Physics 5524
Statistical Mechanics
Problem Set 5
Due: Friday, February 27 (in A. Yuresi's or my Keen mailbox by 5pm)

5.1:
a) Determine the entropy of a Bose gas of N particles as a function of T and V in the limit of $\mu \to -\infty$, i.e. in the limit where it behaves as a classical gas
b) Determine the entropy of a Fermi gas of N particles as a function of T and V in the limit where it behaves as a classical gas, and compare with a)
c) Consider a container of volume $2V$ divided exactly into two halves (each of volume V) by an impenetrable partition. Each half contains $1mol$ of ^4He atoms behaving as a classical gas at temperature T. Calculate the change of the entropy when the partition is removed and the two halves are allowed to mix. Note that the gas constant and Avogadro's number are related by the Boltzmann constant as $R = N_A k_B$.
d) Same as c), but the left half contains ^3He and the right contains ^4He.

5.2: Consider a two body Schrodinger equation for two identical fermions:

$$\mathcal{H} = \frac{\mathbf{p}_1^2}{2m} + \frac{\mathbf{p}_2^2}{2m} + \frac{1}{2}\kappa \left(\mathbf{r}_1 - \mathbf{r}_2\right)^2 , \tag{1}$$

where m is the particle mass, and $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position coordinates of the two fermions carrying spin 1/2.
a) Find the eigenvalues of $\mathcal{H}$.
b) Find the eigenstates of $\mathcal{H}$ consistent with the fermionic exchange statistics. Remember to specify the spin, as well as the center-of-mass, part of the wavefunction.
c) For fixed value of the center-of-mass momentum, and for fixed values of the three harmonic oscillator quantum numbers n_x, n_y and n_z, determine the degeneracy of each state.
d) Calculate the free energy for a gas of such "molecules" using this model, in the limit when $\mu \to -\infty$.
e) Plot the constant volume heat capacity of one mol of such a gas as a function of $k_B T/(\hbar\omega)$, again assuming that $\mu \to -\infty$ throughout the temperature range. Here ω is the natural frequency of the harmonic oscillator.