Solution of problem # 13.1

long solenoid

\[ \oint B \cdot dl = \mu_0 I \text{enc} \]

determine first \( \vec{B} \)

* for an amperian loop outside the solenoid

\[ \oint B \cdot dl = 0 = L(B_1 - B_2) \Rightarrow B_1 = B_2 \]

since \( B = 0 \) at infinity

\[ \vec{B} = 0 \quad s > a \]

* for amperian loop as in the figure \( B_2 = 0 \)

\[ \oint B \cdot dl = (B_1 - B_2)L = \mu_0 n I L \Rightarrow \vec{B} = \mu_0 n I \hat{z} \quad s < a \]

valid in the quasistatic approximation, i.e. slow variation of \( I \) with time.

Inside: \( \vec{E} \) is along \( \hat{\phi} \). For an amperian loop (circle) of radius \( s < a \)

\[ \Phi = B_1 \pi s^2 = \mu_0 n I \pi s^2 \quad ; \quad \oint \vec{E} \cdot dl = E_2 \pi s = -\frac{d\Phi}{dt} = -\mu_0 n \pi s^2 \frac{dI}{dt} \]

Hence,

\[ \vec{E} = -\frac{\mu_0 n s}{2} \frac{dI}{dt} \hat{\phi} \]

Outside: \( \vec{E} \) is along \( \hat{\phi} \). For an amperian loop (circle) of radius \( s > a \)

\[ \Phi = B_1 \pi a^2 = \mu_0 n I \pi a^2 \quad , \text{since there is no flux for } a < r < s \]

\[ \oint \vec{E} \cdot dl = E_2 \pi s = -\frac{d\Phi}{dt} = -\mu_0 n \pi a^2 \frac{dI}{dt} \]

\[ \vec{E} = -\frac{\mu_0 n a^2}{2s} \frac{dI}{dt} \hat{\phi} \]
Solution of problem #13.2

(a) The field inside the solenoid is \( B = \mu_0 n I \).

The flux through the solenoid is \( \Phi = \pi a^2 \mu_0 n I \). The flux through the loop is also \( \Phi \). The induced emf in the loop is

\[
E = -\frac{d\Phi}{dt} = -\pi a^2 \mu_0 n \frac{dI}{dt} \quad ; \quad \frac{dI}{dt} = k = \text{const.}
\]

In magnitude \( E = \pi a^2 \mu_0 n k \). The current in the loop is then

\[
I_L = \frac{\pi a^2 \mu_0 n k}{R}
\]

The field \( B \) in the solenoid is to the right and increasing. Hence, the field in the loop is to the left, and the current is counterclockwise, i.e. to the right, through the resistor.

(b) \( \Delta\Phi = \Phi_1 - \Phi_2 = \pi a^2 \mu_0 n I \); \( I = \frac{dQ}{dt} = \frac{E}{R} = -\frac{1}{R} \frac{d\Phi}{dt} \)

Hence, \( \Delta Q = \frac{1}{R} \Delta\Phi \), in magnitude. So

\[
\Delta Q = \frac{\pi a^2 \mu_0 n I}{R}
\]
Solution of problem #13.3

Field generated by the current: \( \mathbf{B} = \frac{\mu_0 I}{2\pi s} \mathbf{\hat{\phi}} \)

Flux through the square loop:
\[
\Phi = \int \mathbf{B} \cdot d\mathbf{a} = \frac{\mu_0 I}{2\pi} \int_s^{s+a} (ds') \mathbf{\hat{\phi}} \cdot \mathbf{\hat{s}} = \frac{\mu_0 I a}{2\pi} \ln \frac{s+a}{s}
\]

\( E_{\text{loop}} = -\frac{d\Phi}{dt} = I_{\text{loop}} R = \frac{dQ}{dt} R = -\frac{\mu_0 a}{2\pi} \ln \frac{s+a}{s} \frac{dI}{dt} \)

Hence,
\( dQ = -\frac{\mu_0 a}{2\pi R} \ln \frac{s+a}{s} \, dI \)

Integrating:
\( Q = \frac{I \mu_0 a}{2\pi R} \ln \frac{s+a}{s} \)

The field of the wire, at the square loop, is out of the page. It is decreasing, so the field of the induced current in the loop must point out of the page. Hence, the induced current flows counterclockwise.
Solution of problem #13, 4

(a) \( \vec{B} \) points in the \( \hat{z} \) direction (from symmetry)

\[
B(z) = \frac{\mu_0 I}{4\pi} \int \frac{dl'}{r^2} \cos \theta = \frac{\mu_0 I}{4\pi} \frac{\cos \phi}{r^2} \int_{2\pi}^{\pi} b dh
\]

\[
= \frac{\mu_0 I}{2} \frac{b}{(b^2 + z^2)^{3/2}}
\]

Now the flux through the little loop of area \( \pi a^2 \) is

\[
\Phi = \frac{\mu_0 \pi I a^2 b^2}{2 (b^2 + z^2)^{3/2}}
\]

(b) The little loop is so small that the field generated is that of a magnetic dipole:

\[
\vec{B} = \frac{\mu_0 m}{4\pi r^3} (2\cos \hat{r} + \sin \hat{\theta})
\]

\( m = I \pi a^2 \)

We now integrate over the spherical cap, centered at the little loop, and bounded by the big loop. In spherical coordinates \( d\vec{a} = \hat{r} d\vec{a} \)

\[
\Phi = \int \vec{B} \cdot d\vec{a} = \frac{\mu_0}{4\pi} \frac{m}{r^3} \int [2\cos \hat{r} + \sin \hat{\theta}] r^2 d\phi d\theta \sin \hat{\theta} =
\]

\[
= \frac{\mu_0 I \pi a^2}{2r} \left[ \frac{\sin^2 \theta}{2} \right]_{\phi = 0}^{\phi = 2\pi} = \frac{\mu_0 I \pi a^2}{2r} \sin^2 \theta
\]

\( r = \sqrt{b^2 + z^2} \), \( \sin \hat{\theta} = \frac{b}{r} \),

\[
\Phi = \frac{\mu_0 \pi I a^2 b^2}{2 (b^2 + z^2)^{3/2}} \quad \text{same as in (a)}
\]

(c) \( \Phi_1 = H_{12} I_2 \), \( \Phi_2 = H_{21} I \)

\[
H_{12} = H_{21} = \frac{\mu_0 \pi a^2 b^2}{2 (b^2 + z^2)^{3/2}}
\]
Solution of problem 13.5

(a) In the quasistatic approximation for the straight wire

\[ \Phi = \frac{\mu_0 I}{2\pi} \phi \]

Flux through one turn of the toroidal coil:

\[ \Phi_1 = \frac{\mu_0 I}{2\pi} \int_{a}^{b} \frac{1}{2} (hds) = \frac{\mu_0 I h}{2\pi} \ln\left(\frac{b}{a}\right) \]

Flux through \( N \) turns:

\[ \Phi = \frac{\mu_0 N h}{2\pi} \ln\left(\frac{b}{a}\right) I_0 \cos(wt) \]

\[ \text{Emp: } E = -\frac{d\Phi}{dt} = \frac{\mu_0 N h}{2\pi} \ln\left(\frac{b}{a}\right) I_0 w \sin(wt) \]

Insert numbers:

\[ E = \frac{4\pi \times 10^{-7} N/A \times 10^3 \times (10^{-2} \text{ m})}{2\pi} \ln(2) \times 0.5 A \times (2\pi \times 60 \frac{1}{2} \text{ Hz}) \sin(wt) \]

\[ = 2.61 \times 10^{-4} \sin(wt) \frac{Nm}{A^2} = 2.61 \times 10^{-4} \sin(wt) \frac{Nm}{C} = 2.61 \times 10^{-4} \sin(wt) \]

\[ I_R(t) = \frac{E}{R} = \frac{2.61 \times 10^{-6}}{500} \sin(wt) \frac{V}{\Omega} = 5.22 \times 10^{-7} \sin(wt) \]

(b) \( I_b = -L \frac{dI_R}{dt} \)

\[ L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \]

\[ \text{Calculated in class} \]

\[ L = \frac{4\pi \times 10^{-7} N/A \times (10^3)^2 \times (10^{-2})}{2\pi} \ln(2) = 1.39 \times 10^{-3} \frac{Nm}{A^2} = 1.39 \times 10^{-3} \text{ henries} \]

\[ I_b = -1.39 \times 10^{-3} \frac{Nm}{A^2} \times 5.22 \times 10^{-7} A (2\pi \times 60 \frac{1}{2} \text{ Hz}) \cos(wt) = \]

\[ = -2.74 \times 10^{-7} \cos(wt) \]

\[ \text{Ratio of amplitudes: } \frac{E_b}{E} = \frac{2.74 \times 10^{-7}}{2.61 \times 10^{-4}} = 1.05 \times 10^{-3} = \frac{\mu_0 N^2 h \omega}{2\pi R} \ln\left(\frac{b}{a}\right) \]