ELECTRICITY AND MAGNETISM II

Homework set #22: Special Theory of Relativity II

Problem # 22.1 :

(a) In classical mechanics, Newton's law can be written in the more familiar form $\mathbf{F} = m\mathbf{a}$. The relativistic equation, $\mathbf{F} = d\mathbf{p}/dt$, cannot be so simply expressed. Show, rather, that

$$\mathbf{F} = \frac{m}{\sqrt{1 - u^2/c^2}} \left[\mathbf{a} + \frac{\mathbf{u}(\mathbf{u} \cdot \mathbf{a})}{c^2 - u^2} \right] ,$$

where $\mathbf{a} = d\mathbf{u}/dt$ is the **ordinary acceleration**.

(b) Show that it is possible to outrun a light ray, if you're given sufficient head start, and your feet generate a constant force.

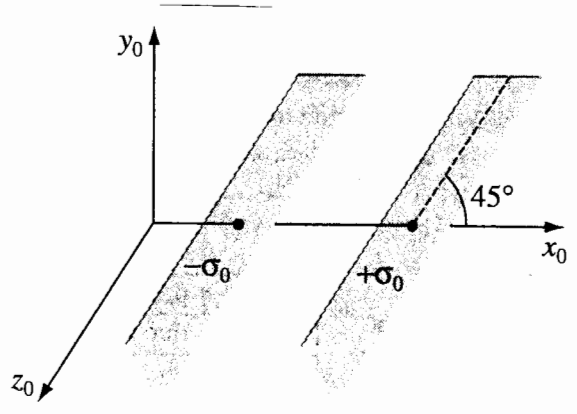
Problem # 22.2 :

Show that the ordinary acceleration of a particle of mass m and charge q, moving at velocity $\mathbf{u}$ under the influence of electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$, is given by

$$\mathbf{a} = \frac{q}{m} \sqrt{1 - u^2/c^2} \left[\mathbf{E} + \mathbf{u} \times \mathbf{B} - \frac{1}{c^2} \mathbf{u}(\mathbf{u} \cdot \mathbf{E}) \right] .$$

Hint: Use the result of problem #22.1.

Problem # 22.3 :



A parallel-plate capacitor, at rest in $\mathcal{S}_0$ and tilted at a 45° angle to the x_0 axis, carries

charge densities $\pm\sigma_0$ on the two plates (see figure). System $\mathcal{S}$ is moving to the right at speed v relative to $\mathcal{S}_0$.

(a) Find $\mathbf{E}_0$, the field in $\mathcal{S}_0$.

(b) Find $\mathbf{E}$, the field in $\mathcal{S}$.

(c) What angle do the plates make with the x axis?

(d) Is the field perpendicular to the plates in $\mathcal{S}$? Find $\mathbf{E}_0$, the field in $\mathcal{S}_0$.

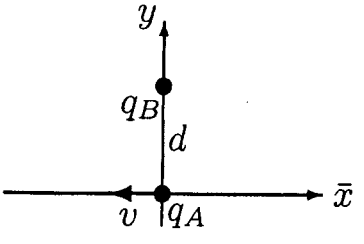
Problem # 22.4 :

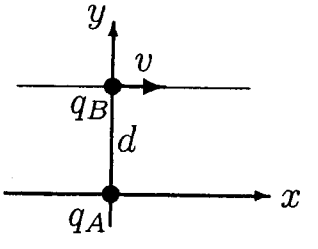
In system $\mathcal{S}_0$, a static uniform line charge λ coincides with the z axis.

(a) Write the electric field $\mathbf{E}_0$ in *Cartesian* coordinates, for the point (x_0, y_0, z_0).

(b) Use the transformation rules for the fields to find the electric field in $\mathcal{S}$, which moves with speed v in the x direction with respect to $\mathcal{S}_0$. The field is still in terms of (x_0, y_0, z_0); express it instead in terms of the coordinates (x, y, z) in $\mathcal{S}$. Finally, write $\mathbf{E}$ in terms of the vector $\mathbf{S}$ from the *present* location of the wire and the angle θ between $\mathbf{S}$ and $\hat{\mathbf{x}}$. Does the field point away from the instantaneous location of the wire, like the field of a uniformly moving point charge?

Problem # 22.5 :





(a) Charge q_A is at rest at the origin in system $\mathcal{S}$; charge q_B flies by at speed v on a trajectory parallel to the x axis, but at $y = d$. What is the electromagnetic force on q_B as it crosses the y axis?

(b) Now study the same problem from system $\overline{\mathcal{S}}$, which moves to the right with speed v.

What is the force on q_B when q_A passes the $\overline{y}$ axis? Do it two ways:

(i) by using your answer to (a) and transforming the force;

(ii) by computing the fields in $\overline{\mathcal{S}}$ and using the Lorentz force law.