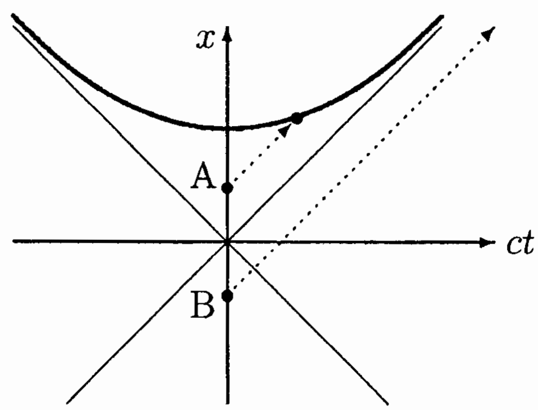
Solution of problem #22.1

(a) $$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\frac{m\vec{u}}{\sqrt{1-u^2/c^2}} = m\left\{\frac{d\vec{u}/dt}{(1-u^2/c^2)^{1/2}} + \vec{u}\left(-\frac{1}{2}\right)\frac{-\frac{1}{c^2}2\vec{u}\cdot\frac{d\vec{u}}{dt}}{(1-u^2/c^2)^{3/2}}\right\}$$

$$= \frac{m}{(1-u^2/c^2)^{1/2}}\left\{\vec{a} + \frac{\vec{u}(\vec{u}\cdot\vec{a})}{c^2-u^2}\right\} \qquad \text{q.e.d.}$$

(b) At constant force you go in "hyperbolic" motion. Photon A, which left the origin at $t<0$, catches up with you, but photon B, which passes the origin at $t>0$, never does.



Solution of problem # 22.2

$$\vec{F} = \frac{m}{\sqrt{1-u^2/c^2}}\left[\vec{a} + \frac{\vec{u}(\vec{u}\cdot\vec{a})}{c^2-u^2}\right] = q\left(\vec{E} + \vec{u}\times\vec{B}\right)$$

$$\vec{a} + \frac{\vec{u}(\vec{u}\cdot\vec{a})}{c^2-u^2} = \frac{q}{m}\sqrt{1-u^2/c^2}\left(\vec{E} + \vec{u}\times\vec{B}\right) \qquad (1)$$

take scalar product with $\vec{u}$

$$\vec{u}\cdot\vec{a} + \frac{u^2(\vec{u}\cdot\vec{a})}{c^2-u^2} = \frac{c^2\,\vec{u}\cdot\vec{a}}{c^2-u^2} = \frac{q}{m}\sqrt{1-u^2/c^2}\left[\vec{u}\cdot\vec{E} + \underbrace{\vec{u}\cdot(\vec{u}\times\vec{B})}_{=0}\right]$$

$$\frac{\vec{u}(\vec{u}\cdot\vec{a})}{c^2-u^2} = \frac{q}{m}\sqrt{1-\frac{u^2}{c^2}}\;\frac{\vec{u}(\vec{u}\cdot\vec{E})}{c^2} \qquad (2)$$

Insert (2) into (1)

$$\vec{a} = \frac{q}{m}\sqrt{1-u^2/c^2}\left[\vec{E} + \vec{u}\times\vec{B} - \frac{\vec{u}(\vec{u}\cdot\vec{E})}{c^2}\right] \qquad q.e.d.$$

Solution of problem # 22.3

(a) The field in S_0 is σ_0/ε_0 and points perpendicular to the positive plate, so:

$$\vec{E}_0 = \frac{\sigma_0}{\varepsilon_0}\left(-\cos 45^\circ\,\hat{x} + \sin 45^\circ\,\hat{y}\right) = \boxed{\frac{\sigma_0}{\sqrt{2}\,\varepsilon_0}\left(-\hat{x}+\hat{y}\right)}$$

(b) speed v in the positive x-direction

$$E_x = E_{x0} = -\frac{\sigma_0}{\sqrt{2}\,\varepsilon_0} \quad , \quad E_y = E_{y0}\gamma = \gamma\frac{\sigma_0}{\sqrt{2}\,\varepsilon_0} \quad , \quad E_z = \gamma E_{z0} = 0$$

$$\boxed{\vec{E} = \frac{\sigma_0}{\sqrt{2}\,\varepsilon_0}\left(-\hat{x}+\gamma\hat{y}\right)}$$

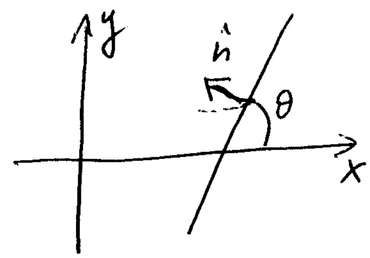
(c) Angle of the plates with the x-axis: In S_0, $\theta_0 = 45^\circ$, $\tan\theta_0 = 1$

In S, the x-component is Lorentz contracted

$$\tan\theta = \frac{y}{x} = \frac{y_0}{x_0\frac{1}{\gamma}} = \tan\theta_0\cdot\gamma = \gamma \qquad \boxed{\theta = \arctan\gamma}$$

(d) Let $\hat{n}$ be a unit vector perpendicular to the plates in S

$$\hat{n} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y} \quad , \quad |\vec{E}| = \frac{\sigma_0}{\sqrt{2}\,\varepsilon_0}\sqrt{1+\gamma^2}$$



The angle ϕ between $\hat{n}$ and $\vec{E}$ is

$$\frac{\vec{E}\cdot\hat{n}}{|\vec{E}|} = \cos\phi = \frac{\sin\theta + \gamma\cos\theta}{\sqrt{1+\gamma^2}} = \frac{\cos\theta}{\sqrt{1+\gamma^2}}\left[\tan\theta + \gamma\right]$$

$$= \frac{2\gamma}{\sqrt{1+\gamma^2}}\cos\theta \quad , \quad \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+\gamma^2}}$$

$$\boxed{\cos\phi = \frac{2\gamma}{1+\gamma^2}}$$

Hence, the field is not perpendicular to the plates in S.

Solution of problem # 22.4

(a) $$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\hat{s}}{s} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\frac{x_0\hat{x} + y_0\hat{y}}{x_0^2 + y_0^2}}$$

(b) $$\bar{E}_x = E_x = \frac{\lambda}{2\pi\varepsilon_0}\frac{x_0}{x_0^2+y_0^2}, \quad \bar{E}_y = \gamma E_y = \gamma\frac{\lambda}{2\pi\varepsilon_0}\frac{y_0}{x_0^2+y_0^2}, \quad \bar{E}_z = \gamma E_z = 0$$

$$\boxed{\bar{\vec{E}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{x_0\hat{x} + \gamma y_0\hat{y}}{x_0^2 + y_0^2}}$$

We have to express x_0, y_0, z_0 in terms of x, y, z. Use inverse Lorentz transformation: $x_0 = \gamma(x + vt)$, $y_0 = y$; Now

$$\bar{\vec{E}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\gamma(x+vt)\hat{x} + \gamma y\hat{y}}{[\gamma^2(x+vt)^2 + y^2]} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\frac{1}{\gamma}\frac{(x+vt)\hat{x} + y\hat{y}}{[(x+vt)^2 + y^2/\gamma^2]}}$$

and $\vec{S} = (x+vt)\hat{x} + y\hat{y}$, $y = S\sin\theta$

$$[(x+vt)^2 + y^2/\gamma^2] = [(x+vt)^2 + y^2(1 - v^2/c^2)] = S^2 - \frac{v^2}{c^2}S^2\sin^2\theta =$$

$$= S^2\left[1 - \left(\frac{v}{c}\right)^2\sin^2\theta\right], \text{ so}$$

$$\bar{\vec{E}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{\gamma}\frac{\vec{S}}{S^2}\frac{1}{1 - v^2\sin^2\theta/c^2} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\frac{\sqrt{1 - v^2/c^2}}{1 - v^2\sin^2\theta/c^2}\frac{\hat{S}}{S}}$$

Yes, the field points away from the present location of the wire. Compare to the electric field of a point charge moving with constant velocity studied in class.

Solution of problem # 22.5

(a) Fields of A at B: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_A}{d^2} \hat{y}$; $\vec{B} = 0$

The force on q_B is $\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{d^2} \hat{y}}$

(b) (i) In $\bar{S}$ the charge q_B is at rest. This is then the opposite transformation of what we studied in class, not

$\bar{F}_\perp \neq \frac{1}{\gamma} F_\perp$, $\bar{F}_\parallel = F_\parallel$, but $\bar{F}_\perp = \gamma F_\perp$ and $\bar{F}_\parallel = F_\parallel$:

$$\boxed{\vec{\bar{F}} = \frac{\gamma}{4\pi\varepsilon_0} \frac{q_A q_B}{d^2} \hat{y}}$$

(ii) We have derived in class that

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_A (1 - v^2/c^2)}{\left[1 - \left(\frac{v^2}{c^2}\right) \sin^2\theta\right]^{3/2}} \frac{\hat{R}}{R^2}$$

Here, $\theta = \frac{\pi}{2}$, $\hat{R} = \hat{y}$ and $R = d$, so that

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_A (1 - v^2/c^2)}{\left[1 - (v^2/c^2)\right]^{3/2}} \frac{\hat{y}}{d^2} = \frac{\gamma}{4\pi\varepsilon_0} \frac{q_A}{d^2} \hat{y}$$

Since q_B is at rest, there are no magnetic forces.

$$\boxed{\vec{\bar{F}} = \frac{\gamma}{4\pi\varepsilon_0} \frac{q_A q_B}{d^2} \hat{y}}$$ (as before)