Problem # 19.1:
A particle of charge $q$ moves in a circle of radius $a$ at constant angular velocity $\omega$. Assume that the circle lies in the $xy$ plane, centered at the origin, and at time $t = 0$ the charge is at $(a,0)$, on the positive $x$ axis. Find the Liénard-Wiechert potentials for points on the $z$ axis.

Problem # 19.2:
Suppose a point charge $q$ is constrained to move along the $x$ axis. Show that the fields at points on the axis to the right of the charge are given by

$$E = \frac{q}{4\pi\varepsilon_0} \frac{1}{R} \left( \frac{c+v}{c-v} \right) \hat{x}, \quad B = 0.$$

Do not assume $v$ is constant. What are the fields on the axis to the left of the charge?

Problem # 19.3:
For a point charge moving at constant velocity, calculate the flux integral $\oint \mathbf{E} \cdot d\mathbf{a}$ over the surface of a sphere centered at the present location of the charge. Use that the electric field is given by

$$E(r,t) = \frac{q}{4\pi\varepsilon_0} \frac{1 - v^2/c^2}{|1 - v^2 \sin^2(\theta)/c^2|^{3/2}} \frac{\hat{R}}{R^2},$$

where $\mathbf{R} = \mathbf{r} - vt$ is the vector from the present location to $\mathbf{r}$, and $\theta$ is the angle between $\mathbf{R}$ and $v$.

Problem # 19.4:
Find the radiation resistance of the wire joining the two ends of the electric dipole. This is the resistance that would give the same average power loss – to heat – as the oscillating dipole in fact puts out in the form of radiation.
(a) Show that \( R = 790(d/\lambda)^2 \Omega \), where \( \lambda \) is the wavelength of the radiation. (\( \Omega = \text{ohm} \)).

(b) For the wires in an ordinary ratio (say, \( d = 5 \, \text{cm} \)) and \( \lambda = 10^3 \, \text{m} \), should you worry about the radiative contribution to the total resistance?

Problem # 19.5:
Check that the retarded potentials of an oscillating dipole (along the \( z \)-axis)

\[
V(r, \theta, t) = \frac{p_0 \cos \theta}{4\pi \varepsilon_0 r} \left\{ -\frac{\omega}{c} \sin[\omega(t - r/c)] + \frac{1}{r} \cos[\omega(t - r/c)] \right\}
\]

\[
A(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin[\omega(t - r/c)] \hat{z}
\]

satisfy the Lorentz gauge condition. Do not use approximation 3, \( r \gg c/\omega \).