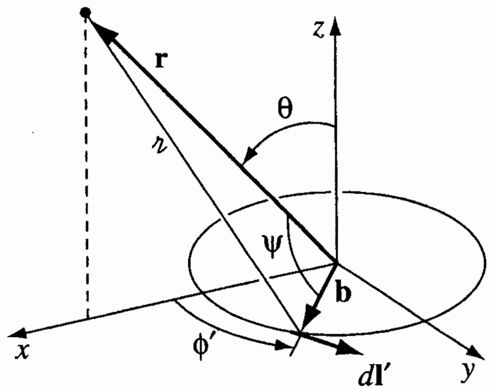
ELECTRICITY AND MAGNETISM II

Homework set #20: Radiation II

Problem # 20.1 :

Find the **radiation resistance** (see problem #19.4) for the oscillating magnetic dipole shown in the figure. Express your answer in terms of the wave length λ and the radius of the loop b. Compare the radiation resistance with that of the electric dipole. [*Answer:* $3 \times 10^5 (b/\lambda)^4\ \Omega$].



Problem # 20.2 :

(a) Calculate the electric and magnetic fields of an oscillating magnetic dipole *without* using approximation 3.

(b) Find the Poynting vector, and show that the intensity of the radiation is exactly the same as we got using approximation 3.

Problem # 20.3 :

A positive charge q is fired head-on at a distant positive charge Q (which is held stationary), with an initial velocity v_0. It comes in, decelerates to $v - 0$, and returns out to infinity. What fraction of its initial energy ($mv_0^2/2$) is radiated away? Assume $v_0 \ll c$, and that you can safely ignore the effect of radiative losses on the motion of the particle. [*Answer:* $(16/45)(q/Q)(v_0/c)^3$.]

Hint: $\int_{x_0}^{\infty} \frac{dx}{x^4\sqrt{(1/x_0)-(1/x)}} = \frac{16}{15x_0^{5/2}}$

Problem # 20.4 :

In Bohr's theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius 5×10^{-11} m, held in orbit by the Coulomb attraction of the proton. According to classical electrodynamics, this electron should radiate, and hence spiral in to the nucleus.

(a) Show that $v \ll c$ for most of the trip (so you can use the Larmor formula).

(b) Calculate the lifespan of Bohr's atom. (Assume each revolution is essentially circular.)

Problem # 20.5 :

Consider an ideal stationary magnetic dipole $\mathbf{m}$ in a static electric field $\mathbf{E}$. Show that the fields carry momentum

$$\mathbf{p} = -\epsilon_0\mu_0(\mathbf{m} \times \mathbf{E}) \ .$$

Hint: Starting with $\mathbf{p} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B}) d\tau$, write $\mathbf{E} = -\vec{\nabla} V$, and use integration by parts to show that

$$\mathbf{p} = \epsilon_0\mu_0 \int V\mathbf{J} d\tau \ .$$

So far this is valid for *any* localized static configuration. For a current confined to an infinitesimal neighborhood of the origin we can approximate $V(\mathbf{r}) \sim V(\mathbf{0}) - \mathbf{E}(\mathbf{0}) \cdot \mathbf{r}$. Treat the dipole as a current loop and use $\oint d\mathbf{l}' = \mathbf{0}$. Note that $\oint (\mathbf{c} \cdot \mathbf{r}) d\mathbf{l} = \mathbf{a} \times \mathbf{c}$, where $\mathbf{c}$ is a constant vector and $\mathbf{a}$ is the area of the loop.