A unified picture of protein hydration: prediction of hydrodynamic properties from known structures

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Received 13 February 2001; received in revised form 17 July 2001; accepted 18 July 2001

Abstract

Hydration is essential for the structural and functional integrity of globular proteins. How much hydration water is required for that integrity? A number of techniques such as X-ray diffraction, nuclear magnetic resonance (NMR) spectroscopy, calorimetry, infrared spectroscopy, and molecular dynamics (MD) simulations indicate that the hydration level is 0.3–0.5 g of water per gram of protein for medium sized proteins. Hydrodynamic properties, when accounted for by modeling proteins as ellipsoids, appear to give a wide range of hydration levels. In this paper we describe an alternative numerical technique for hydrodynamic calculations that takes account of the detailed protein structures. This is made possible by relating hydrodynamic properties (translational and rotational diffusion constants and intrinsic viscosity) to electrostatic properties (capacitance and polarizability). We show that the use of detailed protein structures in predicting hydrodynamic properties leads to hydration levels in agreement with other techniques. A unified picture of protein hydration emerges. There are preferred hydration sites around a protein surface. These sites are occupied nearly all the time, but by different water molecules at different times. Thus, though a given water molecule may have a very short residence time (≈ 100–500 ps from NMR spectroscopy and MD simulations) in a particular site, the site appears fully occupied in experiments in which time-averaged properties are measured. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Hydration; Hydration sites; Residence time; Diffusion constant; Intrinsic viscosity; Rotational correlation time

1. Introduction

Hydrodynamic measurements are one of the oldest techniques for characterizing the size and shape of protein molecules. Historically molecular dimensions were estimated from the expressions of hydrodynamic properties for ellipsoids [1–4]. Owing to the crudeness of ellipsoids as models of globular proteins, hydrodynamic hydration levels have varied widely, from 0.1 to over 1.0 g of water per gram of protein. Now several numerical techniques based on modeling the de-
etailed structures of proteins have been developed to calculate hydrodynamic properties [5–8]. It is time to re-examine the issue of hydrodynamic hydration.

A number of techniques such as X-ray diffraction, nuclear magnetic resonance (NMR) spectroscopy, calorimetry, infrared spectroscopy, and molecular dynamics simulations indicate that the hydration level is 0.3–0.5 g of water per gram of protein for medium sized proteins. For example, a recent high-resolution X-ray diffraction of ribonuclease A identified 258 water molecules in the first hydration shell [9], corresponding to a hydration level of 0.34 g/g. The non-freezable water around a protein can be easily detected by NMR. The hydration levels from such a measurement are 0.34, 0.42 and 0.34 g/g for lysozyme, myoglobin, and chymotrypsinogen A, respectively [10]. Calorimetric measurements of the heat capacity of lysozyme powder at different hydration levels indicate that the amount of water required to hydrate lysozyme is 0.38 g/g (i.e. any additional water beyond this hydration level simply becomes part of the bulk water) [11]. Similarly, infrared spectroscopy indicates that hydration of lysozyme stops at approximately 0.33 g/g [12–14]. Molecular dynamics simulations of myoglobin solvated by different numbers of water molecules show that root-mean-square deviations from the X-ray structure become stable after approximately 350 water molecules (i.e. 0.37 g/g) are included [15].

In earlier work we presented hydrodynamic calculation results to indicate that when the detailed structures of proteins are explicitly modeled, hydration levels in the narrow range of 0.3–0.5 g/g are predicted [7]. Here we present additional data on rotational correlation times to support this conclusion.

2. Relations of hydrodynamic and electrostatic properties

The connection between hydrodynamic and electrostatic properties was recognized from the fact that the Oseen Tensor, i.e. the Green function for the Navier–Stokes equation, when orientationally averaged is proportion to the Green function for the Laplace equation [16]. One has

\[ \tilde{T} = \frac{1}{8\pi \eta_0} \mathbf{R} \left( \tilde{I} + \hat{\mathbf{R}} \hat{\mathbf{R}} \right) \]  

(1a)

\[ \langle \tilde{T} \rangle_0 = \frac{1}{6\pi \eta_0} \mathbf{R} \tilde{I} = \frac{2}{3\eta_0} \gamma(\mathbf{R}) \tilde{I} \]  

(1b)

where \( \mathbf{R} = \mathbf{R} \mathbf{R} = \mathbf{r}' - \mathbf{r} \), \( \tilde{I} \) is the identity tensor, \( \eta_0 \) is the viscosity of the fluid, \( \langle \ldots \rangle_0 \) denotes averaging over a uniform distribution of orientations, and

\[ \gamma(\mathbf{R}) = \frac{1}{4\pi R} \frac{1}{4\pi |r' - r|} \]  

(2)

By approximating the Oseen tensor by its orientational average, Zhou [16] showed that the translational friction is related to the electric capacitance by

\[ \xi = 6\pi \eta_0 C \]  

(3)

In particular this result is exact for triaxial ellipsoids. By the same approximation, the intrinsic viscosity is proportional to the electric polarizability. To make the relation exact for a sphere, Zhou proposed adding a term proportional to the volume of particle under consideration. Thus we have

\[ [\eta] = \frac{3}{4} \alpha + \frac{1}{4} V_p \]  

(4)

The capacitance is the total charge on the surface of the particle if it were a conductor with the surface electric potential maintained at unity. That is,

\[ C = \int_{S_p} d\mathbf{s} \sigma_c(\mathbf{r}) \]  

(5a)

\[ 4\pi \int_{S_p} d\mathbf{s} \gamma(\mathbf{r} - \mathbf{r}') \sigma_c(\mathbf{r}) = 1 \]  

(5b)

where \( S_p \) is the surface of the particle and \( \sigma_c(\mathbf{r}) \) is...
the surface charge density. The electric polarizability tensor $\tilde{\alpha}$ has the components

$$\alpha_{ij} = 4\pi \int_{S_p} d\mathbf{s} \sigma_j(\mathbf{r})$$

where $x_i$ are the Cartesian components of $\mathbf{r}$, and $\sigma_j(\mathbf{r})$ are the charge densities satisfying

$$4\pi \int_{S_p} d\mathbf{s} \gamma(\mathbf{r} - \mathbf{r'} )\sigma_j(\mathbf{r}) = x'_j$$

The origin for the coordinate system should be chosen such that the total charges $\int_{S_p} d\mathbf{s} \sigma_j(\mathbf{r})$ are zero. In practice, one can choose any origin and calculate $\sigma_j(\mathbf{r})$ according to Eq. (7). If the total charges are $\int_{S_p} d\mathbf{s} \sigma_j(\mathbf{r}) = Q_j$, then the charge densities to be used for calculating the polarizability tensor should be $\sigma_j(\mathbf{r}) - (Q_j/C)\sigma_j(\mathbf{r})$. The orientationally averaged polarizability is

$$\alpha = \frac{1}{3} \sum_{i=1}^{3} \alpha_{ii} = \text{Tr}(\tilde{\alpha})/3$$

By the orientational average of the Oseen tensor, Zhou [16] also showed a relationship between the rotational friction tensor $\tilde{\zeta}$ and the polarizability tensor. This is given by

$$\tilde{\zeta} = 3\eta_0(3\alpha \hat{I} - \tilde{\alpha})/2 = 3\eta_0\tilde{\beta}$$

The accuracy of this relation was not as extensively tested as that of Eqs. (3) and (4). In the case of an axially symmetric particle, Zhou did conjecture a relation between the axial rotational friction and the transverse electric polarizability. This is

$$\xi_\parallel = 2\eta_0\alpha_\perp$$

which is proven in Appendix A. For axisymmetric particles, Eq. (9) predicts $\xi_\parallel = 3\eta_0\alpha_\perp$ and $\xi_\perp = 3\eta_0(\alpha_\parallel + \alpha_\perp)/2$. The former differs from the exact result in Eq. (10) by a coefficient of 3 instead of 2. For ellipsoids, cylinders, and dumbbells, the proportionality constant between $\xi_\perp$ and $\eta_0(\alpha_\parallel + \alpha_\perp)/2$ was found to range from 2 for shapes close to a sphere to 4 for needles and disks.

The translational diffusion constant is related to the translational friction via the Stokes–Einstein equation

$$D = k_B T/\xi$$

where $k_B T$ is the product of the Boltzmann constant and the absolute temperature. A similar equation relates the rotational diffusion tensor $D^r$ and the rotational friction tensor. Globular proteins have traditionally been modeled as isotropic diffusers. More recently in interpreting NMR relaxation data on backbone dynamics proteins have been modeled as axisymmetric diffusers, with $D_\parallel/D_\perp = \zeta_\parallel/\zeta_\perp$ ranging from 0.8 to 1.4 [17–24]. The rotational correlation function of a unit vector attached to the particle is [25] is

$$C(t) = (1.5\cos^2 \theta - 0.5)^2 e^{-t/t_1} + 3\sin^2 \theta \cos^2 \theta e^{-t/t_2} + 0.75 \sin^4 \theta e^{-t/t_3}$$

where $\theta$ is the angle between the unit vector and the symmetric axis, $1/t_1 = 6D_\perp = 6k_B T/\xi_\perp$, $1/t_2 = 5D_\perp + D_\parallel = k_B T(\zeta_\parallel/\zeta_\perp + 1/\zeta_\perp)$, and $1/t_3 = 2D_\perp + 4D_\parallel = k_B T(2/\zeta_\perp + 4/\zeta_\parallel)$. For $0.8 < D_\parallel/D_\perp = \zeta_\parallel/\zeta_\perp < 1.4$, Eq. (12) is well approximated by a single exponential. In Fig. 1 we compare Eq. (12) at $D_\parallel/D_\perp = 1.2$ against the single exponential function $\exp(-t/t_c)$ with an effective correlation function

$$t_c = (2\zeta_\perp + \zeta_\parallel)/18k_BT = \text{Tr}(\tilde{\zeta})/18k_BT$$

The agreement is very reasonable, with the area under the curve overestimated by 6.7% at $\theta = 0$ and underestimated by 2.5% at $\theta = 90^\circ$ by the single-exponential function. Comparable amounts of errors are incurred at $D_\parallel/D_\perp = 0.8$. Note that the rotational correlation time in Eq. (13) is not defined in the usual way as $t_c = 1/2(2D_\perp + D_\parallel) = 1/2k_B T(2/\zeta_\perp + 1/\zeta_\parallel)$. The
result in Eq. (13) is larger by approximately 1% at $D_\parallel/D_\perp = 1.2$ and 0.8. The reason for using Eq. (13) is that we want to relate $t_c$ to the orientationally averaged electric polarizability [see Eqs. (9) and (8)]. Considering the small deviations of $D_\parallel/D_\perp$ from 1 observed on globular proteins, we propose the following relation

$$t_c = 2.3\eta_0\alpha/6k_BT$$

(14)

The numerical factor, 2.3, is between the value 2 for a sphere and 3 according to Eq. (9).

Eq. (14) is the main theoretical result of the present paper. Along with the previously proposed Eqs. (3) and (4), we can now obtain the translation diffusion constant, the intrinsic viscosity, and the rotational correlation time simultaneously from a single electrostatic calculation for the capacitance and the polarizability. The details of this calculation are described previously [7]. In the next section we present results on the hydrodynamic properties of proteins predicted on their actual structures.

3. Results

Zhou [7] has used Eqs. (3) and (4) to calculate the translational diffusion constant and the intrinsic viscosity for ribonuclease A, lysozyme, myoglobin, and chymotrypsinogen A. By including a 0.9-Å-thick uniform hydration shell, the experimental results for both hydrodynamic properties are reproduced for all four proteins. Table 1 gives the comparison (data taken from Zhou [7]). The 0.9-Å-thick uniform hydration shell gives hydration levels of 0.40, 0.39, 0.40, 0.38 g of water per gram of protein for the four proteins.

The same calculation results contain other data that can be directly compared with experiments. In Table 2 we compare the volume $V_p$ enclosed by the shell 0.9 Å away from the van der Waals surface of the protein with the hydrated volume as determined by small-angle X-ray scattering. Kumosinski and Pessen [26] compiled data for 18 proteins, among which are ribonuclease A, lysozyme, and chymotrypsinogen A. The values of $V_p$ used for predicting $D$ and $[\eta]$ agree with the experimental data on the hydrated volume to within 5, 10 and 4%, respectively, for the three proteins.

With Eq. (14) proposed in the present paper, we can now predict the rotational correlation time. The results, by using the previously calculated polarizability [7] (listed in Table 1) in Eq. (14), are shown in Table 3. The predicted values of $t_c$ are consistent with experimental results for all four proteins.

4. Discussion

With Eqs. (3), (4) and (14), we now have com-
Table 1
Electrostatic and hydrodynamic properties (at 20°C) of four proteins

<table>
<thead>
<tr>
<th>Proteins</th>
<th>$V_p$ (Å³)</th>
<th>C (Å)</th>
<th>$\alpha$ (Å³)</th>
<th>$\text{D} \times 10^{-7}$ cm²/s</th>
<th>$[\eta]$ (cm³/g)</th>
<th>Hydration (g/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculation</td>
<td>Experiment</td>
<td>Calculation</td>
<td>Experiment</td>
<td>Calculation</td>
<td>Experiment</td>
</tr>
<tr>
<td>Ribonuclease A</td>
<td>20.941</td>
<td>19.24</td>
<td>91.691</td>
<td>11.06</td>
<td>11.2 ± 0.2</td>
<td>3.26</td>
</tr>
<tr>
<td>Lysozyme</td>
<td>21.668</td>
<td>19.03</td>
<td>87.640</td>
<td>11.18</td>
<td>11.2 ± 0.2</td>
<td>2.99</td>
</tr>
<tr>
<td>Myoglobin</td>
<td>27.631</td>
<td>20.52</td>
<td>110.288</td>
<td>10.37</td>
<td>10.3</td>
<td>3.14</td>
</tr>
<tr>
<td>Chymotrypsinogen A</td>
<td>39.190</td>
<td>23.06</td>
<td>153.540</td>
<td>9.23</td>
<td>9.2 ± 0.2</td>
<td>2.93</td>
</tr>
</tbody>
</table>
completed an approach to predict the three hydrodynamic properties, translational diffusion constant, intrinsic viscosity, and rotational correlation time, of globular proteins, without any adjustable parameters. The new data in Tables 2 and 3 demonstrate the accuracy of our approach.

A very important physical property that comes out of the hydrodynamic calculations is the hydration level. This falls within the narrow range of 0.3–0.5 g of water per gram of protein for all the proteins.

<table>
<thead>
<tr>
<th>Proteins</th>
<th>Model</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ribonuclease A</td>
<td>20941</td>
<td>22000</td>
</tr>
<tr>
<td>Lysozyme</td>
<td>21668</td>
<td>24200</td>
</tr>
<tr>
<td>Chymotrypsinogen A</td>
<td>39190</td>
<td>37790</td>
</tr>
</tbody>
</table>

Table 3

Comparison of predicted and experimental rotational correlation times (ns) at 20°C

<table>
<thead>
<tr>
<th>Proteins</th>
<th>Prediction</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ribonuclease A</td>
<td>8.8</td>
<td>8.3(^a)</td>
</tr>
<tr>
<td>Lysozyme</td>
<td>8.4</td>
<td>7.2(^b), 7.5(^c), 10(^d)</td>
</tr>
<tr>
<td>Myoglobin</td>
<td>10.5</td>
<td>8.9(^e), 9.4(^f), 12.7(^g)</td>
</tr>
<tr>
<td>Chymotrypsinogen A</td>
<td>14.7</td>
<td>15(^h)</td>
</tr>
</tbody>
</table>

\(^a\) Converted from the result of 7.3 ns at 25°C in Krause and O’Konski [27].
\(^b\) Cross and Fleming [28].
\(^c\) Dill and Allerhand [29].
\(^d\) Bubin et al. [30].
\(^e\) Converted from the result of 10.3 ns at 15°C in Anderson et al. [31].
\(^f\) Tao [32].
\(^g\) Calculated from the steady-state fluorescence data of Stryer [33] by using a fluorescence lifetime of 16.4 ns for ANS complexed to apomyoglobin, as measured in Anderson et al. [31] and Tao [32].
\(^h\) Stryer [34].
four proteins considered in this paper. By using the
detailed protein structures, we have brought
the hydration levels implicated by hydrodynamic
properties into conformity with those detected by
other experimental techniques.

It now appears reasonable to propose a unified
picture of protein hydration that reconciles obser-
vations from different experimental techniques.
As Fig. 2 illustrates, around the protein surface
are preferred hydration sites. As seen by X-ray
diffraction and in molecular dynamics simula-
tions, these sites are around charged side chains
and polar side chains and the backbone, where
water molecules can make hydrogen bonds with
the protein. Though a given water molecule may
have a residence time of only 100–500 ps in a
particular site as determined by NMR spectro-
scopy [35], the site is almost fully occupied albeit
by different water molecules at different times.
Thus, in experiments e.g. calorimetry and hydro-
dynamic measurements which detect time-aver-
aged properties the hydration water appears as an
integral part of the protein. It is important to
note that hydration water is marked by the per-
manence of the occupation sites, not by the per-
manence of the occupants.

Appendix A

In this appendix we prove Eq. (10), the exact
result relating the axial rotational friction and the
transverse electric polarizability of an axially sym-
metric particle.

For a fluid perturbed by the axial rotation of an
axisymmetric particle, the only non-zero com-
ponent of the fluid velocity is in the azimuthal
direction. This we will denote as \( v \). In cylindrical
coordinates the Navier–Stokes equation for \( v \) is
[36]

\[
\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial v}{\partial \rho} \right) + \frac{\partial^2 v}{\partial z^2} - \frac{v}{\rho^2} = 0
\]  
(A1)

The boundary conditions are

\[ v \bigg|_{s_p} = \Omega \rho \]  
(A2a)

\[ v \left( \sqrt{\rho^2 + z^2} \to \infty \right) = 0 \]  
(A2b)

where \( S_p \) is the surface of the particle and \( \Omega \) is
the rotational velocity. By the transformation
[37,38]

\[ V = \nu \cos \phi \]  
(A3)

Eq. (A1) can be converted to a Laplace equa-
tion

\[ \nabla^2 V = 0 \]  
(A4)

The boundary conditions are now

\[ V \bigg|_{S_p} = \Omega \rho \cos \phi \]  
(A5a)

\[ V \left( \sqrt{\rho^2 + z^2} \to \infty \right) = 0 \]  
(A5b)

The surface stress force in the present case is
[36]

\[ f = n \cdot \vec{\Pi} = -p n + \eta_0 n \rho \nabla (v/\rho) \]  
(A6)

The axial component of the resulting torque is

\[ L = \int_{S_p} ds \vec{r} \times f \cdot \hat{z} = \eta_0 \int_{S_p} ds n \cdot \rho^2 \nabla (v/\rho) \]  
(A7)

The axial rotational friction is

\[ \zeta \parallel = -L/\Omega = -\eta_0 \int_{S_p} ds n \cdot \rho^2 \nabla (v/\rho \Omega) \]  
(A8)

The electrostatic potential \( \varphi \) induced by a uni-
form electric field, \( E \hat{k} \), in the transverse direc-
tion satisfies the Laplace equation

\[ \nabla^2 \varphi = 0 \]  
(A9)

with the boundary conditions

\[ \varphi \bigg|_{S_p} = E \hat{x} = E \rho \cos \phi \]  
(A10a)

\[ \varphi \left( \sqrt{\rho^2 + z^2} \to \infty \right) = 0 \]  
(A10b)
Comparing with Eqs. (4), (5a) and (5b) gives

$$
\varphi = (E/\Omega) V = (E/\Omega) v \cos \phi
$$

(A11)

The transverse electric polarizability is [16]

$$
\alpha_\perp = \int_{S_p} d\mathbf{s} n \cdot (-\nabla \varphi + E \hat{x}) / E
$$

(A12)

Using Eq. (A11) and noting \( \mathbf{n} \cdot \hat{x} = \mathbf{n} \cdot \hat{\rho} \cos \phi \), we have

$$
\alpha_\perp = \int_{S_p} d\mathbf{s} \rho \cos^2 \phi \mathbf{n} \cdot (-\nabla v / \Omega + \hat{\rho})
$$

$$
= -\int_{S_p} d\mathbf{s} \rho \cos^2 \phi \mathbf{n} \cdot \nabla (v / \rho \Omega)
$$

$$
= -\frac{1}{2} \int_{S_p} d\mathbf{s} \rho \mathbf{n} \cdot \nabla (v / \rho \Omega) = \frac{\xi_3}{2 \eta_0}
$$

which is just Eq. (10).

References


[26] T.F. Kumosinski, H. Pessen, Estimation of sedimentation coefficients of globular proteins: An application of


