ELECTRICITY AND MAGNETISM II

Homework set #21: Special Theory of Relativity I

Problem # 21.1 :

A rocket ship leaves earth at a speed of $\frac{3}{5}c$. When a clock on the rocket says 1 hour has elapsed, the rocket ship sends a light signal back to earth.

(a) According to *earth* clocks, when was the signal sent?

(b) According to *earth* clocks, how long after the rocket left did the signal arrive back on earth?

(c) According to the *rocket* observer, how long after the rocket left did the signal arrive back on earth?

Problem # 21.2 :

Check that

$$-\overline{a}^0\overline{b}^0 + \overline{a}^1\overline{b}^1 + \overline{a}^2\overline{b}^2 + \overline{a}^3\overline{b}^3 = -a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3$$

using the Lorentz transformation

$$\overline{a}^0 = \gamma(a^0 - \beta a^1) \ , \quad \overline{a}^1 = \gamma(a^1 - \beta a^0) \ , \quad \overline{a}^2 = a^2 \ , \quad \overline{a}^3 = a^3 \ .$$

[This only proves the invariance of the scalar product for transformations along the x direction. But the scalar product is also invariant under rotations, since the first term is not affected at all, and the last three constitute the three-dimensional dot product $\mathbf{a}\cdot\mathbf{b}$. By suitable rotation, the x direction can be aimed any way you please, so the four-dimensional scalar product is actually invariant under *arbitrary* Lorentz transformations.]

Problem # 21.3 :

Use the notation ct for time coordinate, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$.

(a) Write out the matrix that describes a *Galilean* transformation with velocity along the x axis.

(b) Write out the matrices describing a Lorentz transformation along the x axis and y axis, respectively.

(c) Find the matrix describing a Lorentz transformation with velocity v along the x axis followed by a Lorentz transformation with velocity $\overline{v}$ along the y axis. Does it matter in what order the transformations are carried out?

Problem # 21.4 :

(a) Using the definition of proper velocity in terms of ordinary velocity,

$$\eta = \frac{1}{\sqrt{1 - u^2/c^2}}\mathbf{u} \ ,$$

invert the equation to get the formula for $\mathbf{u}$ in terms of η.

(b) Find the invariant product of the 4-velocity with itself, $\eta^\mu \eta_\mu$. Is η^μ timelike, spacelike, or lightlike?

(c) A cop pulls you over and asks what speed you were going. "Well, officer, I cannot tell a lie: the speedometer read 4×10^8 m/s." He gives you a ticket, because the speed limit on this highway is 2.5×10^8 m/s. In court, your lawyer (who, luckily, has studied physics) points out that a car's speedometer measures *proper* velocity, whereas the speed limit is *ordinary* velocity. Guilty, or innocent?

Problem # 21.5 :

(a) A particle of mass m whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, what is the mass of the resulting composite particle? What is its velocity?

(b) A neutral pion of rest mass m and relativistic momentum $p = (3/4)mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion, and the other in the opposite direction. Find the relativistic energy of each photon.