ELECTRICITY AND MAGNETISM II

Homework set #18: Potentials and Fields I

Problem # 18.1 :

(a) Find the fields, and the charge and current distributions, corresponding to

$$V(\mathbf{r},t) = 0 \quad , \quad \mathbf{A}(\mathbf{r},t) = -\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\hat{\mathbf{r}} \ .$$

(b) Use the gauge function $\lambda = -(1/4\pi\epsilon_0)(qt/r)$ to transform the potentials, and comment on the result.

Problem # 18.2 :

A time-dependent point charge $q(t)$ at the origin, $\rho(\mathbf{r},t) = q(t)\delta^3(\mathbf{r})$, is fed by a current $\mathbf{J}(\mathbf{r},t) = -(1/4\pi)(\dot{q}/r^2)\hat{\mathbf{r}}$, where $\dot{q} = dq/dt$.

(a) Check that charge is conserved, by confirming that the continuity equation is obeyed.

(b) Find the scalar and vector potentials in the Coulomb gauge. If you get stuck, try working on (c) first.

(c) Find the fields, and check that they satisfy all of Maxwell's equations.

Problem # 18.3 :

Suppose $V = 0$ and $\mathbf{A} = A_0 \sin(kx - \omega t)\hat{\mathbf{y}}$, where A_0, ω, and k are constants. Find $\mathbf{E}$ and $\mathbf{B}$, and check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω and k?

Problem # 18.4 :

The vector potential for a uniform magnetostatic field is $\mathbf{A} = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$. Show that $d\mathbf{A}/dt = -\frac{1}{2}(\mathbf{v} \times \mathbf{B})$, in this case, and confirm that

$$\frac{d}{dt}(\mathbf{p} + q\mathbf{A}) = -\vec{\nabla} U_{vel} \quad , \quad U_{vel} = q(V - \mathbf{v} \cdot \mathbf{A})$$

yields the correct equation of motion.

Problem # 18.5 :

A piece of wire bent into a loop, as shown in the figure, carries a current that increases linearly with time:

$$I(t) = kt \quad (-\infty < t < \infty) \; .$$

Calculate the retarded vector potential $\mathbf{A}$ at the center. Find the electric field at the center. Why does this neutral wire produce an *electric* field? Why can't you determine the *magnetic* field from this expression for $\mathbf{A}$?