Homework #4
phy 5246
due: Wednesday, September 24 (in class)

Goldstein Poole, and Safko, Classical Mechanics (Third Edition)
Chap. 2; Problem 14 Pg. 67.
Chap. 2; Problem 18 Pg. 67.
Chap. 2; Problem 23 Pg. 68.

P4: A particle of mass m is constrained to move under the influence of gravity on the surface of a paraboloid of revolution whose axis is vertical. Taking the z direction to be up, this surface is described by the equation

$$z = \alpha(x^2 + y^2) \quad \alpha > 0.$$

(a) Write the Lagrangian for this system using as generalized coordinates r and θ, the polar coordinates in the $x - y$ plane.
(b) Reduce the problem to an effective one-dimensional problem for the radial coordinate r.
(c) Determine the condition on the particles initial velocity required to produce circular motion.
(d) Find the period of small oscillations about this circular motion.

P5: A particle of mass m moves in a central force field given by the potential

$$V(r) = -k\frac{e^{-ar}}{r}; \quad k > 0,\ a > 0.$$

(a) Determine the effective one-dimensional problem for the radial coordinate. Sketch the effective potential for various values of the angular momentum ℓ and discuss qualitatively the possible orbits of the system.
(b) Determine the condition for a stable circular orbit. This condition should be expressed in terms of the radius of the orbit and the constant a.
(c) Find the period of small radial oscillations about these circular orbits.