Physics 5524

Statistical Mechanics

Problem Set 1

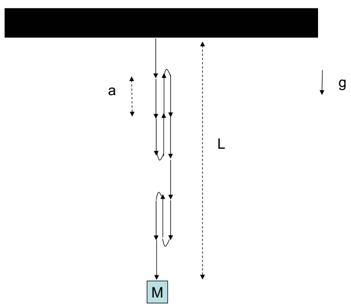
Due: Monday, Jan. 26

1.1

A simple statistical mechanical model of a rubber band consists of N connected rigid segments, each of length a, which can either point up or down. The total length L of the rubber band is then

$$L = a\Big((\text{\# of segments pointing down}) - (\text{\# of segments pointing up})\Big).$$

One end of this rubber band is fixed, and a mass M is attached to the other end so that the rubber band hangs vertically (see figure). We will ignore the mass of the rubber band, so the total energy of this system is just the gravitational potential energy of the mass, $E = -MgL$.



(a) For a given macrostate, characterized by fixed N and E, determine the number of microstates $\Omega(N, E)$ for this simple model.

(b) Obtain an expression for the entropy $S(N, E)$ for this system valid in the limit $N \gg 1$ using Stirling's approximation.

(c) Use the result of (b) to obtain an expression for the energy $E(N, T)$ where T is the temperature.

(d) Since $E = -MgL$, the result of (c) also gives you an expression for the length of the rubber band, L, as a function of T. Sketch L vs. T.

1.2 Pathria, Problem 1.2, pg. 26

Assuming that the entropy S and the number of microstates Ω are related through an arbitrary functional form

$$S = f(\Omega),$$

show that the additive character of S and the multiplicative character of Ω *necessarily* require that the function $f(\Omega)$ be of the form $S = k_B \ln \Omega$.

1.3 Pathria, Problem 1.4, pg. 27

In a classical gas of hard spheres (of diameter σ), the spatial distribution of the particles is no longer uncorrelated. Roughly speaking, the presence of n particles in the system leaves only a volume $(V - nv_0)$ available for the $(n+1)^{\text{st}}$ particle; clearly, v_0 would be proportional to σ^3. Assuming that $Nv_0 \ll V$, determine the dependence of $\Omega(N, V, E)$ on V (i.e., find an expression analogous to the result that $\Omega(N, V, E) \propto V^N$ for an ideal gas) and show that, as a result of this, V in the ideal gas law ($PV = Nk_BT$) gets replaced by $(V - b)$, where b is four times the actual space occupied by the particles.