NAME:

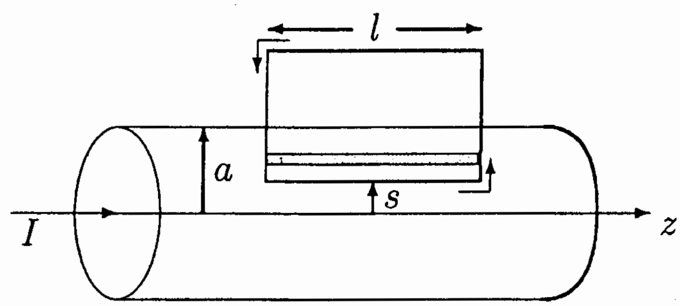
ELECTRICITY AND MAGNETISM II

First Midterm Exam (Spring 2017)

Show all your work to receive full credit

 # 1 : (9 points)

An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius a.



(a) In what *direction* does the induced electric field point (radial, circumferential, or longitudinal)? (3 pts.)

(b) Assuming the field is zero for $s > a$, find $\mathbf{E}(s, t)$. (6 pts.)

 # 2 : (6 points)

Find the self-inductance per unit length of a long solenoid, of radius R, carrying n turns per unit length.

 # 3 : (10 points)

An infinitely long cylindrical tube, of radius a, moves at constant speed v along its axis. It carries a net charge per unit length λ, uniformly distributed over its surface. Surrounding it, at radius b, is another cylinder, moving with the same velocity but carrying the opposite charge $(-\lambda)$.

(a) Find the electric and magnetic fields between the cylinders. (2 pts.)

(b) Find the energy per unit length stored in the fields. (3 pts.)

(c) Find the momentum per unit length in the fields. (3 pts.)

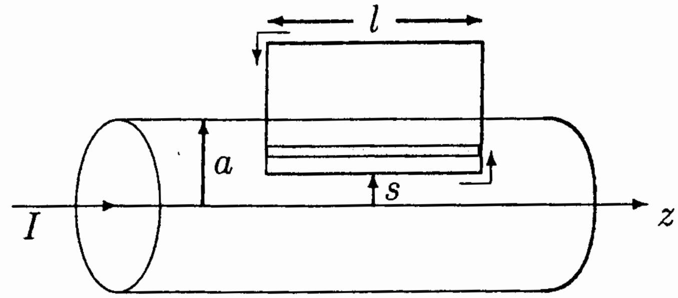
(d) Find the energy per unit time transported by the fields across a plane perpendicular to the cylinders. (2 pts.)

Hint: $\mathbf{g} = \epsilon_0(\mathbf{E} \times \mathbf{B})$

Solution of problem #1

(a) In the quasistatic approximation the magnetic field is "circumferentia[l]. The induced electric field acts on the carriers along the z-direction and is hence $\boxed{\text{"longitudinal"}}$.

(b) We now use the amperian loop shown in the figure. Outside the coaxial tube, $\vec{B}=0$ and $\vec{E}=0$. (since $\vec{B}=0$ there cannot be any induction). Inside: $\vec{B}=\frac{\mu_0 I}{2\pi s}\hat{\phi}$.



Hence, $\oint \vec{E}\cdot d\vec{l} = E l = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = -\frac{\mu_0}{2\pi}\frac{dI}{dt} l \int_s^a \frac{ds'}{s'} =$

$= -\frac{\mu_0}{2\pi} l \ln\left(\frac{a}{s}\right)\frac{dI}{dt}$; since $\frac{dI}{dt} = -I_0 \omega \sin(\omega t)$

$$\boxed{\vec{E} = \frac{\mu_0 I_0 \omega}{2\pi}\sin(\omega t)\ln\left(\frac{a}{s}\right)\hat{z}}$$

Solution of problem #2

Inside the solenoid $\vec{B} \parallel \hat{z}$: $\vec{B} = \mu_0 n I \hat{z}$,

the flux through a single turn is $\Phi_1 = \mu_0 n I \pi R^2$

In a length l there are nl such turns, so that the total flux is

$\Phi = \mu_0 n^2 l I \pi R^2 = LI$, where L is the self-inductance.

The self-inductance per unit length is then

$$\boxed{\mathcal{L} = \frac{L}{l} = \mu_0 n^2 \pi R^2}$$

Solution of problem #3

(a) The fields are zero for $s<a$ and $s>b$. Between the cylinders,

$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s}\hat{s}$ cylinder of radius s as Gaussian surface

$$2\pi s l E = \lambda l/\varepsilon_0$$

$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{s}\hat{\phi}$ Amperian loop of radius s

$$2\pi s B = \mu_0 I = \mu_0 \lambda v$$

(b) $$u = \frac{1}{2}\left[\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right] = \frac{1}{2}\left[\varepsilon_0\left(\frac{\lambda}{2\pi\varepsilon_0 s}\right)^2 + \frac{1}{\mu_0}\left(\frac{\mu_0}{2\pi}\frac{I}{s}\right)^2\right] =$$

$$\boxed{= \frac{\lambda^2}{8\pi^2\varepsilon_0}\left(1+\varepsilon_0\mu_0 v^2\right)\frac{1}{s^2}}$$

Integrating over the volume between the cylinders

$$\frac{W}{l} = \frac{\lambda^2}{8\pi^2\varepsilon_0}\left(1+\frac{v^2}{c^2}\right)\int_a^b \frac{1}{s^2}(2\pi s)\,ds = \boxed{\frac{\lambda^2}{4\pi\varepsilon_0}\left(1+\frac{v^2}{c^2}\right)\ln\frac{b}{a}}$$

(c) $$\vec{g} = \varepsilon_0(\vec{E}\times\vec{B}) = \varepsilon_0\left(\frac{\lambda}{2\pi\varepsilon_0 s}\right)\left(\frac{\mu_0}{2\pi}\frac{I}{s}\right)\hat{z} = \frac{\mu_0\lambda^2 v}{4\pi^2 s^2}\hat{z}$$

$$\frac{\vec{P}}{l} = \frac{\mu_0\lambda^2 v}{4\pi^2}\hat{z}\int_a^b \frac{1}{s^2}2\pi s\,ds = \boxed{\frac{\lambda^2 v\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)\hat{z}}$$

(d) $$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\mu_0\varepsilon_0}\vec{g} = c^2\vec{g}$$

$$\frac{dW}{dt} = \int \vec{S}\cdot d\vec{a} = \frac{\mu_0\lambda^2 v}{4\pi^2\varepsilon_0\mu_0}\int_a^b \frac{ds}{s^2}(2\pi s) = \boxed{\frac{\lambda^2 v}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)}$$