ELECTRICITY AND MAGNETISM II

Homework set #17: Electromagnetic Waves II

Problem # 17.1 :

(a) Show that the skin depth in a poor conductor ($\sigma \ll \omega\epsilon$) is $(2/\sigma)\sqrt{\epsilon/\mu}$ (independent of frequency). Find the skin depth (in meters) for (pure) water. Use the static values $\epsilon = 80.1\epsilon_0$, $\mu \sim \mu_0$ and $\sigma = 1/(2.5 \times 10^5)$ (ohm - m)$^{-1}$. Your answers will be valid, then, only at relatively low frequencies.

(b) Show that the skin depth in a good conductor ($\sigma \gg \omega\epsilon$) is $\lambda/2\pi$, where λ is the wavelength *in the conductor.* Find the skin depth (in nanometers) for a typical metal ($\sigma \sim 10^7$ (ohm - m)$^{-1}$) in the visible range ($\omega \sim 10^{15}$/s), assuming $\epsilon \sim \epsilon_0$ and $\mu \sim \mu_0$. Why are the metals opaque?

(c) Show that in a good conductor the magnetic field lags the electric field by 45^o, and find the ratio of their amplitudes. For a numerical example, use the "typical metal" in part (b).

Problem # 17.2 :

Calculate the reflection coefficient for light at an air-to-silver interface ($\mu_1 = \mu_2 = \mu_0$, $\epsilon_1 = \epsilon_0$, $\sigma = 6 \times 10^7 (\Omega \text{ m})^{-1}$), at optical frequencies ($\omega = 4 \times 10^{15}$/s).

Problem # 17.3 :

Starting from Maxwell's equations (without free charges and free currents) show that TEM waves cannot occur in a hollow wave guide. Show first that the vector $\tilde{\mathbf{E}}_0$ has zero divergence and zero curl. It can therefore be written as the gradient of a scalar potential that satisfies Laplace's equation. Show that this potential is constant throughout, i.e. the electric field is zero.

Problem # 17.4 :

Show that the mode TE_{00} cannot occur in a rectangular wave guide. Starting from Maxwell's equations without charges and currents, show first that B_z is a constant. Then, applying Faraday's law in integral form to a cross section of the wave guide, show that $B_z = 0$. Under these conditions the TE_{00} mode is a TEM mode, which we already know cannot exist in an empty wave guide.

Problem # 17.5 :

Work out the theory of TM modes for a rectangular wave guide. In particular, find the longitudinal electric field, the cutoff frequencies, and the wave and group velocities. Find the ratio of the lowest TM cutoff frequency to the lowest TE cutoff frequency, for a given wave guide. What is the lowest TM mode?