(b-a)\left(-\frac{1}{r}\right)\Big|_\infty^b - \frac{k}{\varepsilon_0}\ln\frac{a}{b} + \frac{k}{\varepsilon_0}a\left(-\frac{1}{r}\right)\Big|_b^a =$$

$$= \cancel{+\frac{k}{\varepsilon_0}\frac{b-a}{b}} + \frac{k}{\varepsilon_0}\ln\frac{b}{a} + \cancel{\frac{k}{\varepsilon_0}\left(\frac{a}{b}-1\right)} = \boxed{\frac{k}{\varepsilon_0}\ln\frac{b}{a}}$$

Solution of problem #2



(a) $r<a$ $\quad Q_{enc}=0 \quad \vec{E}=0$

$a<r<b$ $\quad Q_{enc}=q \quad \vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$b<r$ $\quad Q_{enc}=0 \quad \vec{E}=0$

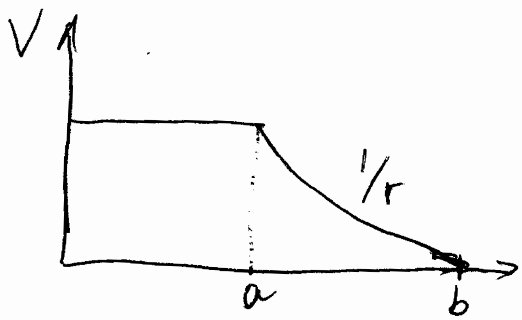
(b) $$W=\frac{\varepsilon_0}{2}\int d\tau\, E^2=\frac{\varepsilon_0}{2}4\pi\int_a^b dr\, r^2\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2=\frac{q^2}{8\pi\varepsilon_0}\int_a^b dr\frac{1}{r^2}=$$

$$\boxed{=\frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

(c) $$V(\vec{r})=-\int_\infty^{\vec{r}} d\vec{l}\cdot\vec{E}=-\int_b^r dr\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right) \quad \text{for } a<r<b$$

$$V(\vec{r})=-\int_\infty^{\vec{r}} d\vec{l}\cdot\vec{E}=-\int_b^a dr\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}-\int_a^{\vec{r}} dr\underbrace{\vec{E}}_{0}=$$

$$=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) \quad \text{for } r<a$$



(d) $$C=\frac{Q}{V}=4\pi\varepsilon_0\frac{1}{\frac{1}{a}-\frac{1}{b}}=4\pi\varepsilon_0\frac{ab}{b-a}$$